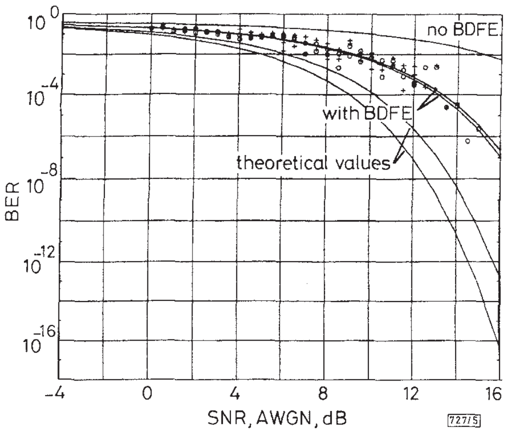

Fig. 5 *Improvement of response of PPM (+) and IPPM(○) in AWGN with a BDFE equaliser in the receiver*

Measures: The results of the previous Figures have been obtained on an emitting/receiving 4-IPPM 2Mbit/s circuit prototype. A standard PPM codifier is added to a pulse conformation block that takes eight samples stored in a look-up table to give the form to the pulse. It has been implemented on an FPGA ALTERA EPM 5064 JC-1. The D/A is a simple and inexpensive DAC08. As the optical emitter we use a Siemens SFH 477 IRED with an analogue driver. As the receiver we used a Hamamatsu C5331 APD. The decissor is a block maximum likelihood one.

Conclusions: An alternative to the classic PPM signal has been outlined. IPPM requires a bandwidth that is 30% of the basic PPM bandwidth. IPPM improves the response of PPM against jitter. The global performances of the system against AWGN and low frequency interference are similar to PPM and can also be improved, introducing a BDFE equaliser in the receiver.

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Serial concatenation of block and convolutional codes

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Indexing terms: Convolutional codes, Block codes

Parallel concatenated coding schemes employing convolutional codes as constituent codes linked by an interleaver have been proposed in the literature as 'turbo codes'. They yield very good performance in connection with simple suboptimum decoding algorithms. The authors propose an alternative scheme consisting in the serial concatenation of block or convolutional codes and evaluate its average performance in terms of bit error probability.

Introduction: Since their appearance in [1], 'turbo codes' have been the object of great interest, and consequently of wide investigation, in the coding community.

In two previous Letters [2, 3] we have shown how to evaluate the performance of parallel concatenated coding schemes using as constituent codes both block and convolutional codes. Here, we analyse an alternative to the parallel concatenation, which consists in the serial concatenation of two constituent codes (CCs) separated by an interleaver of length N. We call the obtained concatenated codes SCBC (serial concatenated block codes) or SCCC (serial concatenated convolutional codes) according to the nature of the CCs. We derive an upper bound to the maximum likelihood bit error probability of SCBC and SCCC codes and show with examples that the new scheme outperforms turbo codes. In a companion letter, still in preparation, we will deal with the algorithms for iterative decoding.

For simplicity of the exposition, we will present the methodology in connection with SCBC, and then extend it to the more complicated case of SCCC.

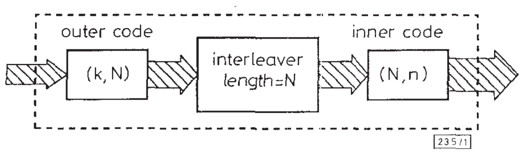

Fig. 1 *Serially concatenated block code*

Serially concatenated block codes: The scheme of two serially concatenated block codes is shown in Fig. 1. It is composed of two cascaded CCs, the outer (k, N) code and the inner (N, n) code, linked by an interleaver of length N. The overall SCBC is then a (k, n) code.

As in [2, 3], a crucial step in the analysis consists in replacing the actual interleaver that performs a permutation of the N input bits with an abstract interleaver called a uniform interleaver, defined as a probabilistic device that maps a given input word of weight l into all distinct $\binom{N}{l}$ permutations of it with equal probability $1/\binom{N}{l}$. Use of the uniform interleaver leads to a much easier computation of the average performance of SCBC, intended as the expectation over the ensemble of all interleavers of a given length of the performance of any SCBC with the same CCs.

Let us define the input-output weight enumerating function (IOWEF) of the SCBC as

$$A^{C_S}(W, H) = \sum_{w,h} A^{C_S}_{w,h} W^w H^h \qquad (1)$$

where $A_{w,h}{}^{C_S}$ is the number of codewords of the SCBC with weight h associated to an input word of weight w.

We also define the conditional weight enumerating function (CWEF) $A^{C_S}(w, H)$ of the SCBC as the weight distribution of the codewords of the SCBC conditioned to a given weight w of the input word. It is related to the IOWEF by

$$A^{C_S}(w, H) = \frac{1}{w!} \frac{\partial^w A^{C_S}(W, Z)}{\partial W^w}\bigg|_{W=0} \qquad (2)$$

Knowledge of the CWEF permits us to obtain an upper bound to the bit error probability of the SCBC in the form [3]

$$P_b(e) \leq \sum_{w=1}^{k} \frac{w}{k} A^{C_S}(w, H)\big|_{H=e^{-R_c E_b/N_0}} \qquad (3)$$

where $R_c = k/n$ is the code rate and E_b/N_0 is the signal/noise ratio per bit.

The problem thus consists in the evaluation of the CWEF of the SCBC from the knowledge of the CWEFs of the two CCs, which we call $A^{C_1}(w, L)$ and $A^{C_2}(l, H)$. To do this, we exploit the properties of the uniform interleaver, which transforms a codeword of weight l at the output of the first encoder into all its distinct $\binom{N}{l}$ permutations. As a consequence, each codeword of the outer code C_1 of weight l generates $\binom{N}{l}$ codewords of the inner code C_2 so that the number $A_{w,h}^{C_S}$ of codewords of the SCBC of weight h associated with an input word of weight w is given by

$$A_{w,h}^{C_S} = \sum_{l=1}^{N} \frac{A_{w,l}^{C_1} \times A_{l,h}^{C_2}}{\binom{N}{l}} \qquad (4)$$

From eqn. 4 we easily derive the expressions of the IOWEF and CWEF of the SCBC:

$$A^{C_S}(W, H) = \sum_{l=1}^{N} \frac{A^{C_1}(W, l) \times A^{C_2}(l, H)}{\binom{N}{l}} \qquad (5)$$

$$A^{C_S}(w, H) = \sum_{l=1}^{N} \frac{A_{w,l}^{C_1} \times A^{C_2}(l, H)}{\binom{N}{l}} \qquad (6)$$

where $A^{C_1}(W, l)$ is the conditional weight distribution of the input word that generates codewords of the outer code of weight l.

Eqns. 5 and 6 can be generalised easily to the case of an interleaver of length mN, that is an integer multiple of the length of the outer codewords [2] and to the case of more than two concatenated codes.

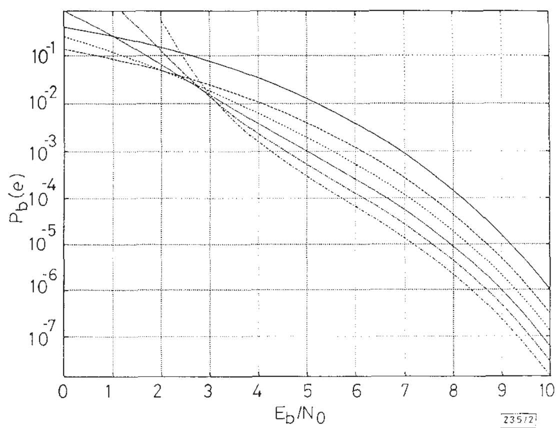

Fig. 2 *Performance of serially concatenated block codes with different interleaver lengths*

——— no interleaver ——·—— $m = 4$
– – – – $m = 1$ — · — $m = 8$
·········· $m = 2$ - · - · - · - $m = 16$

As an example, consider the parity check code (4, 3) concatenated through an interleaver of length $4m$ to a Hamming code (7, 4). From the IOWEF $A^{C_1}(W, L)$ and $A^{C_2}(L, H)$ of the outer and inner code first and second we have derived the upper bounds to the bit error probability plotted in Fig. 2 for various values of the integer m. For the sake of comparison, the curve relative to the simple concatenation of the two codes with the identity interleaver of length 4 is also shown. The curves show the gain obtainable with the interleaver, and do not exhibit the fast gain saturation of the parallel concatenation of block codes [2].

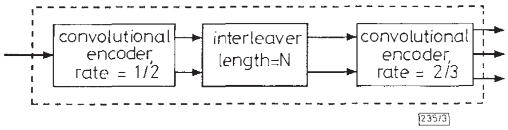

Fig. 3 *Serially concatenated convolutional code*

SCCC rate = 1/3

Serially concatenated convolutional codes: The structure of a serially concatenated convolutional code (SCCC) is shown in Fig. 3. It refers to the case of two convolutional CCs of rate 1/2 and 2/3 joined by an interleaver of length N generating a rate 1/3 SCCC. The exact analysis of this scheme requires, as illustrated in [3], the use of a hypertrellis having as states pairs of states of the outer and inner code. A much simpler approximation can be used, however, when the length of the interleaver is much greater than the constraint length of the CCs: it consists in retaining only the branch of the hypertrellis joining the all-zero hyperstates.

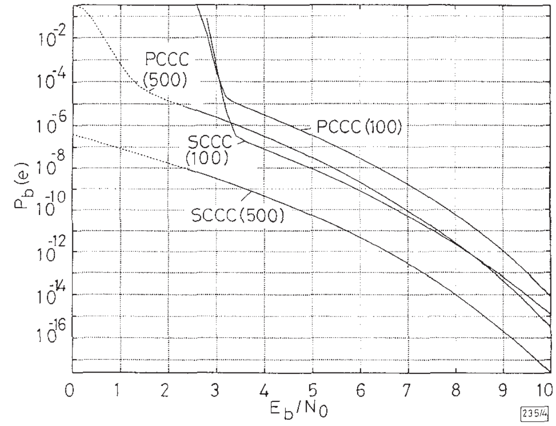

Fig. 4 *Performance of serially and parallel concatenated convolutional codes*

As an example, consider a rate 1/2 SCCC formed by an outer rate 1/2 4-state convolutional code and an inner 4-state rate 2/3 convolutional code, joined by a uniform interleaver of length N = 100 and 500. Using the previously outlined analysis, we have obtained the bit error probability bound shown in Fig. 4, where we have also reported the performance of a turbo code with the same overall rate of 1/3 obtained through the concatenation of two equal 4-state CCs of rate 1/2 and an interleaver of the same length as for the SCCC. A comparison of the two curves leads to the conclusion that the SCCC outperforms the turbo code by more than 1 dB. Moreover, the coding gain tends to increase more sensibly for the SCCC with the interleaving length. The dotted part of the curves reflects the uncertainty due to the limited number of terms considered in the computation of the union bound.

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