Advanced Topics in Statistical Process Control

The Power of Shewhart's Charts

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Chapter Five

Three-Sigma Limits

"As indicated … the method of attack is to establish limits of variability …, such that, when [a value] is found outside these limits, looking for an assignable cause is worth while."

"We usually choose a symmetric range characterized by limits $\mu_\theta \pm t\,\sigma_\theta$."

"If more than one statistic is used, then the limits on all the statistics should be chosen so that the probability of looking for trouble when any one of the chosen statistics falls outside its own limits is economic."

"Experience indicates that $t = 3$ seems to be an acceptable economic value."

"Hence the method for establishing allowable limits of variation in a statistic depends upon theory to furnish the expected value and the standard deviation of the statistic and upon empirical evidence to justify the choice of limits

[expected value] ± t [standard deviation]

"Construct control charts with limits $\bar{\theta} \pm 3\,\hat{\sigma}_\theta$ for each statistic. If an observed point falls outside [these] limits, take this fact as an indication of trouble or lack of control."

W.A. Shewhart [25]

One of the foundations of Shewhart's control charts is the use of control limits which are set at a distance of three standard deviations on either side of the appropriate central line. Such limits are commonly referred to as "three-sigma" limits. Dr. Shewhart carefully

[25] *Economic Control of Quality of Manufactured Product* pp. 147-148, 277, 276, 277, and 304.

explained the rationale behind this choice in *Economic Control of Quality of Manufactured Product*. As shown by the quotations, this choice was neither arbitrary nor accidental. It was a deliberate choice, made because three-sigma limits provided the needed sensitivity without causing an unacceptable number of false alarms. In short, three-sigma limits were chosen because they provided an economic balance between the consequences of the two mistakes one can make when interpreting data. This choice has been thoroughly validated in practice. The purpose of this chapter is to provide some insight to why and how three-sigma limits work.

5.1 Why Three-Sigma Limits?

Three-sigma limits are not probability limits. While we will resort to some theory to demonstrate some of the properties of three-sigma limits, it is important to remember that there are other considerations which were used by Shewhart in selecting this criterion. As indicated by the quotations at the beginning of this chapter, the strongest justification of three-sigma limits is the empirical evidence that three-sigma limits work well in practice—that they provide effective action limits when applied to real world data. Thus, the following arguments cannot further justify the use of three-sigma limits, but they can reveal one of the reasons why they work so well.

While it is not a rigorous probabilistic argument, the Empirical Rule provides a useful way of characterizing data using a measure of location and a measure of dispersion.

THE EMPIRICAL RULE: Given a homogeneous set of data:

Part One: Roughly 60% to 75% of the data will be located within a distance of one standard deviation on either side of the mean.

Part Two: Usually 90% to 98% of the data will be located within a distance of two standard deviations on either side of the mean.

Part Three: Approximately 99% to 100% of the data will be located within a distance of three standard deviations on either side of the mean.

In order to display the robustness of the Empirical Rule six different probability models are used. All are constructed so as to have $MEAN(X) = 0$ and $SD(X) = 1.0$. Therefore, the interval defined by Part One of the Empirical Rule will go from –1.0 to 1.0, the interval defined by Part Two will range from –2.0 to 2.0, while the interval defined by Part Three will range from –3.0 to 3.0.

The three parts of the Empirical Rule are illustrated in Figures 5.1, 5.2, and 5.3.

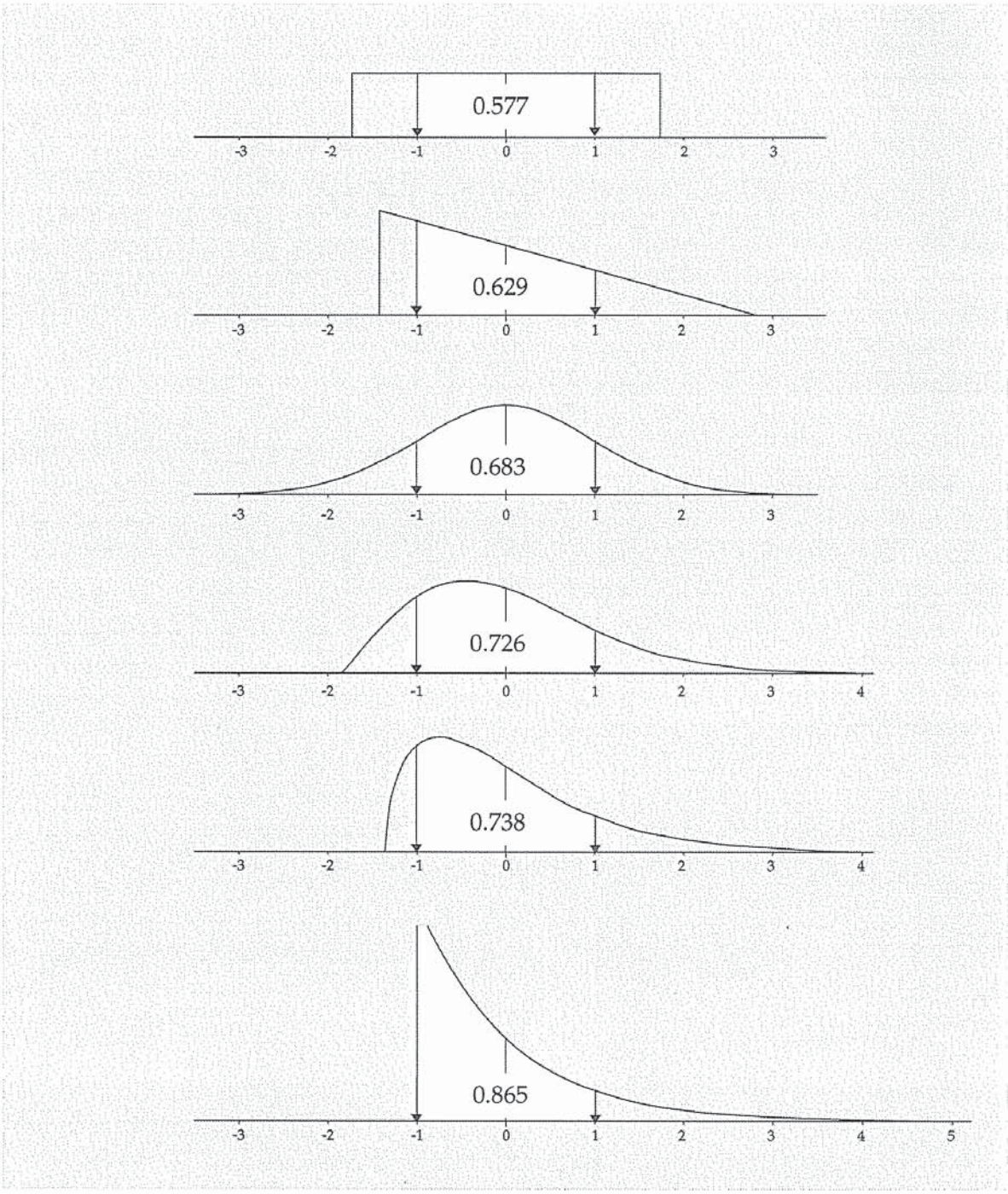

Figure 5.1: Part One of the Empirical Rule

Part One of the Empirical Rule is the weakest part. Only four of the six distributions shown in Figure 5.1 satisfy Part One. Nevertheless, Part One is still a useful guide for describing where the bulk of the distribution (or the data) will be.

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