## Thomas

## DOCKET

# CALCULUS <br> AND <br> ANALYTIC <br> GEOMETRY 

FOURTH EDITION

ADDISON-WESLEY PUBLISHING COMPANY
Reading, Massachusetts
Menlo Park, California - London - Don Mills, Ontario

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### 11.1 THE POLAR COORDINATE SYSTEM

We know that a point can be located in a plane by giving its abscissa and ordinate relative to a given coordinate system. Such $x$ - and $y$-coordinates are called Cartesian coordinates, in honor of the French mathematician-philosopher René Descartes* (15961650 ), who is credited with discovering this method of fixing the position of a point in a plane.

11.1

## POLAR

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Another useful way to locate a point in a plane is by polar coordinates (see Fig. 11.1). First, we fix an origin $O$ and an initial ray $\dagger$ from $O$. The point $P$ has polar coordinates $r, \theta$, with

$$
\begin{equation*}
r=\text { directed distance from } O \text { to } P \tag{1a}
\end{equation*}
$$

and
$\theta=$ directed angle from initial ray to $O P$.
As in trigonometry, the angle $\theta$ is positive when measured counterclockwise and negative when measured clockwise (Fig. 11.1). But the angle associated with a given point is not unique (Fig. 11.2). For instance, the point 2 units from the origin, along the ray $\theta=30^{\circ}$, has polar coordinates $r=2, \theta=30^{\circ}$. It also has coordinates $r=2, \theta=-330^{\circ}$, or $r=2$, $\theta=390^{\circ}$.
There are occasions when we wish to allow $r$ to be negative. That's why we say "directed distance"

[^0]
11.2 The :3y $\theta=30^{\circ}$ is the same as the ray $\theta=-330^{\circ}$.

11.4 The terminal ray $\theta=\pi / 6$ and its negative.
in Eq. (1a). The ray $\theta=30^{\circ}$ and the ray $\theta=210^{\circ}$ together make up a complete line through $O$ (see Fig. 11.3). The point $P\left(2,210^{\circ}\right) 2$ units from $O$ on the ray $\theta=210^{\circ}$ has polar coordinates $r=2$, $\theta=210^{\circ}$. It can be reached by a person standing at $O$ and facing out along the initial ray, if he first turns $210^{\circ}$ counterclockwise, and then goes forward

2 units. He would reach the same point by turning only $30^{\circ}$ counterclockwise from the initial ray and then going backward 2 units. So we say that the point also has polar coordinates $r=-2, \theta=30^{\circ}$.
Whenever the angle between two rays is $180^{\circ}$, the rays actually make a straight line. We then say tha: either ray is the negative of the other. Points of the ray $\theta=\alpha$ have polar coordinates $(r, \alpha)$ witi $r \geq 0$. Points on the negative ray, $\theta=\alpha+180^{\circ}$ have coordinates $(r, \alpha)$ with $r \leq 0$. The origin $:$ $r=0$. (See Fig. 11.4 for the ray $\theta=30^{\circ}$ and $i t z$ negative. A word of caution: The "negative" the ray $\theta=30^{\circ}$ is the ray $\theta=30^{\circ}+180^{\circ}=210^{\circ}$ and not the ray $\theta=-30^{\circ}$. "Negative" refers to the directed distance $r$.)

There is a great advantage in being able to use both polar and Cartesian coordinates at once. Ts do this, we use a common origin and take the initis ray as the positive $x$-axis, and take the ray $\theta=g^{2}$ as the positive $y$-axis. The coordinates, shown $i=$ Fig. 11.5, are then related by the equations

$$
\begin{equation*}
x=r \cos \theta, \quad y=r \sin \theta \tag{2}
\end{equation*}
$$

These are the equations that define $\sin \theta$ and $\cos 5$ when $r$ is positive. They are also valid if $r$ is negtive, because

$$
\begin{aligned}
& \cos \left(\theta+180^{\circ}\right)=-\cos \theta \\
& \sin \left(\theta+180^{\circ}\right)=-\sin \theta
\end{aligned}
$$

so positive $r$ 's on the $\left(\theta+180^{\circ}\right)$-ray correspond at negative $r$ 's associated with the $\theta$-ray. When $r=\pi$ then $x=y=0$, and $P$ is the origin.

If we impose the condition

$$
r=a \quad(a \text { constant })
$$

then the locus of $P$ is a circle with center $O$ and radius $a$, and $P$ describes the circle once as $\theta$ varie from 0 to $360^{\circ}$ (see Fig. 11.6). On the other hadé if we let $r$ vary and hold $\theta$ fixed, say

$$
\theta=30^{\circ}
$$

the locus of $P$ is the straight line shown in Fig. 114.

5 Polar and C :

- 6 The circle $r=$

We adopt the c monber, $-\infty<$ ? $\mathrm{bl} z=0, y=0 \mathrm{ir}$
$r=$
the origin, $x=1$ The same point iiferent ways in
life point $\left(2,30^{\circ}\right)$ Hzresentations: (
$-2,-150^{\circ}$ ). The in the two formula

$$
\begin{aligned}
& \qquad \begin{array}{r}
\left(2,30^{\circ}+n\right. \\
-2,210^{\circ}+n \\
\text { of we represent } \\
\text { rus ulas }
\end{array}
\end{aligned}
$$

(2, $\frac{1}{6} \pi+2 r$ $\left(-2, \frac{7}{6} \pi+2 n\right.$

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[^0]:    * For an interesting biographical account together with an excerpt from Descartes' own writings, see World of Mathematics, Vol. 1, pp. 235-253.
    $\dagger$ A ray is a half-line consisting of a vertex and points of a line on one side of the vertex. For example, the origin and positive $x$-axis is a ray. The points on the line $y=2 x+3$ with $x \geq 1$ is another ray; its vertex is $(1,5)$.

