



GEORGE B. THOMAS, JR.

Department of Mathematics

Massachusetts Institute of Technology

CALCULUS AND ANALYTIC GEOMETRY

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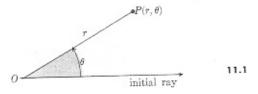


POLAR OORDINATES

CHAPTER 11

11.1 THE POLAR COORDINATE SYSTEM

We know that a point can be located in a plane by giving its abscissa and ordinate relative to a given coordinate system. Such x- and y-coordinates are called Cartesian coordinates, in honor of the French mathematician-philosopher René Descartes* (1596–1650), who is credited with discovering this method of fixing the position of a point in a plane.



Another useful way to locate a point in a plane is by polar coordinates (see Fig. 11.1). First, we fix an origin O and an initial ray† from O. The point P has polar coordinates r, θ , with

$$r =$$
 directed distance from O to P , (1a)

and

$$\theta$$
 = directed angle from initial ray to OP . (1b)

As in trigonometry, the angle θ is positive when measured counterclockwise and negative when measured clockwise (Fig. 11.1). But the angle associated with a given point is not unique (Fig. 11.2). For instance, the point 2 units from the origin, along the ray $\theta = 30^{\circ}$, has polar coordinates r = 2, $\theta = 30^{\circ}$. It also has coordinates r = 2, $\theta = -330^{\circ}$, or r = 2, $\theta = 390^{\circ}$.

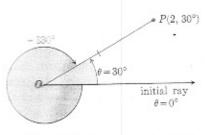
There are occasions when we wish to allow r to be negative. That's why we say "directed distance"



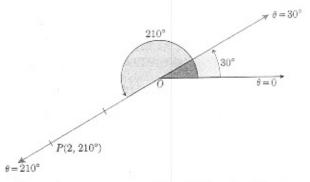
^{*} For an interesting biographical account together with an excerpt from Descartes' own writings, see World of Mathematics, Vol. 1, pp. 235-253.

[†] A ray is a half-line consisting of a vertex and points of a line on one side of the vertex. For example, the origin and positive x-axis is a ray. The points on the line y = 2x + 3 with $x \ge 1$ is another ray; its vertex is (1, 5).

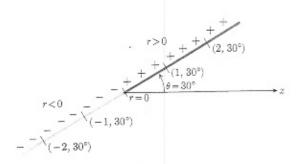




11.2 The ray $\theta = 30^{\circ}$ is the same as the ray $\theta = -330^{\circ}$.



11.3 The rays $\theta = 30^{\circ}$ and $\theta = 210^{\circ}$ make a line.



11.4 The terminal ray $\theta = \pi/6$ and its negative.

in Eq. (1a). The ray $\theta=30^\circ$ and the ray $\theta=210^\circ$ together make up a complete line through O (see Fig. 11.3). The point $P(2,210^\circ)$ 2 units from O on the ray $\theta=210^\circ$ has polar coordinates r=2, $\theta=210^\circ$. It can be reached by a person standing at O and facing out along the initial ray, if he first turns 210° counterclockwise, and then goes forward

2 units. He would reach the same point by turning only 30° counterclockwise from the initial ray and then going backward 2 units. So we say that the point also has polar coordinates r=-2, $\theta=30^\circ$.

Whenever the angle between two rays is 180° , the rays actually make a straight line. We then say that either ray is the negative of the other. Points on the ray $\theta = \alpha$ have polar coordinates (r, α) with $r \geq 0$. Points on the negative ray, $\theta = \alpha + 180^\circ$, have coordinates (r, α) with $r \leq 0$. The origin is r = 0. (See Fig. 11.4 for the ray $\theta = 30^\circ$ and its negative. A word of caution: The "negative" of the ray $\theta = 30^\circ$ is the ray $\theta = 30^\circ + 180^\circ = 210^\circ$ and not the ray $\theta = -30^\circ$. "Negative" refers to the directed distance r.)

There is a great advantage in being able to use both polar and Cartesian coordinates at once. To do this, we use a common origin and take the initial ray as the positive x-axis, and take the ray $\theta = 90^{\circ}$ as the positive y-axis. The coordinates, shown in Fig. 11.5, are then related by the equations

$$x = r \cos \theta, \quad y = r \sin \theta.$$
 (2)

These are the equations that define $\sin \theta$ and $\cos \theta$ when r is positive. They are also valid if r is negtive, because

$$cos (\theta + 180^{\circ}) = -cos \theta,$$

 $sin (\theta + 180^{\circ}) = -sin \theta,$

so positive r's on the $(\theta + 180^{\circ})$ -ray correspond to negative r's associated with the θ -ray. When r = 0 then x = y = 0, and P is the origin.

If we impose the condition

$$r = a$$
 (a constant),

then the locus of P is a circle with center O and radius a, and P describes the circle once as θ varies from 0 to 360° (see Fig. 11.6). On the other hand, if we let r vary and hold θ fixed, say

$$\theta = 30^{\circ}$$
,

the locus of P is the straight line shown in Fig. 114.

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r =

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t if we represent

 $(2, \frac{1}{6}\pi + 2n)$ $(-2, \frac{7}{6}\pi + 2n)$



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