

# Graph-based Codes and Iterative Decoding

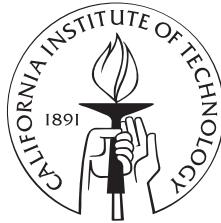
Thesis by

Aamod Khandekar

In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy



California Institute of Technology

Pasadena, California

2002

(Submitted June 10, 2002)

© 2002

Aamod Khandekar

All Rights Reserved

## Acknowledgements

I would like to begin by expressing my heartfelt gratitude to my advisor, Prof. Robert McEliece, for his active guidance and support throughout my stay at Caltech. His infectious enthusiasm and his many insights have been invaluable for the progress of my research career.

I would like to thank Prof. Robert McEliece, Prof. P. P. Vaidyanathan, Prof. Jehoshua (Shuki) Bruck, Prof. John Preskill, Prof. Babak Hassibi and Dr. Dariush Divsalar for serving on my candidacy and defense committees, and for their advice on this thesis.

I am grateful to all the current and former occupants of Moore 155, and to current and former residents of Braun graduate house, for making my stay at Caltech a very pleasant one.

## Abstract

The field of error correcting codes was revolutionized by the introduction of turbo codes [7] in 1993. These codes demonstrated dramatic performance improvements over any previously known codes, with significantly lower complexity. Since then, much progress has been made towards understanding the performance of these codes, as well as in using this understanding to design even better codes.

This thesis takes a few more steps in both these directions. We develop a new technique, called the typical set bound, for analyzing the asymptotic performance of code ensembles based on their weight enumerators. This technique yields very tight bounds on the maximum-likelihood decoding threshold of code ensembles, and is powerful enough to reproduce Shannon's noisy coding theorem for the class of binary-input symmetric channels.

We also introduce a new class of codes called irregular repeat-accumulate (IRA) codes, which are adapted from the previously known class of repeat-accumulate (RA) codes. These codes are competitive in terms of decoding performance with the class of irregular low-density parity-check (LDPC) codes, which are arguably the best class of codes known today, at least for long block lengths. In addition, IRA codes have a significant advantage over irregular LDPC codes in terms of encoding complexity.

We also derive an analytical bound regarding iterative decoding thresholds of code ensembles on general binary-input symmetric channels, an area in which theoretical results are currently lacking.

# Contents

Acknowledgements	iii
Abstract	iv
<b>1 Introduction</b>	<b>1</b>
1.1 Some Basic Concepts . . . . .	2
1.1.1 Channel Models . . . . .	2
1.1.2 Codes and Code Ensembles . . . . .	6
1.1.3 Decoding Algorithms . . . . .	8
1.2 Some Graphical Code Ensembles . . . . .	9
1.2.1 Parallel Concatenation of Convolutional Codes (PCCC) . . . . .	10
1.2.2 Serial Concatenation of Convolutional Codes (SCCC) . . . . .	11
1.2.3 Codes Defined on Tanner Graphs . . . . .	12
1.2.4 Decoding on Tanner Graphs . . . . .	13
1.2.5 Low-Density Parity-Check (LDPC) Codes . . . . .	15
1.2.6 Repeat Accumulate (RA) Codes . . . . .	15
1.3 Density Evolution . . . . .	17
1.3.1 Density Evolution on the BEC . . . . .	18
1.4 Thesis Outline . . . . .	19
<b>2 The Typical Set Bound</b>	<b>21</b>
2.1 The Union Bound . . . . .	22
2.2 The Typical Set Decoder . . . . .	24
2.3 The Typical Set Bound . . . . .	28

# Explore Litigation Insights

Docket Alarm provides insights to develop a more informed litigation strategy and the peace of mind of knowing you're on top of things.

## Real-Time Litigation Alerts



Keep your litigation team up-to-date with **real-time alerts** and advanced team management tools built for the enterprise, all while greatly reducing PACER spend.

Our comprehensive service means we can handle Federal, State, and Administrative courts across the country.

## Advanced Docket Research



With over 230 million records, Docket Alarm's cloud-native docket research platform finds what other services can't. Coverage includes Federal, State, plus PTAB, TTAB, ITC and NLRB decisions, all in one place.

Identify arguments that have been successful in the past with full text, pinpoint searching. Link to case law cited within any court document via Fastcase.

## Analytics At Your Fingertips



Learn what happened the last time a particular judge, opposing counsel or company faced cases similar to yours.

Advanced out-of-the-box PTAB and TTAB analytics are always at your fingertips.

## API

Docket Alarm offers a powerful API (application programming interface) to developers that want to integrate case filings into their apps.

## LAW FIRMS

Build custom dashboards for your attorneys and clients with live data direct from the court.

Automate many repetitive legal tasks like conflict checks, document management, and marketing.

## FINANCIAL INSTITUTIONS

Litigation and bankruptcy checks for companies and debtors.

## E-DISCOVERY AND LEGAL VENDORS

Sync your system to PACER to automate legal marketing.