UNITED STATES PATENT AND TRADEMARK OFFICE
BEFORE THE PATENT TRIAL AND APPEAL BOARD

In re: Inter Partes Review of:	:	
U.S. Pat. No. 7,116,710	:	
U.S. Pat. No. 7,421,032	:	
U.S. Pat. No. 7,421,781	:	
and U.S. Pat. No. 8,284,833	:	
Inventor: Hui Jin, et al	:	IPR No. Unassigned
Assignee: California Institute of Technology		

Common Title: **Serial Concatenation of Interleaved Convolutional Codes Forming Turbo-Like Codes**

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Submitted Electronically via the Patent Review Processing System

DECLARATION OF PAUL H. SIEGEL

<u>Declaration of Paul H. Siegel</u>

I, **Paul H. Siegel**, declare as follows:

1. I am over the age of 18 and am legally competent to make this declaration. I make this declaration based upon my own personal knowledge.

2. I am currently a Professor at the University of California, San Diego, in the Department of Electrical and Computer Engineering, the Jacobs School of Engineering, and the Center for Memory and Recording Research.

3. I have been asked to provide a statement of certain facts related to work I did together with Henry D. Pfister relating to the serial concatenation of rate-1 codes through uniform random interleavers.

4. Beginning in early 1999, I collaborated with Dr. Pfister, then a Ph.D student of mine, to show that improved error-correcting codes can be constructed from simple components, *e.g.*, by serially concatenating an arbitrary outer code of rate $r < 1$ and m identical rate-1 inner codes.

5. The result of that collaboration was a presentation at the 1999 Allerton Conference on Communications, Control and Computing, in Allerton, Illinois ("1999 Allerton Conference") in September 1999. This conference was held September 22-24, 1999, and was open to the public for attendance. Any person who wanted to attend and was able to pay the attendance fee could

attend. The 1999 Allerton Conference was considered one of two primary conferences on the topic of iterative decoding during the time. The 1999 Allerton conference itself, as well as the proceedings that took place there, were publicized and generally known to those who were interested in topics relating to error-correcting codes and iterative decoding.

6. At the 1999 Allerton Conference, I presented a series of slides relating to my work with Dr. Pfister on a class of codes based on serial concatenation of a rate-r code with $m \geq 1$, uniformly interleaved rate-1 codes. In these slides, I discussed an example of these codes, called a "Repeat-Accumulate-Accumulate" (or "RAA") code. An RAA code is is similar to an RA code (Divsalar, et al., Allerton '98) to which an additional accumulator has been added. I compared RAA codes to RA codes and concluded that the RAA codes were able to achieve a lower word-error-probability than the RA codes, demonstrating that the performance of RA codes could be improved by including an additional accumulator. This presentation was entitled "The Serial Concatenation of Rate-1 Codes Through Uniform Random Interleavers," and I presented it in the IIA: Coding Theory: Iterative Decoding and Turbo Codes Session on September 22, 1999, the first day of the conference. A true and accurate copy of the slides that I presented at the 1999 Allerton Conference is attached as Exhibit 1.

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information or belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code.

Dated: 11/9/16 ______________ Paul H. Siegel ______________

Paul H. Siegel

Exhibit 1

The Serial Concatenation of Rate-1 Codes Through Uniform Random Interleavers

Henry Pfister and Paul H. Siegel
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University of California, San Diego

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Allerton Conference
September 22-24 , 1999

University of California
San Diego

Outline

- Union Bounds and Code Performance
- Serial Concatenation and Repeat-Accumulate (RA) Codes
- Serial Concatenation of Rate-1 Codes
- Repeat-Accumulate-Accumulate (RAA) Codes
- Summary

Union Bounds on Performance

- Rate $r = k/n$, linear block code C
- Input Output Weight Enumerator Function (IOWEF) :

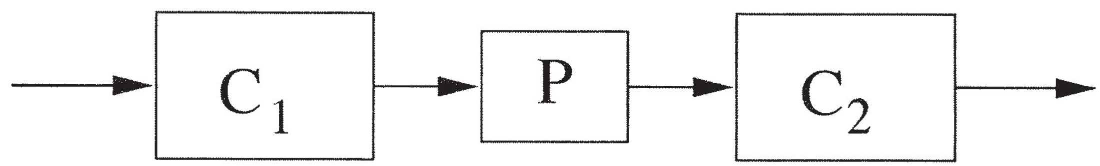
$$A_{w,h} \stackrel{\text{def}}{=} \#\text{ codewords, input weight } w\text{, output weight } h$$

- Union bound on word error probability P_W
(binary-input, memoryless channel, maximum-likelihood decoding):

$$P_W \leq \sum_{h=1}^{n} \sum_{w=1}^{k} A_{w,h} z^h$$

- z is channel dependent; e.g., for Gaussian channel, $z = e^{-r(E_b/N_0)}$.
- For ensembles, replace $A_{w,h}$ by average IOWEF $\overline{A_{w,h}}$

Serial Concatenation through a Uniform Interleaver

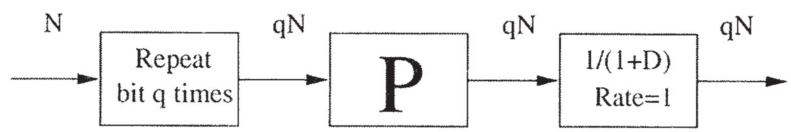


- Let C_1, C_2 be (n_1, k_1), (n_2, k_2) linear block codes with $n_1 = k_2$, and IOWEFs $A^{(1)}_{w,h}$, $A^{(2)}_{w,h}$.

- Let C be the (n_2, k_1) code obtained by serial concatenation of C_1 and C_2 through a uniform interleaver of size n_1, with average IOWEF $A_{w,h}$:

$$A_{w,h} = \sum_{h_1=0}^{n_1} A^{(1)}_{w,h_1} \cdot \frac{A^{(2)}_{h_1,h}}{\binom{n_1}{h_1}}$$

Repeat-Accumulate (RA) Codes
(Divsalar, et al., Allerton'98)



- Repeat input block $x_1 x_2 \cdots x_N$ a total of q times.
- Permute with random interleaver P of size $n = qN$.
- Accumulate over block:

$$u_1 u_2 \cdots u_n \longrightarrow v_1 v_2 \cdots v_n$$

$$v_1 = u_1$$

$$v_2 = u_1 + u_2$$

$$\vdots$$

$$v_n = u_1 + \cdots + u_n$$

A Simple Example

- Rate-1, $(n, k) = (3, 3)$ Accumulate code:

Input Bits	Input Weight w	Output Bits	Output Weight h
000	0	000	0
001	1	001	1
010	1	011	2
100	1	111	3
011	2	010	1
101	2	110	2
110	2	100	1
111	3	101	2

RA Weight Enumerator
(Divsalar, et al., Allerton'98)

- (qN, N) Repeat Code

$$A^{(1)}_{w,h} = \begin{cases} 0 & if \quad h \neq qw \\ \binom{qN}{w} & if \quad h = qw \end{cases}$$

(n, n) Accumulate Code (cf. 'Oberg and Siegel, Allerton'98)

$$A^{(2)}_{w,h} = \binom{n-h}{\lfloor w/2 \rfloor} \binom{h-1}{\lceil w/2 \rceil - 1}$$

- (qN, N) RA Code

$$\overline{A}_{w,h} = \frac{\binom{N}{w}\binom{qN-h}{\lfloor qw/2 \rfloor}\binom{h-1}{\lceil qw/2 \rceil - 1}}{\binom{qN}{qw}} \; if \; w, h > 0$$

RA Coding Theorem

(Divsalar, et al., Allerton'98)

- **Theorem:** For $q \geq 3$, there exists $\gamma_q > 0$ such that, for any $E_b/N_0 > \gamma_q$, as the block length N becomes large,

$$P_W^{UB} = O(N^\beta),$$

where $\beta = -\left\lceil \frac{q-2}{2} \right\rceil$. Hence $P_W \rightarrow 0$ as $N \rightarrow \infty$.

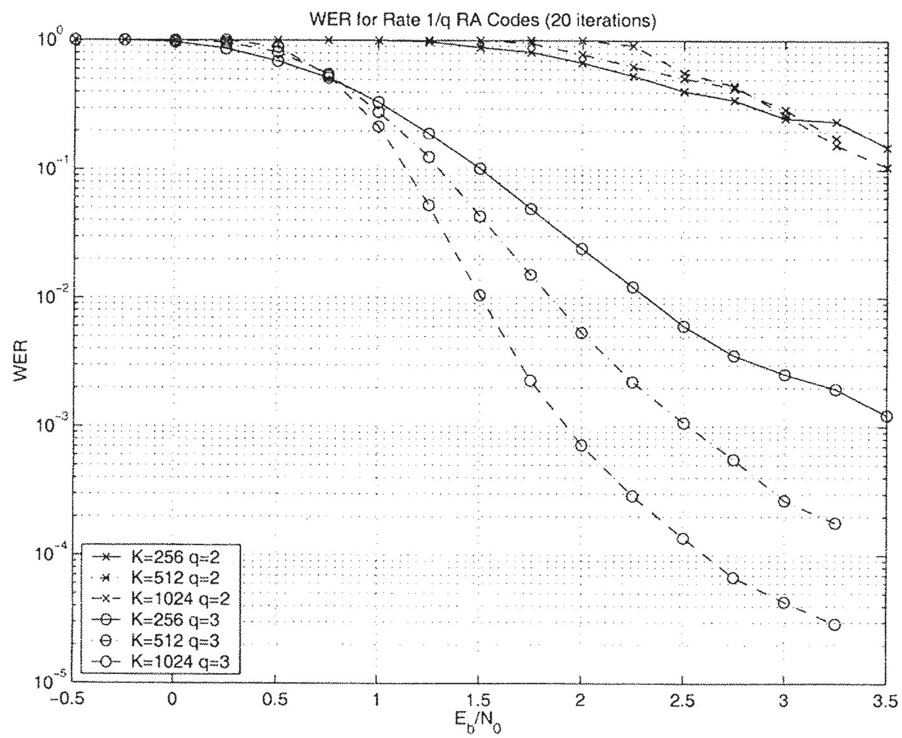
- Example γ_q estimates (from Viterbi-Viterbi Bound):

$$\gamma_3 \approx 1.112 \text{ dB}$$

$$\gamma_4 \approx 0.313 \text{ dB}$$

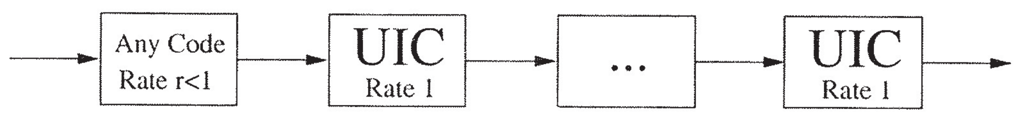
Performance of RA Codes

- RA codes, $r = \frac{1}{3}$ and $r = \frac{1}{2}$, with iterative decoding.



Serial Concatenation

- Consider an encoder architecture of the following form:



- UIC represents a Uniform Interleaver in cascade with a rate-1 Code (e.g., an (n, n) Accumulate Code).

- We want to characterize the average IOWEF when the code is concatenated with m UIC's.

Serial Concatenation with UIC's

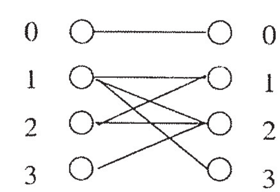
- Let IOWEF for rate-1, (n, n) code be $A = [A_{i,j}], \quad 0 \le i, j \le n.$

- Define IOW Transition Probabilities (IOWTP): $P = [P_{i,j}], \quad 0 \le i, j \le n,$

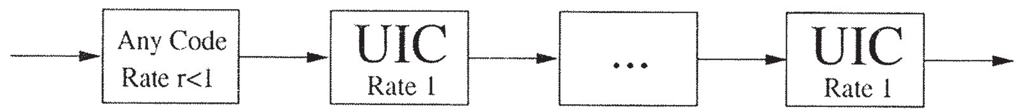
$$P_{i,j} = Pr(h = j | w = i) = \frac{A_{i,j}}{\binom{n}{i}}.$$

- Rate-1, $(n, k) = (3, 3)$ Accumulate code:

$$[P_{i,j}] = [Pr(h = j | w = i)] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \\ 0 & 0.\overline{6} & 0.\overline{3} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



Serial Concatenation with UIC's



- Outer (n, k) code C, IOWEF $C_{w,h}$.
- Cascade of m, rate-1 (n, n) UIC's, each with IOWTP $P = [P_{i,j}]$.
- Average IOWEF $\overline{A_{w,h}}$ of serial concatenation:

$$\overline{A_{w,h}} = \sum_{h_1=0}^{n} C_{w,h_1} [P^m]_{h_1,h}$$

Perron-Frobenius Theory

- **Theorem:** An irreducible stochastic matrix P has a unique stationary distribution π s.t. $\pi P = \pi$ and $\sum \pi_i = 1$.

- **Theorem:** An irreducible aperiodic (primitive) stochastic matrix P with unique stationary distribution π satisfies:

$$\lim_{m \to \infty} P^m = \begin{bmatrix} \pi \\ \vdots \\ \pi \end{bmatrix}$$

- A linear rate-1 code is *primitive* if $P^+ = [P_{i,j}],\ i,\ j \neq 0,$ is a primitive matrix.

A Simple Example (cont.)

- Rate-1, $(n, k) = (3, 3)$ Accumulate Code:

$$\begin{pmatrix} 0 & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \\ 0 & 0.\overline{6} & 0.\overline{3} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1+2}{7} & \frac{1+1+1}{7} & \frac{1}{7} \end{pmatrix}$$

$$\lim_{m \to \infty} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \\ 0 & 0.\overline{6} & 0.\overline{3} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}^m = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 3/7 & 3/7 & 1/7 \\ 0 & 3/7 & 3/7 & 1/7 \\ 0 & 3/7 & 3/7 & 1/7 \end{pmatrix}$$

Stationary Distributions and Rate-1 Codes

- **Proposition:** Consider the SC of m primitive rate-1 codes through uniform interleavers. For non-zero input weights and as $m \rightarrow \infty$, the output weight distribution is independent of the input weight distribution and is

$$\pi_h = \frac{\binom{n}{h}}{2^n - 1}, \quad for \quad h \neq 0.$$

- The ensemble averaged OWEF for any rate $r < 1$ code SC with $m \rightarrow \infty$ primitive rate-1 codes is

$$\overline{A_h} = \frac{2^{rn} - 1}{2^n - 1}\binom{n}{h}$$

Iterated Rate-1 Codes Make Good Codes

- **Proposition:** For a code randomly chosen from an ensemble with average OWE $\overline{A_h}$,

$$Pr(d_{min} < d) \leq \sum_{h=1}^{d-1} \overline{A_h}.$$

(cf. Gallager, 1963)

- For length n, rate $r < 1$,and $\epsilon > 0$, let $d^*(n, r, \epsilon)$ be the largest value of d satisfying

$$\sum_{h=0}^{d-1} \binom{n}{h} \leq \frac{2^n}{2^{rn} - 1}\epsilon.$$

Iterated Rate-1 Codes Make Good Codes

- **Proposition:** For the ensemble of length-n codes obtained by serial concatenation of a rate-r linear block code with $m \to \infty$ primitive rate-1 linear codes through uniform random interleavers,

$$Pr(d_{min} < d^*(n, r, \epsilon)) \leq \epsilon.$$

- **Corollary:** Let $\epsilon = (2^{(1-r)n} - 1)(2^{rn} - 1)/(2^n - 1)$. Then,

$$Pr(d_{min} < d^*) \leq \epsilon < 1,$$

where $d^*$is the largest value of d satisfying

$$\sum_{h=0}^{d-1} \binom{n}{h} \leq 2^{n(1-r)} = 2^{n-k}$$

(i.e., at least one code satisfies the Gilbert-Varshamov Bound).

Bounds for CA^m Codes

- Rate r<1, outer code C.
- Serial concatenation with m uniformly interleaved Accumulate codes.
- Compute largest value d^* satisfying

$$\sum_{h=1}^{d-1} \overline{A_h} < \frac{1}{2}.$$

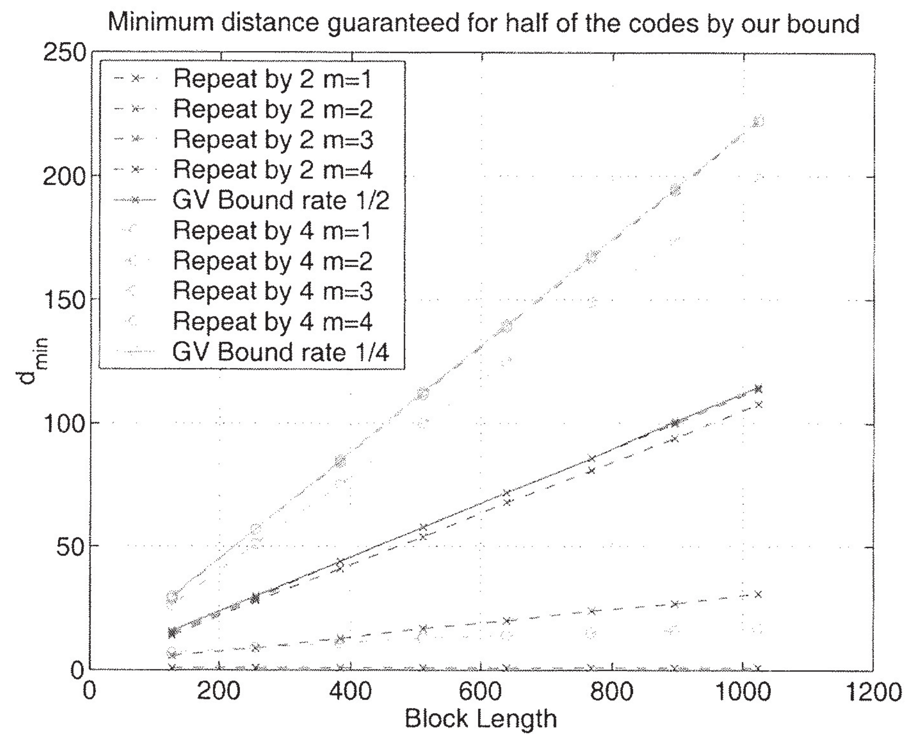
- Then, by the bound,

$$Pr(d_{min} < d^*) < \frac{1}{2},$$

(i.e., at least half the codes satisfy $d_{min} \geq d^*$).

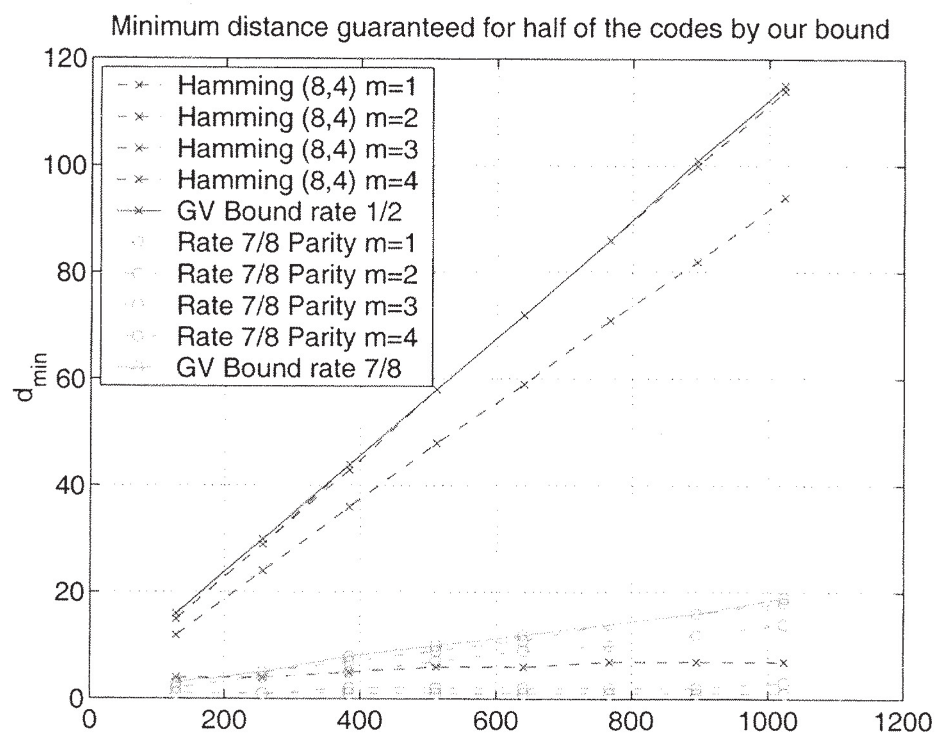
Numerical Results

- CA^m bounds for Repeat-2 and Repeat-4 codes, $1 \leq m \leq 4$.

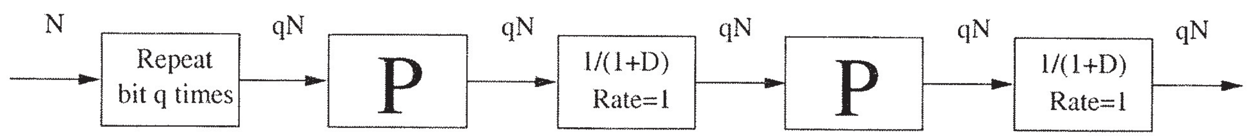


Numerical Results

- For (8,4) Hamming and rate 7/8 parity-check codes, $1 \leq m \leq 4$.



Rate 1/2, RAA Codes

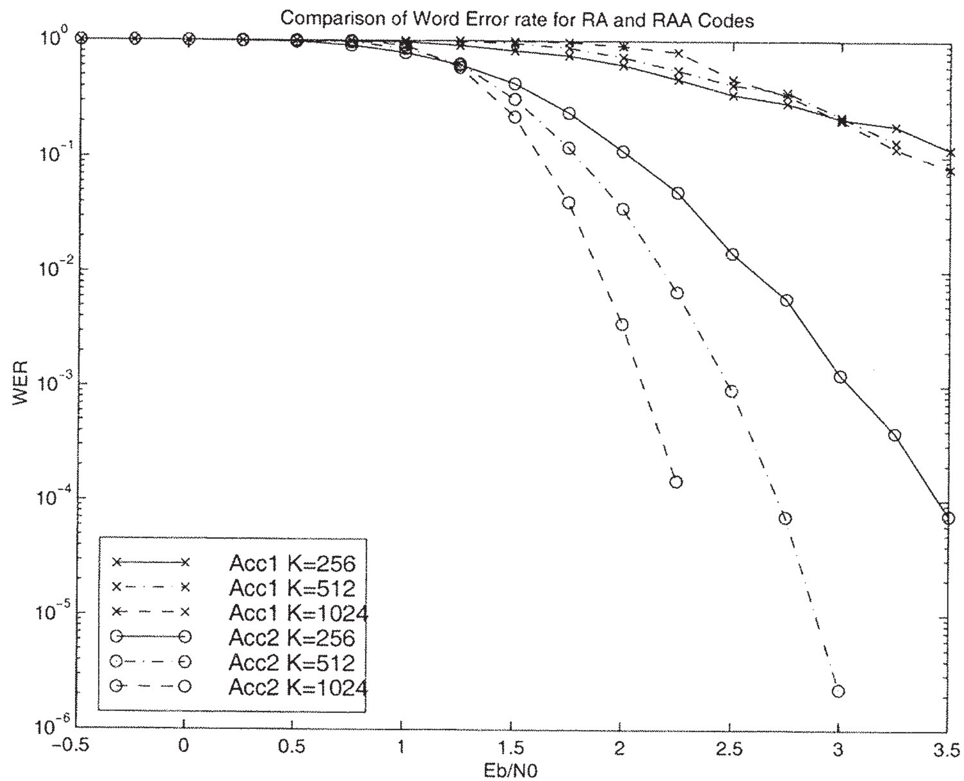


- For rate $r = 1/2$ RA code, analysis and simulations suggest that ***there is no word-error-probability interleaving gain***.

- For rate $r = 1/2$ RAA code, analysis and simulations suggest that ***there is...***

Rate 1/2, RA vs. RAA

- Word-error-probability P_W , from computer simulation.



Summary

- We investigated a new class of codes based upon serial concatenation of a rate-r code with $m \geq 1$, uniformly interleaved rate-1 codes.
- We analyzed the output weight enumerator function $\overline{A_h}$ for finite m, as well as asymptotically for $m \to \infty$.
- We evaluated the "goodness" of these ensembles in terms of the Gilbert-Varshamov Bound.
- We compared $r = 1/2$ RAA codes to RA codes.

  Question: For $r = 1/2$, RAA codes, is there a γ such that , for $E_b/N_0 > \gamma$, $P_W \to 0$ as $N \to \infty$?