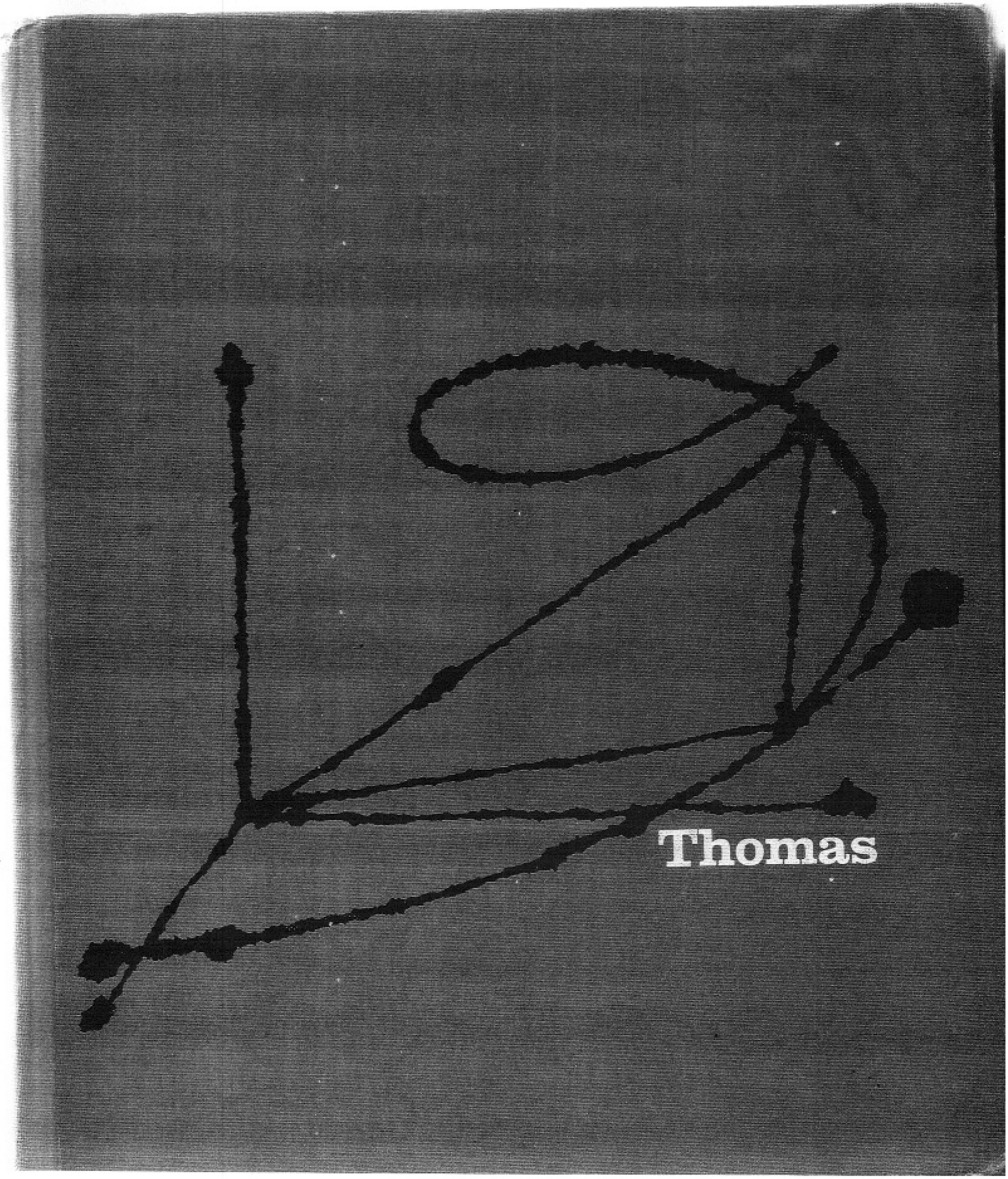

Thomas

GEORGE B. THOMAS, JR.
Department of Mathematics
Massachusetts Institute of Technology

CALCULUS AND ANALYTIC GEOMETRY

FOURTH EDITION

ADDISON-WESLEY PUBLISHING COMPANY
Reading, Massachusetts
Menlo Park, California · London · Don Mills, Ontario

This book is in the
Addison-Wesley Series in Mathematics

Second printing, December 1969

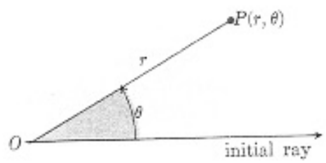
 Library of Congress Catalog Card No. 68-17568.

POLAR COORDINATES

CHAPTER 11

11.1 THE POLAR COORDINATE SYSTEM

We know that a point can be located in a plane by giving its abscissa and ordinate relative to a given coordinate system. Such x- and y-coordinates are called *Cartesian* coordinates, in honor of the French mathematician-philosopher René Descartes* (1596–1650), who is credited with discovering this method of fixing the position of a point in a plane.



11.1

Another useful way to locate a point in a plane is by *polar coordinates* (see Fig. 11.1). First, we fix an *origin* O and an *initial ray*† from O. The point P has polar coordinates r, θ, with

$$r = \text{directed distance from } O \text{ to } P, \quad (1a)$$

and

$$\theta = \text{directed angle from initial ray to } OP. \quad (1b)$$

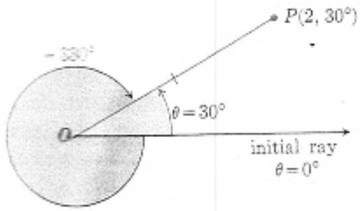
As in trigonometry, the angle θ is *positive* when measured counterclockwise and negative when measured clockwise (Fig. 11.1). But the angle associated with a given point is not unique (Fig. 11.2). For instance, the point 2 units from the origin, along the ray $\theta = 30°$, has polar coordinates $r = 2$, $\theta = 30°$. It also has coordinates $r = 2$, $\theta = -330°$, or $r = 2$, $\theta = 390°$.

There are occasions when we wish to allow r to be negative. That's why we say "directed distance"

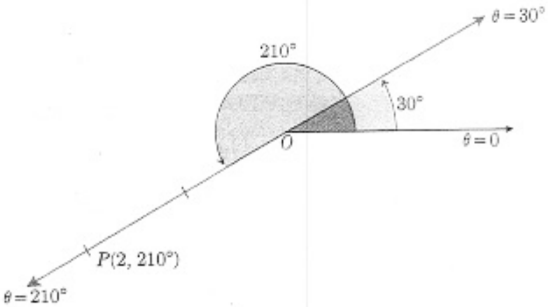
---

* For an interesting biographical account together with an excerpt from Descartes' own writings, see *World of Mathematics*, Vol. 1, pp. 235–253.

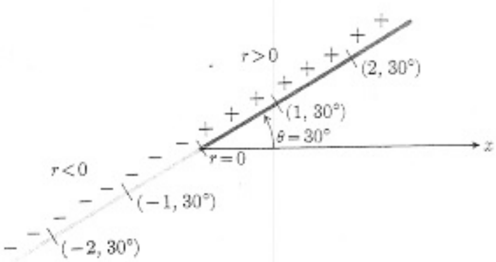
† A *ray* is a half-line consisting of a vertex and points of a line on one side of the vertex. For example, the origin and positive x-axis is a ray. The points on the line $y = 2x + 3$ with $x \geq 1$ is another ray; its vertex is (1, 5).



11.2 The ray $\theta = 30°$ is the same as the ray $\theta = -330°$.



11.3 The rays $\theta = 30°$ and $\theta = 210°$ make a line.



11.4 The terminal ray $\theta = \pi/6$ and its negative.

in Eq. (1a). The ray $\theta = 30°$ and the ray $\theta = 210°$ together make up a complete line through O (see Fig. 11.3). The point $P(2, 210°)$ 2 units from O on the ray $\theta = 210°$ has polar coordinates $r = 2$, $\theta = 210°$. It can be reached by a person standing at O and facing out along the initial ray, if he first turns 210° counterclockwise, and then goes forward 2 units. He would reach the same point by turning only 30° counterclockwise from the initial ray and then going *backward* 2 units. So we say that the point also has polar coordinates $r = -2$, $\theta = 30°$.

Whenever the angle between two rays is 180°, the rays actually make a straight line. We then say that either ray is the negative of the other. Points on the ray $\theta = \alpha$ have polar coordinates (r, α) with $r \geq 0$. Points on the negative ray, $\theta = \alpha + 180°$ have coordinates (r, α) with $r \leq 0$. The origin is $r = 0$. (See Fig. 11.4 for the ray $\theta = 30°$ and its negative. A word of caution: The "negative" of the ray $\theta = 30°$ is the ray $\theta = 30° + 180° = 210°$ and *not* the ray $\theta = -30°$. "Negative" refers to the directed distance r.)

There is a great advantage in being able to use both polar and Cartesian coordinates at once. To do this, we use a common origin and take the initial ray as the positive x-axis, and take the ray $\theta = 90°$ as the positive y-axis. The coordinates, shown in Fig. 11.5, are then related by the equations

$$x = r \cos \theta, \quad y = r \sin \theta. \qquad (2)$$

These are the equations that define $\sin \theta$ and $\cos \theta$ when r is positive. They are also valid if r is negative, because

$$\cos(\theta + 180°) = -\cos \theta,$$
$$\sin(\theta + 180°) = -\sin \theta,$$

so positive r's on the $(\theta + 180°)$-ray correspond to negative r's associated with the θ-ray. When $r = 0$, then $x = y = 0$, and P is the origin.

If we impose the condition

$$r = a \quad (a \text{ constant}), \qquad (3)$$

then the locus of P is a circle with center O and radius a, and P describes the circle once as θ varies from 0 to 360° (see Fig. 11.6). On the other hand, if we let r vary and hold θ fixed, say

$$\theta = 30°, \qquad (4)$$

the locus of P is the straight line shown in Fig. 11.4.

11.5 Polar and C…

11.6 The circle r =…

We adopt the c… number, $-\infty < r$… $x = 0, y = 0$ in…

$r =$

the origin, $x =$… The same point… different ways in… the point $(2, 30°)$… representations: (… $(-2, -150°)$. The… the two formula…

$(2, 30° + n$…
$(-2, 210° + n$…

…if we represent… formulas

$(2, \frac{1}{6}\pi + 2n$…
$(-2, \frac{7}{6}\pi + 2n$…

DOCKET ALARM

Explore Litigation Insights

Docket Alarm provides insights to develop a more informed litigation strategy and the peace of mind of knowing you're on top of things.

Real-Time Litigation Alerts

Keep your litigation team up-to-date with **real-time alerts** and advanced team management tools built for the enterprise, all while greatly reducing PACER spend.

Our comprehensive service means we can handle Federal, State, and Administrative courts across the country.

Advanced Docket Research

With over 230 million records, Docket Alarm's cloud-native docket research platform finds what other services can't. Coverage includes Federal, State, plus PTAB, TTAB, ITC and NLRB decisions, all in one place.

Identify arguments that have been successful in the past with full text, pinpoint searching. Link to case law cited within any court document via Fastcase.

Analytics At Your Fingertips

Learn what happened the last time a particular judge, opposing counsel or company faced cases similar to yours.

Advanced out-of-the-box PTAB and TTAB analytics are always at your fingertips.

API

Docket Alarm offers a powerful API (application programming interface) to developers that want to integrate case filings into their apps.

LAW FIRMS

Build custom dashboards for your attorneys and clients with live data direct from the court.

Automate many repetitive legal tasks like conflict checks, document management, and marketing.

FINANCIAL INSTITUTIONS

Litigation and bankruptcy checks for companies and debtors.

E-DISCOVERY AND LEGAL VENDORS

Sync your system to PACER to automate legal marketing.

WHAT WILL YOU BUILD? | **sales@docketalarm.com** | **1-866-77-FASTCASE**

fastcase®
Smarter legal research.