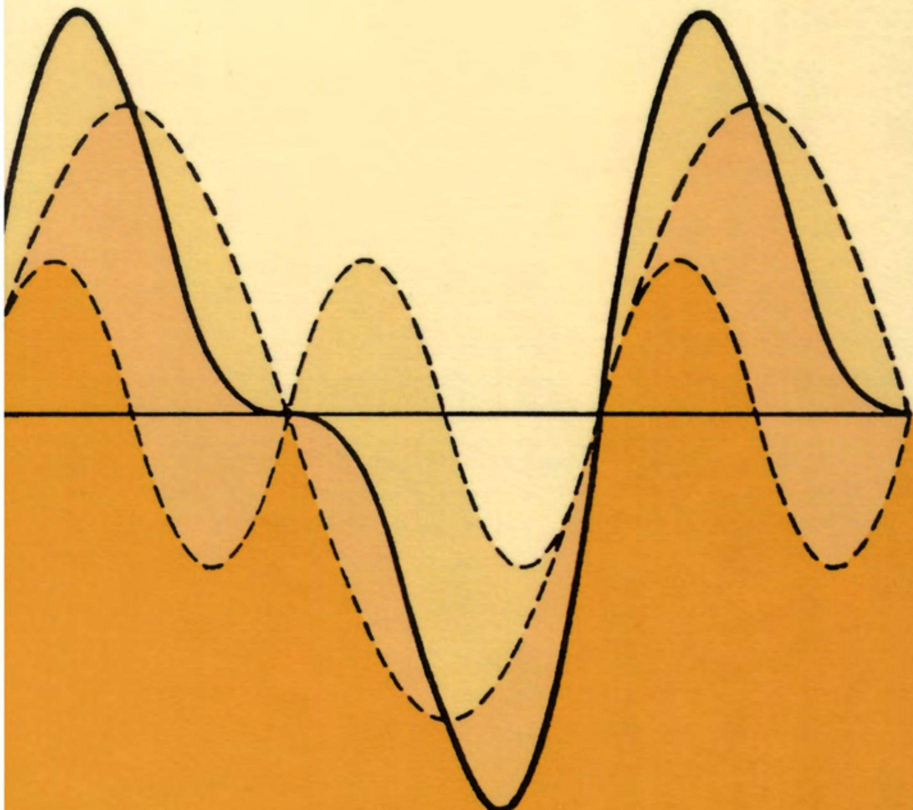
MECHANICAL VIBRATIONS



J. P. DEN HARTOG

This Dover edition, first published in 1985, is an unabridged, slightly corrected republication of the fourth edition (1956) of the work first published by the McGraw-Hill Book Company, Inc., New York, in 1934. A brief Preface has been added to this edition.

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PREFACE TO THE FC

This book grew from a course of l
Design School of the Westinghouse Cc
period from 1926 to 1932, when the sul
into the curriculum of our technical
beginning of the war, it became a regula
ing School, and the book was written f
course, being first published in 1934. I
entirely by the author's industrial expe
editions have brought modifications a
problems published in the literature, b
by service during the war in the Burea

The book aims to be as simple as
complete treatment of the subject. M
but in all cases the mathematical ap
available.

In the fourth edition the number of p
substantially, rising from 81 in the fir
second and third, and to 230 in this pr
have been made in every chapter to bri
to keep the size of the volume within l
deletions as well as additions.

During the life of this book, from
engineering has grown at an astonishing
has expanded with it. While in 1934
covered more or less what was know
such claim can be made for this fourth
our subject has become the parent of t
each of which now stands on its own f
body of literature. They are (1) elect
the theory and practice of instrumentati
trol or systems engineering, (3) aircraft f

No attempt has been made to cover
a superficial treatment would have mad
However, all three subjects are offshoo

ght pull. By changing the position of the nuts the I is changed while the mass m remains constant. djustment of the nuts the two natural frequencies can ly the same value. Then by pulling down and releas- n motion of the mass without twist is initiated. After ng occurs without vertical motion, and so on.

ustrated in Fig. 3.3*e*, is the electrical analogue of this pages 27, 28). Two equal masses (inductances) L main springs (condensers) C are coupled with a weak ge coupling condenser C_3 since k is equivalent to $1/C$. in one mesh will after a time be completely transferred and so on. Electrically minded readers may reason ts flow in each of the two "natural modes" and what , and may also construct a figure similar to 3.4 or 3.5

. bar of mass m and length $2l$ is supported by two springs, one c). The springs are *not* equally stiff, their constants being k espectively. Find the two natural frequencies and the shape nodes of vibration.

e upward displacement of the center of the bar and φ its (clock- . Then the displacement of the left end is $x + l\varphi$ and that of The spring forces are $k(x + l\varphi)$ and $2k(x - l\varphi)$, respectively.

$$m\ddot{x} + k(x + l\varphi) + 2k(x - l\varphi) = 0$$
$$\tfrac{1}{3}ml^2)\ddot{\varphi} + kl(x + l\varphi) - 2kl(x - l\varphi) = 0$$

ations. With the assumption of Eq. (3.3) we obtain

$$(-m\omega^2 + 3k)x_0 - kl\varphi_0 = 0$$
$$-klx_0 + (-\tfrac{1}{3}m\omega^2l^2 + 3kl^2)\varphi_0 = 0$$

frequency equation

$$m\omega^2 + 3k)(-\tfrac{1}{3}m\omega^2l^2 + 3kl^2) - k^2l^2 = 0$$
$$\omega^4 - 12\frac{k}{m}\omega^2 + 24\left(\frac{k}{m}\right)^2 = 0$$

$$\omega_1^2 = 2.54\frac{k}{m} \quad \text{and} \quad \omega_2^2 = 9.46\frac{k}{m}$$

notion corresponding to these frequencies are found from the ation, which can be written as

$$\frac{x_0}{l\varphi_0} = -\frac{1}{3}\frac{m}{k}\omega^2 + 3$$

s for ω^2 just found, this becomes

$$\left(\frac{x_0}{l\varphi_0}\right)_1 = +2.16 \qquad \left(\frac{x_0}{l\varphi_0}\right)_2 = -0.15$$

This means a rotary vibration of the bar about a point which lies at a distance of $2.16l$ to the right of the center of the bar for the first natural frequency and about a point at $0.15l$ to the left of the center for the second natural frequency.

3.2. The Undamped Dynamic Vibration Absorber. A machine or machine part on which a steady alternating force of *constant* frequency is acting may take up obnoxious vibrations, especially when it is close to resonance. In order to improve such a situation, we might first attempt to eliminate the force. Quite often this is not practical or even possible. Then we may change the mass or the spring constant of the system in an attempt to get away from the resonance condition, but in some cases this also is impractical. A third possibility lies in the application of the *dynamic vibration absorber*, invented by Frahm in 1909.

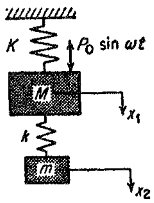

FIG. 3.6. The addition of a small k-m system to a large machine K-M prevents vibration of that machine in spite of the alternating force $P_0 \sin \omega t$.

In Fig. 3.6 let the combination K, M be the schematic representation of the machine under consideration, with the force $P_0 \sin \omega t$ acting on it. The vibration absorber consists of a comparatively small vibratory system k, m attached to the main mass M. *The natural frequency* $\sqrt{k/m}$ *of the attached absorber is chosen to be equal to the frequency* ω *of the disturbing force. It will be shown that then the main mass* M *does not vibrate at all*, and that the small system k, m vibrates in such a way that its spring force is at all instants equal and opposite to $P_0 \sin \omega t$. Thus there is no net force acting on M and therefore that mass does not vibrate.

To prove this statement, write down the equations of motion. This is a simple matter since Fig. 3.6 is a special case of Fig. 3.1 in which k_2 is made zero. Moreover, there is the external force $P_0 \sin \omega t$ on the first mass M. Equations (3.1) and (3.2) are thus modified to

$$\left.\begin{aligned} M\ddot{x}_1 + (K + k)x_1 - kx_2 &= P_0 \sin \omega t \\ m\ddot{x}_2 + k(x_2 - x_1) &= 0 \end{aligned}\right\} \qquad (3.10)$$

The *forced* vibration of this system will be of the form

$$\left.\begin{aligned} x_1 &= a_1 \sin \omega t \\ x_2 &= a_2 \sin \omega t \end{aligned}\right\} \qquad (3.11)$$

This is evident since (3.10) contains only x_1, $\ddot{x}_1$, and x_2, $\ddot{x}_2$, but *not* the first derivatives $\dot{x}_1$ and $\dot{x}_2$. A sine function remains a sine function after two differentiations, and consequently, with the assumption (3.11), all terms in (3.10) will be proportional to $\sin \omega t$. Division by $\sin \omega t$ transforms the *differential* equations into *algebraic* equations as was seen before with Eqs. (3.1) to (3.4). The result is that

$$\left.\begin{aligned} a_1(-M\omega^2 + K + k) - ka_2 &= P_0 \\ -ka_1 + a_2(-m\omega^2 + k) &= 0 \end{aligned}\right\} \tag{3.12}$$

For simplification we want to bring these into a dimensionless form and for that purpose we introduce the following symbols:

$x_{st} = P_0/K$ = static deflection of main system
$\omega_a^2 = k/m$ = natural frequency of absorber
$\Omega_n^2 = K/M$ = natural frequency of main system
$\mu = m/M$ = mass ratio = absorber mass/main mass

Then Eq. (3.12) becomes

$$\left.\begin{aligned} a_1\left(1 + \frac{k}{K} - \frac{\omega^2}{\Omega_n^2}\right) - a_2\frac{k}{K} &= x_{st} \\ a_1 &= a_2\left(1 - \frac{\omega^2}{\omega_a^2}\right) \end{aligned}\right\} \tag{3.13}$$

or, solving for a_1 and a_2,

$$\left.\begin{aligned} \frac{a_1}{x_{st}} &= \frac{1 - \dfrac{\omega^2}{\omega_a^2}}{\left(1 - \dfrac{\omega^2}{\omega_a^2}\right)\left(1 + \dfrac{k}{K} - \dfrac{\omega^2}{\Omega_n^2}\right) - \dfrac{k}{K}} \\ \frac{a_2}{x_{st}} &= \frac{1}{\left(1 - \dfrac{\omega^2}{\omega_a^2}\right)\left(1 + \dfrac{k}{K} - \dfrac{\omega^2}{\Omega_n^2}\right) - \dfrac{k}{K}} \end{aligned}\right\} \tag{3.14}$$

From the first of these equations the truth of our contention can be seen immediately. The amplitude a_1 of the main mass is zero when the numerator $1 - \dfrac{\omega^2}{\omega_a^2}$ is zero, and this occurs when the frequency of the force is the same as the natural frequency of the absorber.

Let us now examine the second equation (3.14) for the case that $\omega = \omega_a$. The first factor of the denominator is then zero, so that this equation reduces to

$$a_2 = -\frac{K}{k}x_{st} = -\frac{P_0}{k}$$

With the main mass standing still and the damper mass having a motion $-P_0/k \cdot \sin \omega t$ the force in the damper spring varies as $-P_0 \sin \omega t$, which is actually equal and opposite to the external force.

These relations are true for any value of the ratio ω/Ω_n. It was seen, however, that the addition of an absorber has not much reason unless the

original system is in resonance or at least near it. In what follows, the case for which

$$\omega_a = \Omega_n \qquad \text{or} \qquad \frac{k}{m} = \frac{K}{M} \qquad \text{or}$$

The ratio

$$\mu = \frac{m}{M}$$

then defines the size of the damper as compared t
system. For this special case, (3.14) becomes

$$\frac{x_1}{x_{st}} = \frac{1 - \dfrac{\omega^2}{\omega_a^2}}{\left(1 - \dfrac{\omega^2}{\omega_a^2}\right)\left(1 + \mu - \dfrac{\omega^2}{\omega_a^2}\right) - \mu}$$

$$\frac{x_2}{x_{st}} = \frac{1}{\left(1 - \dfrac{\omega^2}{\omega_a^2}\right)\left(1 + \mu - \dfrac{\omega^2}{\omega_a^2}\right) - \mu}$$

A striking peculiarity of this result and of Eq. (
denominators are equal. This is no coincidence b
ical reason. When multiplied out, it is seen that
tains a term proportional to $(\omega^2/\omega_a^2)^2$, a term pr
and a term independent of this ratio. When equat
inator is a quadratic equation in ω^2/ω_a^2 which nec
Thus for two values of the external frequency ω
(3.15) become zero, and consequently x_1 as well as
large. These two frequencies are the *resonant* or
the system. If the two denominators of (3.15) w
other, it could occur that one of them was zero at a
one not zero. This would mean that x_1 would be
not. But, if x_1 is infinite, the extensions and comp
spring k become infinite and necessarily the forc
Thus we have the impossible case that the amplit
mass m is finite while an infinite force $k(x_1 - x_2)$ is
therefore, if one of the amplitudes becomes infinit
and consequently the two denominators in (3.15)
The natural frequencies are determined by sett
equal to zero:

$$\left(1 - \frac{\omega^2}{\omega_a^2}\right)\left(1 + \mu - \frac{\omega^2}{\omega_a^2}\right) - \mu =$$

or

$$\left(\frac{\omega}{\omega_a}\right)^4 - \left(\frac{\omega}{\omega_a}\right)^2 (2 + \mu) + 1 =$$

:he result is that

$$\left.\begin{aligned} {}^2 + K + k) - ka_2 &= P_0 \\ {}_1 + a_2(-m\omega^2 + k) &= 0 \end{aligned}\right\} \qquad (3.12)$$

to bring these into a dimensionless form and ıce the following symbols:

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$$\left.\begin{aligned} \cdot \frac{k}{K} - \frac{\omega^2}{\Omega_n^2}\right) - a_2 \frac{k}{K} &= x_{st} \\ :\left(1 - \frac{\omega^2}{\omega_n^2}\right) & \end{aligned}\right\} \qquad (3.13)$$

$$\left.\begin{aligned} &\frac{1 - \dfrac{\omega^2}{\omega_a^2}}{-\dfrac{\omega^2}{\omega_a^2}\Big)\left(1 + \dfrac{k}{K} - \dfrac{\omega^2}{\Omega_n^2}\right) - \dfrac{k}{K}} \\ &\frac{1}{-\dfrac{\omega^2}{\omega_a^2}\Big)\left(1 + \dfrac{k}{K} - \dfrac{\omega^2}{\Omega_n^2}\right) - \dfrac{k}{K}} \end{aligned}\right\} \qquad (3.14)$$

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original system is in resonance or at least near it. We therefore consider, in what follows, the case for which

$$\omega_a = \Omega_n \quad \text{or} \quad \frac{k}{m} = \frac{K}{M} \quad \text{or} \quad \frac{k}{K} = \frac{m}{M}$$

The ratio
$$\mu = \frac{m}{M}$$

then defines the size of the damper as compared to the size of the main system. For this special case, (3.14) becomes

$$\left.\begin{aligned} \frac{x_1}{x_{st}} &= \frac{1 - \dfrac{\omega^2}{\omega_a^2}}{\left(1 - \dfrac{\omega^2}{\omega_a^2}\right)\left(1 + \mu - \dfrac{\omega^2}{\omega_a^2}\right) - \mu} \sin \omega t \\ \frac{x_2}{x_{st}} &= \frac{1}{\left(1 - \dfrac{\omega^2}{\omega_a^2}\right)\left(1 + \mu - \dfrac{\omega^2}{\omega_a^2}\right) - \mu} \sin \omega t \end{aligned}\right\} \qquad (3.15a, b)$$

A striking peculiarity of this result and of Eq. (3.14) is that the two denominators are equal. This is no coincidence but has a definite physical reason. When multiplied out, it is seen that the denominator contains a term proportional to $(\omega^2/\omega_a^2)^2$, a term proportional to $(\omega^2/\omega_a^2)^1$ and a term independent of this ratio. When equated to zero, the denominator is a quadratic equation in ω^2/ω_a^2 which necessarily has two roots. Thus for two values of the external frequency ω both denominators of (3.15) become zero, and consequently x_1 as well as x_2 becomes infinitely large. These two frequencies are the *resonant* or *natural* frequencies of the system. If the two denominators of (3.15) were *not* equal to each other, it could occur that one of them was zero at a certain ω and the other one not zero. This would mean that x_1 would be infinite and x_2 would not. But, if x_1 is infinite, the extensions and compressions of the damper spring k become infinite and necessarily the force in that spring also. Thus we have the impossible case that the amplitude x_2 of the damper mass m is finite while an infinite force $k(x_1 - x_2)$ is acting on it. Clearly, therefore, if one of the amplitudes becomes infinite, so must the other, and consequently the two denominators in (3.15) must be the same.

The natural frequencies are determined by setting the denominators equal to zero:

$$\left(1 - \frac{\omega^2}{\omega_a^2}\right)\left(1 + \mu - \frac{\omega^2}{\omega_a^2}\right) - \mu = 0$$

or

$$\left(\frac{\omega}{\omega_a}\right)^4 - \left(\frac{\omega}{\omega_a}\right)^2 (2 + \mu) + 1 = 0$$

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