Adaptive Antenna Systems

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Abstract—A system consisting of an antenna array and an adaptive processor can perform filtering in both the space and the frequency domains, thus reducing the sensitivity of the signal-receiving system to interfering directional noise sources.

Variable weights of a signal processor can be automatically adjusted by a simple adaptive technique based on the least-mean-squares (LMS) algorithm. During the adaptive process an injected pilot signal simulates a received signal from a desired "look" direction. This allows the array to be "trained" so that its directivity pattern has a main lobe in the previously specified look direction. At the same time, the array processing system can reject any incident noises, whose directions of propagation are different from the desired look direction, by forming appropriate nulls in the antenna directivity pattern. The array adapts itself to form a main lobe, with its direction and bandwidth determined by the pilot signal, and to reject signals or noises occurring outside the main lobe as well as possible in the minimum mean-square error sense.

Several examples illustrate the convergence of the LMS adaptation procedure toward the corresponding Wiener optimum solutions. Rates of adaptation and misadjustments of the solutions are predicted theoretically and checked experimentally. Substantial reductions in noise reception are demonstrated in computer-simulated experiments. The techniques described are applicable to signal-receiving arrays for use over a wide range of frequencies.

Introduction

THE SENSITIVITY of a signal-receiving array to interfering noise sources can be reduced by suitable processing of the outputs of the individual array elements. The combination of array and processing acts as a filter in both space and frequency. This paper describes a method of applying the techniques of adaptive filtering[1] to the design of a receiving antenna system which can extract directional signals from the medium with minimum distortion due to noise. This system will be called an *adaptive array*. The adaptation process is based on minimization of mean-square error by the LMS algorithm.[2]–[4] The system operates with knowledge of the direction of arrival and spectrum of the signal, but with no knowledge of the noise field. The adaptive array promises to be useful whenever there is interference that possesses some degree of spatial correlation; such conditions manifest themselves over the entire spectrum, from seismic to radar frequencies.

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The term "adaptive antenna" has previously been used by Van Atta[5] and others[6] to describe a self-phasing antenna system which reradiates a signal in the direction from which it was received. This type of system is called adaptive because it performs without any prior knowledge of the direction in which it is to transmit. For clarity, such a system might be called an adaptive *transmitting* array; whereas the system described in this paper might be called an adaptive *receiving* array.

The term "adaptive filter" has been used by Jakowatz, Shuey, and White[7] to describe a system which extracts an unknown signal from noise, where the signal waveform recurs frequently at random intervals. Davisson[8] has described a method for estimating an unknown signal waveform in the presence of white noise of unknown variance. Glaser[9] has described an adaptive system suitable for the detection of a pulse signal of fixed but unknown waveform.

Previous work on array signal processing directly related to the present paper was done by Bryn, Mermoz, and Shor. The problem of detecting Gaussian signals in additive Gaussian noise fields was studied by Bryn,[10] who showed that, assuming K antenna elements in the array, the Bayes optimum detector could be implemented by either K^2 linear filters followed by "conventional" beam-forming for each possible signal direction, or by K linear filters for each possible signal direction. In either case, the measurement and inversion of a $2K$ by $2K$ correlation matrix was required at a large number of frequencies in the band of the signal. Mermoz[11] proposed a similar scheme for narrowband known signals, using the signal-to-noise ratio as a performance criterion. Shor[12] also used a signal-to-noise-ratio criterion to detect narrowband pulse signals. He proposed that the sensors be switched off when the signal was known to be absent, and a pilot signal injected as if it were a noise-free signal impinging on the array from a specified direction. The need for specific matrix inversion was circumvented by calculating the gradient of the ratio between the output power due to pilot signal and the output power due to noise, and using the method of steepest descent. At the same time, the number of correlation measurements required was reduced, by Shor's procedure, to $4K$ at each step in the adjustment of the processor. Both Mermoz and Shor have suggested the possibility of real-time adaptation.

This paper presents a potentially simpler scheme for obtaining the desired array processing improvement in real time. The performance criterion used is minimum mean-square error. The statistics of the signal are assumed to be known, but no prior knowledge or direct measurements of the noise field are required in this scheme. The adaptive array processor considered in the study may be automatically adjusted (adapted) according to a simple iterative algorithm, and the procedure does not directly involve the computation of any correlation coefficients or the inversion of matrices. The input signals are used only once, as they occur, in the adaptation process. There is no need to store past input data; but there is a need to store the processor adjustment values, i.e., the processor weighting coefficients ("weights"). Methods of adaptation are presented here, which may be implemented with either analog or digital adaptive circuits, or by digital-computer realization.

Directional and Spatial Filtering

An example of a linear-array receiving antenna is shown in Fig. 1(a) and (b). The antenna of Fig. 1(a) consists of seven isotropic elements spaced $\lambda_0/2$ apart along a straight line, where λ_0 is the wavelength of the center frequency f_0 of the array. The received signals are summed to produce an array output signal. The directivity pattern, i.e., the relative sensitivity of response to signals from various directions, is plotted in this figure in a plane over an angular range of $-\pi/2<\theta<\pi/2$ for frequency f_0. This pattern is symmetric about the vertical line $\theta=0$. The main lobe is centered at $\theta=0$. The largest-amplitude side lobe, at $\theta=24°$, has a maximum sensitivity which is 12.5 dB below the maximum main-lobe sensitivity. This pattern would be different if it were plotted at frequencies other than f_0.

The same array configuration is shown in Fig. 1(b); however, in this case the output of each element is delayed in time before being summed. The resulting directivity pattern now has its main lobe at an angle of ψ radians, where

$$\psi = \sin^{-1}\left(\frac{\lambda_0 \delta f_0}{d}\right) = \sin^{-1}\left(\frac{c\delta}{d}\right) \quad (1)$$

in which

f_0 = frequency of received signal
λ_0 = wavelength at frequency f_0
δ = time-delay difference between neighboring-element outputs
d = spacing between antenna elements
c = signal propagation velocity $=\lambda_0 f_0$.

The sensitivity is maximum at angle ψ because signals received from a plane wave source incident at this angle, and delayed as in Fig. 1(b), are in phase with one another and produce the maximum output signal. For the example illustrated in the figure, $d=\lambda_0/2$, $\delta=(0.12941/f_0)$, and therefore $\psi=\sin^{-1}(2\delta f_0)=15°$.

There are many possible configurations for phased arrays. Fig. 2(a) shows one such configuration where each of the antenna-element outputs is weighted by two weights in parallel, one being preceded by a time delay of a quarter of a cycle at frequency f_0 (i.e., a 90° phase shift), denoted by $1/(4f_0)$. The output signal is the sum of all the weighted signals, and since all weights are set to unit values, the directivity pattern at frequency f_0 is by symmetry the same as that of Fig. 1(a). For purposes of illustration, an interfering directional sinusoidal "noise" of frequency f_0 incident on the array is shown in Fig. 2(a), indicated by the dotted arrow. The angle of incidence (45.5°) of this noise is such that it would be received on one of the side lobes of the directivity pattern with a sensitivity only 17 dB less than that of the main lobe at $\theta=0°$.

If the weights are now set as indicated in Fig. 2(b), the directivity pattern at frequency f_0 becomes as shown in that figure. In this case, the main lobe is almost unchanged from that shown in Figs. 1(a) and 2(a), while the particular side lobe that previously intercepted the sinusoidal noise in Fig. 2(a) has been shifted so that a null is now placed in the direction of that noise. The sensitivity in the noise direction is 77 dB below the main lobe sensitivity, improving the noise rejection by 60 dB.

A simple example follows which illustrates the existence and calculation of a set of weights which will cause a signal from a desired direction to be accepted while a "noise" from a different direction is rejected. Such an example is illustrated in Fig. 3. Let the signal arriving from the desired direction $\theta=0°$ be called the "pilot" signal $p(t)=P\sin\omega_0 t$, where $\omega_0 \triangleq 2\pi f_0$, and let the other signal, the noise, be chosen as $n(t)=N\sin\omega_0 t$, incident to the receiving array at an angle $\theta=\pi/6$ radians. Both the pilot signal and the noise signal are assumed for this example to be at exactly the same frequency f_0. At a point in space midway between the antenna array elements, the signal and the noise are assumed to be in phase. In the example shown, there are two identical omnidirectional array elements, spaced $\lambda_0/2$ apart. The signals received by each element are fed to two variable weights, one weight being preceded by a quarter-wave time delay of $1/(4f_0)$. The four weighted signals are then summed to form the array output.

The problem of obtaining a set of weights to accept $p(t)$ and reject $n(t)$ can now be studied. Note that with any set of nonzero weights, the output is of the form $A\sin(\omega_0 t+\phi)$, and a number of solutions exist which will make the output be $p(t)$. However, the output of the array must be independent of the amplitude and phase of the noise signal if the array is to be regarded as rejecting the noise. Satisfaction of this constraint leads to a unique set of weights determined as follows.

The array output due to the pilot signal is

$$P[(w_1 + w_3)\sin\omega_0 t + (w_2 + w_4)\sin(\omega_0 t - \pi/2)]. \quad (2)$$

For this output to be equal to the desired output of $p(t)=P\sin\omega_0 t$ (which is the pilot signal itself), it is necessary that

$$\left.\begin{aligned} w_1 + w_3 &= 1 \\ w_2 + w_4 &= 0 \end{aligned}\right\}. \quad (3)$$

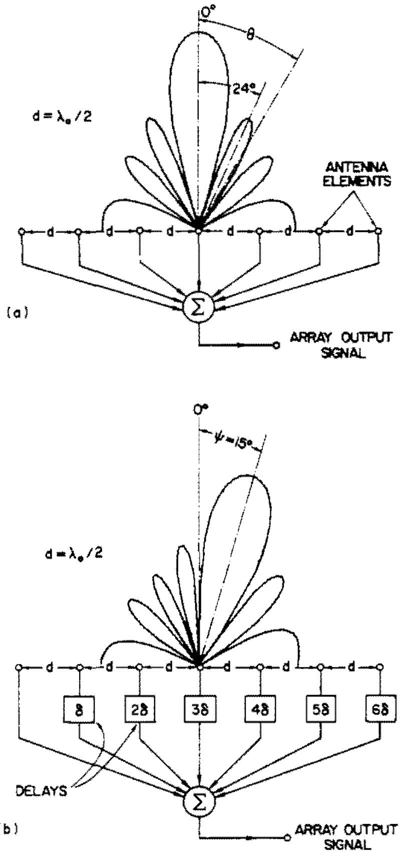

Fig. 1. Directivity pattern for a linear array. (a) Simple array. (b) Delays added.

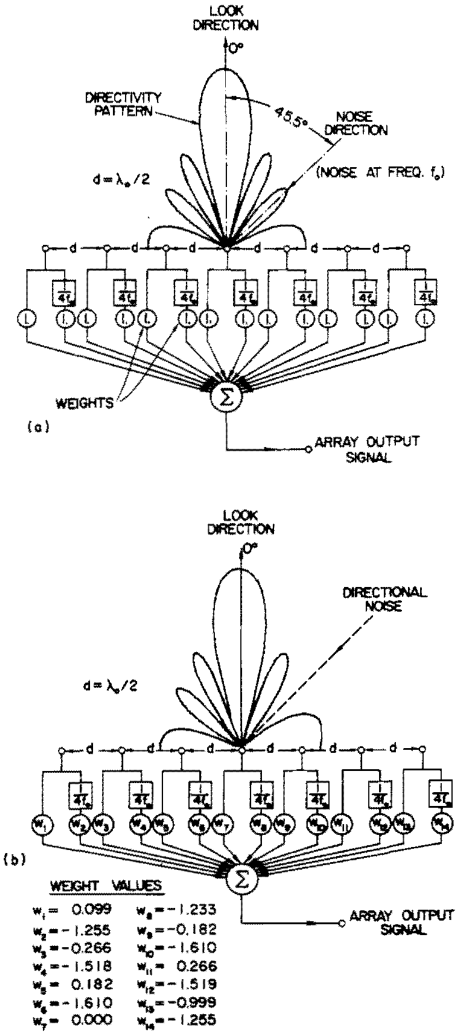

Fig. 2. Directivity pattern of linear array. (a) With equal weighting. (b) With weighting for noise elimination.

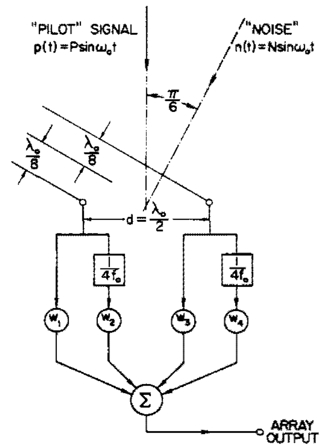

Fig. 3. Array configuration for noise elimination example.

With respect to the midpoint between the antenna elements, the relative time delays of the noise at the two antenna elements are $\pm[1/(4f_0)]\sin\pi/6 = \pm 1/(8f_0) = \pm\lambda_0/(8c)$, which corresponds to phase shifts of $\pm\pi/4$ at frequency f_0. The array output due to the incident noise at $\theta=\pi/6$ is then

$$N\left[w_1 \sin\left(\omega_0 t - \frac{\pi}{4}\right) + w_2 \sin\left(\omega_0 t - \frac{3\pi}{4}\right) + w_3 \sin\left(\omega_0 t + \frac{\pi}{4}\right) + w_4 \sin\left(\omega_0 t - \frac{\pi}{4}\right)\right]. \quad (4)$$

For this response to equal zero, it is necessary that

$$\left.\begin{aligned} w_1 + w_4 &= 0 \\ w_2 - w_3 &= 0 \end{aligned}\right\}. \quad (5)$$

Thus the set of weights that satisfies the signal and noise response requirements can be found by solving (3) and (5) simultaneously. The solution is

$$w_1 = \tfrac{1}{2},\ w_2 = \tfrac{1}{2},\ w_3 = \tfrac{1}{2},\ w_4 = -\tfrac{1}{2}. \quad (6)$$

With these weights, the array will have the desired properties in that it will accept a signal from the desired direction, while rejecting a noise, even a noise which is at the same frequency f_0 as the signal, because the noise comes from a different direction than does the signal.

The foregoing method of calculating the weights is more illustrative than practical. This method is usable when there are only a small number of directional noise sources, when the noises are monochromatic, and when the directions of the noises are known *a priori*. A practical processor should not require detailed information about the number and the nature of the noises. The adaptive processor described in the following meets this requirement. It recursively solves a sequence of simultaneous equations, which are generally overspecified, and it finds solutions which minimize the mean-square error between the pilot signal and the total array output.

Configurations of Adaptive Arrays

Before discussing methods of adaptive filtering and signal processing to be used in the adaptive array, various spatial and electrical configurations of antenna arrays will be considered. An adaptive array configuration for processing narrowband signals is shown in Fig. 4. Each individual antenna element is shown connected to a variable weight and to a quarter-period time delay whose output is in turn connected to another variable weight. The weighted signals are summed, as shown in the figure. The signal, assumed to be either monochromatic or narrowband, is received by the antenna element and is thus weighted by a complex gain factor $Ae^{j\phi}$. Any phase angle $\phi = -\tan^{-1}(w_2/w_1)$ can be chosen by setting the two weight values, and the magnitude of this complex gain factor $A=\sqrt{w_1^2+w_2^2}$ can take on a wide range of values limited only by the range limitations of the two individual weights. The latter can assume a continuum of both positive and negative values.

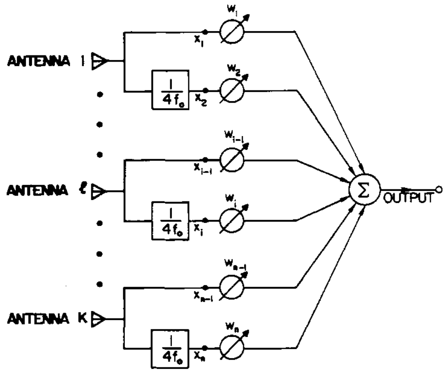

Fig. 4. Adaptive array configuration for receiving narrowband signals.

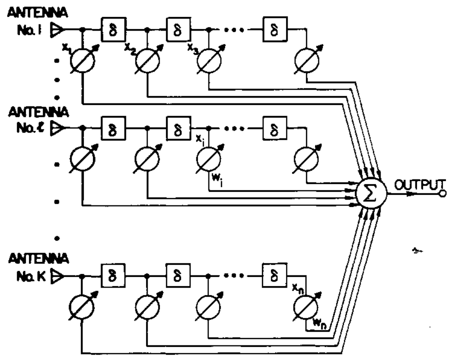

Fig. 5. Adaptive array configuration for receiving broadband signals.

Thus the two weights and the $1/(4f_0)$ time delay provide completely adjustable linear processing for narrowband signals received by each individual antenna element.

The full array of Fig. 4 represents a completely general way of combining the antenna-element signals in an adjustable linear structure when the received signals and noises are narrowband. It should be realized that the same generality (for narrowband signals) can be achieved even when the time delays do not result in a phase shift of exactly $\pi/2$ at the center frequency f_0. Keeping the phase shifts close to $\pi/2$ is desirable for keeping required weight values small, but is not necessary in principle.

When one is interested in receiving signals over a wide band of frequencies, each of the phase shifters in Fig. 4 can be replaced by a tapped-delay-line network as shown in Fig. 5. This tapped delay line permits adjustment of gain and phase as desired at a number of frequencies over the band of interest. If the tap spacing is sufficiently close, this network approximates the ideal filter which would allow complete control of the gain and phase at each frequency in the passband.

Adaptive Signal Processors

Once the form of network connected to each antenna element has been chosen, as shown for example in Fig. 4 or Fig. 5, the next step is to develop an adaptation procedure which can be used to adjust automatically the multiplying weights to achieve the desired spatial and frequency filtering.

The procedure should produce a given array gain in the specified look direction while simultaneously nulling out interfering noise sources.

Fig. 6 shows an adaptive signal-processing element. If this element were combined with an output-signal quantizer, it would then comprise an adaptive threshold logic unit. Such an element has been called an "Adaline"[13] or a threshold logic unit (TLU).[14] Applications of the adaptive threshold element have been made in pattern-recognition systems and in experimental adaptive control systems.[2],[3],[14]–[17]

In Fig. 6 the input signals $x_1(t), \cdots, x_i(t), \cdots, x_n(t)$ are the same signals that are applied to the multiplying weights $w_1, \cdots, w_i, \cdots, w_n$ shown in Fig. 4 or Fig. 5. The heavy lines show the paths of signal flow; the lighter lines show functions related to weight-changing or adaptation processes.

The output signal $s(t)$ in Fig. 6 is the weighted sum

$$s(t) = \sum_{i=1}^{n} x_i(t) w_i \tag{7}$$

where n is the number of weights; or, using vector notation

$$s(t) = \boldsymbol{W}^T \boldsymbol{X}(t) \tag{8}$$

where $\boldsymbol{W}^T$ is the transpose of the weight vector

$$\boldsymbol{W} \triangleq \begin{bmatrix} w_1 \\ \vdots \\ w_i \\ \vdots \\ w_n \end{bmatrix}$$

and the signal-input vector is

$$\boldsymbol{X}(t) \triangleq \begin{bmatrix} x_1(t) \\ \vdots \\ x_i(t) \\ \vdots \\ x_n(t) \end{bmatrix}.$$

For digital systems, the input signals are in discrete-time sampled-data form and the output is written

$$s(j) = \boldsymbol{W}^T \boldsymbol{X}(j) \tag{9}$$

where the index j indicates the jth sampling instant.

In order that adaptation take place, a "desired response" signal, $d(t)$ when continuous or $d(j)$ when sampled, must be supplied to the adaptive element. A method for obtaining this signal for adaptive antenna array processing will be discussed in a following section.

The difference between the desired response and the output response forms the error signal $\varepsilon(j)$:

$$\varepsilon(j) = d(j) - \boldsymbol{W}^T \boldsymbol{X}(j). \tag{10}$$

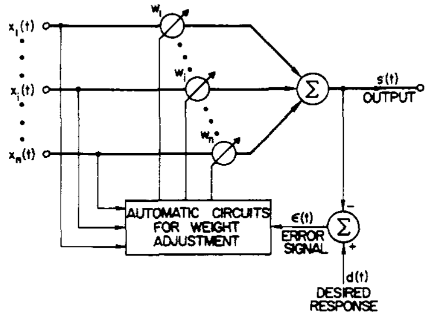

Fig. 6. Basic adaptive element.

This signal is used as a control signal for the "weight adjustment circuits" of Fig. 6.

Solving Simultaneous Equations

The purpose of the adaptation or weight-changing processes is to find a set of weights that will permit the output response of the adaptive element at each instant of time to be equal to or as close as possible to the desired response. For each input-signal vector $\boldsymbol{X}(j)$, the error $\varepsilon(j)$ of (10) should be made as small as possible.

Consider the finite set of linear simultaneous equations

$$\begin{cases} \boldsymbol{W}^T \boldsymbol{X}(1) = d(1) \\ \boldsymbol{W}^T \boldsymbol{X}(2) = d(2) \\ \quad \vdots \qquad\quad \vdots \\ \boldsymbol{W}^T \boldsymbol{X}(j) = d(j) \\ \quad \vdots \qquad\quad \vdots \\ \boldsymbol{W}^T \boldsymbol{X}(N) = d(N) \end{cases} \tag{11}$$

where N is the total number of input-signal vectors; each vector is a measurement of an underlying n-dimensional random process. There are N equations, corresponding to N instants of time at which the output response values are of concern; there are n "unknowns," the n weight values which form the components of $\boldsymbol{W}$. The set of equations (11) will usually be overspecified and inconsistent, since in the present application, with an ample supply of input data, it is usual that $N \gg n$. [These equations did have a solution in the simple example represented in Fig. 3. The solution is given in (6). Although the simultaneous equations (3) in that example appear to be different from (11), they are really the same, since those in (3) are in a specialized form for the case when all inputs are deterministic sinusoids which can be easily specified over all time in terms of amplitudes, phases, and frequencies.]

When N is very large compared to n, one is generally interested in obtaining a solution of a set of N equations [each equation in the form of (10)] which minimizes the sum of the squares of the errors. That is, a set of weights W is found to minimize

$$\sum_{j=1}^{N} \varepsilon^2(j). \tag{12}$$

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