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CONTENTS

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Dissemination of research results and of engineering developments to European signal processing groups and individuals;
Presentation of practical solutions to current signal processing problems in engineering and science.
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SIMPLE FFT AND DCT ALGORITHMS WITH REDUCED NUMBER OF OPERATIONS

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Abstract. A simple algorithm for the evaluation of discrete Fourier transforms (DFT) and discrete cosine transforms (DCT) is presented. This approach, based on the divide and conquer technique, achieves a substantial decrease in the number of additions when compared to currently used FFT algorithms (30% for a DFT on real data, 15% for a DFT on complex data and 25% for a DCT) and keeps the same number of multiplications as the best known FFT algorithms. The simple structure of the algorithm and the fact that it is best suited for real data (one does not have to take a transform of two real sequences simultaneously anymore) should lead to efficient implementations and to a wide range of applications.

Zusammenfassung. Ein einfacher Algorithmus zur Berechnung von diskreten Fourier Transformationen (DFT) und diskreten Cosinus Transformationen (DCT) wird vorgeschlagen. Diese Methode, basierend auf der "Teilen und Lösen" Technik, erlaubt eine Verkleinerung der Anzahl Additionen gegenüber gebraüchlichen FFT Algorithmen (30% für eine DFT von einem reellen Signal, 15% für eine DFT von einem complexen Signal und 25% für eine DCT) und braucht gleichviel Multiplikationen wie die besten bekannten FFT Algorithmen. Die einfache Struktur des Algorithmus und der Fakt dass er am besten für reelle Signale geeignet ist (man braucht nicht mehr gleichzeitig zwei reelle Signale zu transformieren) sollten zu effizienter Implementierung und zu zahlreichen Applikationen führen.

Résumé. Un algorithme simple pour l'évaluation de la transformée de Fourier discrète (DFT) et de la transformée en cosinus discrète (DCT) est proposé. Cette approche, basée sur la méthode de la "division et solution", permet une diminution substantielle du nombre d'additions par rapport aux algorithmes de FFT courants (30% pour une DFT de signaux réels, 15% pour une DFT de signaux complexes et 25% pour une DCT) tout en gardant un nombre de multiplications égal à celui des meilleurs algorithmes de FFT connus. La structure simple de l'algorithme ainsi que le fait qu'il s'applique bien aux signaux réels (il n'y a plus besoin de prendre la transformée de deux signaux réels simultanément) devraient conduire à une implantation efficace ainsi qu'à un large champ d'applications.

Keywords. Fast Fourier transform, fast cosine transform, transforms of real data.

1. Introduction

Since the rediscovery of the fast Fourier transform (FFT) algorithm [1, 2] for the evaluation of discrete Fourier transforms, several improvements have been made to the basic divide and conquer scheme as for example the mixed radix FFT [3] and the real factor FFT [4, 5]. The introduction of the Winograd Fourier transform (WFTA) [6], although a beautiful result in complexity theory, did not bring the expected improvements once implemented on real life computers [7], essentially due to the large total number of operations and to the structural complexity of the algorithm.

The fact that most FFT's are taken on real data is seldom fully taken into account. The algorithm using a FFT of half dimension for the computation of a DFT on a real sequence [8] uses substantially more operations than the method of computing a single FFT on two real sequences simultaneously [9]. The

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latter method has the disadvantage that one has to take two DFT's at once and that the sorting of the output uses additional adds. The fact that the input and output sequences are real is used explicitly in a real convolution algorithm [10] where the DFT and inverse DFT are computed with a single complex FFT.

Another transform that is mostly applied to real data is the discrete cosine transform. Since the introduction of the DCT [11], the search for a fast algorithm followed two main different approaches. One was to compute the DCT through a FFT of same dimension [12], where one is bound to take two transforms simultaneously. The other was a direct approach, leading to rather involved algorithms [13]. It should be noted that the former technique outperforms all the latter ones when using optimal FFT's, a fact often left in the dark [14].

Recently, evaluation of signal processing algorithms has shifted away from multiplication counts alone to the counting of the total number of operations, including data transfers [15]. This is due to the fact that the ratios (multiplication time)/(addition time) and (multiplication time)/(load time) are close to one on most computers and signal processors. Another growing concern has been the generation of time efficient software [16], and finally, the efficiency of an algorithm turns out to be a non-trivial combination of the various operation counts as well as of its structural complexity [17].

In this communication, we address an old problem, namely, the efficient evaluation of DFT's and DCT's of real data. Efficiency is meant in the sense of minimal number of multiplications and additions as well as in the sense of structural simplicity. As it turns out, the two problems are closely related, since a DFT of dimension N can be evaluated with two DCT's of size $N/4$ and since a DCT of size N can be evaluated with a DFT of size N and additional operations. The same technique can be applied again to the reduced DCT of size $N/4$ and to the DFT of size N, and this until only trivial transforms are left over ($N=1, 2$).

This leads to an elegant recursive formulation of the two algorithms and to a number of multiplications identical to the best FFT's while diminishing substantially the number of additions (typically 30%). Interestingly, this last saving is partly kept when computing complex DFT's, and as an example, the total number of operations for a 1024-point transform is nearly 10% below the number of operations required for a 1008-point WFTA. The prime factor FFT (PFA) requires about the same number of operations [18], but has a more complex structure.

Note that the algorithms below were developed while searching for an efficient way to compute DCT's of real data. The derived FFT algorithm for real data that follows immediately requires a number of multiplications identical to the one found in [19] (which is a variation of the Rader–Brenner algorithm), and a total number of operations that can be found in [20]. While obtaining an identical complexity, the derivations are quite different and the algorithm below seems more suitable for programming.

Section 2 is used to derive the general algorithm and Section 3 evaluates its computational complexity. In Section 4, the results are compared to other algorithms and some implementation considerations are addressed.

2. Derivation of the algorithms

Let us define the following transforms of the length-N real vector $\boldsymbol{x}$ with elements $x(0), x(1) \cdots x(N-1)$:

Discrete Fourier transform

$$\mathrm{DFT}(k, N, \boldsymbol{x}) := \sum_{n=0}^{N-1} x(n) \cdot \mathrm{e}^{-\mathrm{j}2\pi nk/N}, \quad k = 0, \ldots, N-1, \tag{1}$$

where $\mathrm{j} = +\sqrt{-1}$.

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