

**CKET LARM** Find authenticated court documents without watermarks at <u>docketalarm.com</u>.

Δ

#### SIGNAL PROCESSING

#### CONTENTS

Volume 6 (1984) No. 4

#### Regular Papers

M.	Vetterli and H. Nussbaumer, Simple FFT and DCT algorithms with reduced number of operations	267
J.   -	3. Martens, Convolution algorithms, based on the CRT (Chinese Remainder Theorem)	279
Т.	Güngen and N. C. Geçkini, An algorithm for formant tracking	293
E.	I. Plotkin, L. M. Roytman and N. M. S. Swamy, Unbiased estimation of an initial phase	301
F.	F. Liedtke, Computer simulation of an automatic classification procedure for digitally modulated communication signals with unknown parameters	311

#### Short Communications

)

D

Δ

O)

R

М

G. H. Allen, Programming an efficient radix-four FFT algorithm	325
J. Le Roux, Non symmetric lattice structure for pole zero filters	331
Book Review	337
Announcements + Call for Papers	339
Author Index	343

JEP - 5 1984

#### LIBRARY

#### HUAWEI EX. 1021 -2/15

3 1

Find authenticated court documents without watermarks at <u>docketalarm.com</u>.

LIGINEERING SUCIETES

#### SIGNAL PROCESSING

A European Journal devoted to the methods and applications of signal processing A publication of the European Association for Signal Processing (EURASIP)

Editor-in-Chief Murat Kunt Laboratoire de traitement de signaux Ecole polytechnique fédérale de Lausanne 16, Chemin de Bellerive CH - 1007 Lausanne Switzerland Tel: 021 47 26 26 Telex: 24478 Editorial Board M. Bellanger (Le Plessis Robinson, France) R. Boite (Mons, Belgium) C. Braccini (Genova, Italy) V. Cappellini (Florence, Italy) G. Carayannis (Athens, Greece) A. G. Constantinides (London, U.K.) F. de Coulon (Lausanne, Switzerland) T. S. Durrani (Glasgow, U.K.) W. Endres (Darmstadt, West Germany) B. Escueidé (Lyon, France)	<ul> <li>O. D. Faugeras (Paris, France)</li> <li>A. L. Fawe (Sart Tilman, Belgium)</li> <li>A. Fettweis (Bochum, West Germany)</li> <li>J. G. Gander (Solothurn, Switzerland)</li> <li>C. Gazanhes (Marseille, France)</li> <li>D. Godard (La Gaude, France)</li> <li>G. H. Granlund (Linkoeping, Sweden)</li> <li>C. Gueguen (Paris, France)</li> <li>T. S. Huang (West Lafayette, USA)</li> <li>Z. Kulpa (Warsaw, Poland)</li> <li>J. L. Lacoume (Grenoble, France)</li> <li>M. A. Lagunan-Hernandes</li> <li>(Barcelona, Spain)</li> <li>D. S. Lebedev (Moscow, USSR)</li> <li>S. Levialdi (Napoli, Italy)</li> <li>C. E. Liedtke (Hannover, West Germany)</li> <li>J. Max (Grenoble, France)</li> <li>R. De Mori (Montreal, Canada)</li> </ul>	M. Nagao (Kyol H. Niemann (Er F. Pellandini (N- B. Picinbono (G B. S. Ramakrish F. Rocca (Milan B. Sankur (Istar L. L. Scharf (Kir J. C. Simon (Pa C. Y. Suen (Mo Ju-Wei Tai (Pel H. Tiziani (Heel D. Wolf (Frankf G. Winkler (Kar I. T. Young (De P. Zamperoni ( Germany) Secretary: I. Be
--	--	--

Editorial Policy. SIGNAL PROCESSING is a European Journal presenting the theory and practice of signal processing. Its primary objectives are:

Dissemination of research results and of engineering developments to European signal processing groups and individuals; Presentation of practical solutions to current signal processing problems in engineering and science.

The editorial policy and the technical content of the Journal are the responsibility of the Editor-in-Chief and the Editorial Board. The Journal is self-supporting from subscription income and contains a minimum amount of advertisements. Advertisements are subject to the prior approval of the Editor-in-Chief.

Scope. SIGNAL PROCESSING incorporates all aspects of the theory and practice of signal processing (analogue and digital). It features original research work, tutorial and review articles, and accounts of practical developments. It is intended for a rapid dissemination of knowledge and experience to engineers and scientists working in signal processing research, development or practical application.

Subject coverage. Subject areas covered by the Journal include:

Signal Theory Stochastic Processes Detection and Estimation Spectral Analysis Filtering

Signal Processing Systems Software Developments Image Processing Pattern Recognition Optical Signal Processing

o, Japan) langen, West Germany) euchatel, Switzerland) if-sur-Yvette, France) nna (Bangalore, India) no, Italy) nbul, Turkey) ngston, USA) aris, France) ntreal, Canada) king, China) rbrugg, Switzerland) urt, West Germany) Isruhe, West Germany) Ift, The Netherlands) Braunschweig, West

ezzi (Miss)

Data Processing Remote Sensing Communication Signal Processing **Biomedical Signal Processing** Geophysical and Astrophysical Signal Processing Earth Resources Signal Processing Acoustic and Vibration Signal Processing Signal Processing Technology Sonar Signal Processing Speech Processing Special Signal Processing Radar Signal Processing New Applications Industrial Applications

#### Membership and Subscription Information.

Signal Processing (ISSN 0165-1684) is published in two volumes (8 issues) a year. The subscription price for 1984 (Volumes 6, 7) is Dfl. 350.00 + Dfl. 42.00 p.p.h. = Dfl. 392.00. Our p.p.h. (postage, packing and handling) charge includes surface delivery of all issues, except to subscribers in the USA/Canada and India, who receive all issues by air delivery (S.A.L. – Surface Airl Lifted) at no extra cost. For the rest of the world, airmail and S.A.L. charges are available upon request. Claims for missing issues will be honoured free of charge within three months after the publication date of the issues. Mail orders and inquiries to: Elsevier Science Publishers B.V., Journals Department, P.O. Box 211, 1000 AE Amsterdam, The Netherlands. For full membership information of the Association, including a subscription at a reduced rate, please contact: EURASIP, P.O. Box 134, CH-1000 Lausanne 13, Switzerland.

© 1984, Elsevier Science Publishers B.V. (North-Holland)

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publisher, Elsevier Science Publishers B.V. (North-Holland), P.O. Box 1991, 1000 BZ Amsterdam, The Netherlands.

Amsterdam, The Netherlands. Submission of an article for publication implies the transfer of the copyright from the author(s) to the publisher and entails the author's irrevocable and exclusive authorization of the publisher to collect any sums or considerations for copying or reproduction payable by third parties (as mentioned in article 17 paragraph 2 of the Dutch Copyright Act of 1912 and in the Royal Decree of June 20, 1974 (S. 351) pursuant to article 16b of the Dutch Copyright Act of 1912) and/or to act in or out of Court in connection therewith

connection therewith. Special regulations for readers in the U.S.A. - This journal has been registered with the Copyright Clearance Center, Inc. Consent is given for copying of articles for personal or internal use, or for the personal use of specific clients. This consent is given on the condition that the copier pays through the Center the per-copy fee stated in the code on the first page of each article for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law. The appropriate fee should be forwarded with the code on the first page of the article to the Copyright Clearance Center, Inc. 21 Congress Street, Salem, MA 01970, U.S.A. If no code appears in an article, the author has a copy of the first page of the article to the Copyright Clearance Center, Inc. 21 Congress Street, Salem, MA 01970, U.S.A. If no code appears in an article, the author has not given broad consent to copy and permission to copy must be obtained directly from the author. All articles published prior to 1981 may be copied for a per-copy fee of to US \$2.25, also payable through the Center. (N.B. For review journals this fee is \$0.25 per copy per page.) This consent does not extend to other kinds of copying, such as US \$2.25, also cover and the author has a cover the cover of the center in the publisher for general distribution, resale, advertising and promotion purposes, or for creating new collective works. Special written permission must be obtained from the publisher for such covering.

Special regulations for authors in the U.S.A. - Upon acceptance of an article by the journal, the author(s) will be asked to transfer copyright of the article to the publisher. This transfer will ensure the widest possible dissemination of information under the U.S. Copyright Law.

Published bimonthly

HUAWEI EX. 1021 -3/15

Printed in The Netherlands

Find authenticated court documents without watermarks at docketalarm.com.

Signal Processing 6 (1984) 267–278 North-Holland

## SIMPLE FFT AND DCT ALGORITHMS WITH REDUCED NUMBER OF OPERATIONS

#### Martin VETTERLI, member EURASIP, and Henri J. NUSSBAUMER

Laboratoire d'Informatique Technique, Ecole Polytechnique Fédérale de Lausanne, 16 Chemin de Bellerive, CH-1007 Lausanne, Switzerland

Received 2 November 1983 Revised 20 February 1984

**Abstract.** A simple algorithm for the evaluation of discrete Fourier transforms (DFT) and discrete cosine transforms (DCT) is presented. This approach, based on the divide and conquer technique, achieves a substantial decrease in the number of additions when compared to currently used FFT algorithms (30% for a DFT on real data, 15% for a DFT on complex data and 25% for a DCT) and keeps the same number of multiplications as the best known FFT algorithms. The simple structure of the algorithm and the fact that it is best suited for real data (one does not have to take a transform of two real sequences simultaneously anymore) should lead to efficient implementations and to a wide range of applications.

Zusammenfassung. Ein einfacher Algorithmus zur Berechnung von diskreten Fourier Transformationen (DFT) und diskreten Cosinus Transformationen (DCT) wird vorgeschlagen. Diese Methode, basierend auf der "Teilen und Lösen" Technik, erlaubt eine Verkleinerung der Anzahl Additionen gegenüber gebraüchlichen FFT Algorithmen (30% für eine DFT von einem reellen Signal, 15% für eine DFT von einem complexen Signal und 25% für eine DCT) und braucht gleichviel Multiplikationen wie die besten bekannten FFT Algorithmen. Die einfache Struktur des Algorithmus und der Fakt dass er am besten für reelle Signale geeignet ist (man braucht nicht mehr gleichzeitig zwei reelle Signale zu transformieren) sollten zu effizienter Implementierung und zu zahlreichen Applikationen führen.

**Résumé.** Un algorithme simple pour l'évaluation de la transformée de Fourier discrète (DFT) et de la transformée en cosinus discrète (DCT) est proposé. Cette approche, basée sur la méthode de la "division et solution", permet une diminution substantielle du nombre d'additions par rapport aux algorithmes de FFT courants (30% pour une DFT de signaux réels, 15% pour une DFT de signaux complexes et 25% pour une DCT) tout en gardant un nombre de multiplications égal à celui des meilleurs algorithmes de FFT connus. La structure simple de l'algorithme ainsi que le fait qu'il s'applique bien aux signaux réels (il n'y a plus besoin de prendre la transformée de deux signaux réels simultanément) devraient conduire à une implantation efficace ainsi qu'à un large champ d'applications.

Keywords. Fast Fourier transform, fast cosine transform, transforms of real data.

#### 1. Introduction

DOCKE

Since the rediscovery of the fast Fourier transform (FFT) algorithm [1, 2] for the evaluation of discrete Fourier transforms, several improvements have been made to the basic divide and conquer scheme as for example the mixed radix FFT [3] and the real factor FFT [4, 5]. The introduction of the Winograd Fourier transform (WFTA) [6], although a beautiful result in complexity theory, did not bring the expected improvements once implemented on real life computers [7], essentially due to the large total number of operations and to the structural complexity of the algorithm.

The fact that most FFT's are taken on real data is seldom fully taken into account. The algorithm using a FFT of half dimension for the computation of a DFT on a real sequence [8] uses substantially more operations than the method of computing a single FFT on two real sequences simultaneously [9]. The

0165-1684/84/\$3.00 © 1984, Elsevier Science Publishers B.V. (North-Holland)

ter ter

HUAWEI EX. 1021 -4/15

267

#### M. Vetterli, H. J. Nussbaumer / Simple FFT and DCT Algorithms

latter method has the disadvantage that one has to take two DFT's at once and that the sorting of the output uses additional adds. The fact that the input and output sequences are real is used explicitly in a real convolution algorithm [10] where the DFT and inverse DFT are computed with a single complex FFT.

Another transform that is mostly applied to real data is the discrete cosine transform. Since the introduction of the DCT [11], the search for a fast algorithm followed two main different approaches. One was to compute the DCT through a FFT of same dimension [12], where one is bound to take two transforms simultaneously. The other was a direct approach, leading to rather involved algorithms [13]. It should be noted that the former technique outperforms all the latter ones when using optimal FFT's, a fact often left in the dark [14].

Recently, evaluation of signal processing algorithms has shifted away from multiplication counts alone to the counting of the total number of operations, including data transfers [15]. This is due to the fact that the ratios (multiplication time)/(addition time) and (multiplication time)/(load time) are close to one on most computers and signal processors. Another growing concern has been the generation of time efficient software [16], and finally, the efficiency of an algorithm turns out to be a non-trivial combination of the various operation counts as well as of its structural complexity [17].

In this communication, we address an old problem, namely, the efficient evaluation of DFT's and DCT's of real data. Efficiency is meant in the sense of minimal number of multiplications and additions as well as in the sense of structural simplicity. As it turns out, the two problems are closely related, since a DFT of dimension N can be evaluated with two DCT's of size N/4 and since a DCT of size N can be evaluated with a DFT of size N and additional operations. The same technique can be applied again to the reduced DCT of size N/4 and to the DFT of size N, and this until only trivial transforms are left over (N = 1, 2).

This leads to an elegant recursive formulation of the two algorithms and to a number of multiplications identical to the best FFT's while diminishing substantially the number of additions (typically 30%). Interestingly, this last saving is partly kept when computing complex DFT's, and as an example, the total number of operations for a 1024-point transform is nearly 10% below the number of operations required for a 1008-point WFTA. The prime factor FFT (PFA) requires about the same number of operations [18], but has a more complex structure.

Note that the algorithms below were developed while searching for an efficient way to compute DCT's of real data. The derived FFT algorithm for real data that follows immediately requires a number of multiplications identical to the one found in [19] (which is a variation of the Rader-Brenner algorithm), and a total number of operations that can be found in [20]. While obtaining an identical complexity, the derivations are quite different and the algorithm below seems more suitable for programming.

Section 2 is used to derive the general algorithm and Section 3 evaluates its computational complexity. In Section 4, the results are compared to other algorithms and some implementation considerations are addressed.

#### 2. Derivation of the algorithms

Let us define the following transforms of the length-N real vector x with elements  $x(0), x(1) \cdots x(N-1)$ :

Discrete Fourier transform

DFT
$$(k, N, x) := \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nk/N}, \quad k = 0, ..., N-1,$$

(1)

where  $j = +\sqrt{-1}$ .

Signal Processing

DOCKE

HUAWEI EX. 1021 -5/15

Find authenticated court documents without watermarks at docketalarm.com

268

# DOCKET



## Explore Litigation Insights

Docket Alarm provides insights to develop a more informed litigation strategy and the peace of mind of knowing you're on top of things.

## **Real-Time Litigation Alerts**



Keep your litigation team up-to-date with **real-time** alerts and advanced team management tools built for the enterprise, all while greatly reducing PACER spend.

Our comprehensive service means we can handle Federal, State, and Administrative courts across the country.

## **Advanced Docket Research**



With over 230 million records, Docket Alarm's cloud-native docket research platform finds what other services can't. Coverage includes Federal, State, plus PTAB, TTAB, ITC and NLRB decisions, all in one place.

Identify arguments that have been successful in the past with full text, pinpoint searching. Link to case law cited within any court document via Fastcase.

## **Analytics At Your Fingertips**



Learn what happened the last time a particular judge, opposing counsel or company faced cases similar to yours.

Advanced out-of-the-box PTAB and TTAB analytics are always at your fingertips.

### API

Docket Alarm offers a powerful API (application programming interface) to developers that want to integrate case filings into their apps.

#### LAW FIRMS

Build custom dashboards for your attorneys and clients with live data direct from the court.

Automate many repetitive legal tasks like conflict checks, document management, and marketing.

#### **FINANCIAL INSTITUTIONS**

Litigation and bankruptcy checks for companies and debtors.

#### **E-DISCOVERY AND LEGAL VENDORS**

Sync your system to PACER to automate legal marketing.

