

# DISCRETE-TIME SIGNAL PROCESSING



Alan V. Oppenheim · Ronald W. Schaffer

PRENTICE HALL SIGNAL PROCESSING SERIES  
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# **Discrete-Time Signal Processing**

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response of a certain class of discrete-time filters, the signal value at any time index is a linear function of (and thus linearly predictable from) previous values. Consequently, efficient signal representations can be obtained by estimating these prediction parameters and using them along with the prediction error to represent the signal. The signal can then be regenerated when needed from the model parameters. This class of signal coding techniques has been particularly effective in speech coding and is described in considerable detail in Jayant and Noll (1984), Markel and Gray (1976), and Rabiner and Schafer (1978).

Another advanced topic of considerable importance is adaptive signal processing. In this text the emphasis is almost entirely on linear time-invariant systems. Adaptive systems represent a particular class of time-varying and, in some sense, nonlinear systems with broad application and with established and effective techniques for their design and analysis. Again, many of these techniques build from the fundamentals of discrete-time signal processing covered in this text. Details of adaptive signal processing are given by Haykin (1986) and Widrow and Stearns (1985).

These represent only a few of the many advanced topics that extend from the topics covered in this text. Others include advanced and specialized filter design procedures, a variety of specialized algorithms for evaluation of the Fourier transform, specialized filter structures, and various advanced multirate signal processing techniques. An introduction to many of these advanced topics is contained in Lim and Oppenheim (1988).

It is often said that the purpose of a fundamental textbook should be to uncover rather than cover a subject, and in choosing the topics and depth of coverage in this book we have been guided by this philosophy. The preceding brief discussion of advanced topics and the Bibliography at the end of the book should be strongly suggestive of the rich variety of directions that these fundamentals begin to uncover.

### **Historical Perspective**

Discrete-time signal processing has a rich history. It has advanced in uneven steps over a long period of time. Since the invention of calculus in the 17th century, scientists and engineers have developed models to represent physical phenomena in terms of functions of continuous variables and differential equations. Numerical techniques have been used to solve these equations when analytical solutions are not possible. Indeed, Newton used finite-difference methods that are special cases of some of the discrete-time systems that we present in this text. Mathematicians of the 18th century, such as Euler, Bernoulli, and Lagrange, developed methods for numerical integration and interpolation of functions of a continuous variable. Interesting historical research by Heideman, Johnson, and Burrus (1984) showed that Gauss discovered the fundamental principle of the fast Fourier transform (discussed in Chapter 9) as early as 1805 — even before the publication of Fourier's treatise on harmonic series representation of functions.

Until the early 1950s, signal processing as we have defined it was typically done with analog systems that were implemented with electronic circuits or even with

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