Carrier and Bit Synchronization in Data Communication—A Tutorial Review

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Abstract—**This paper examines the problems of carrier phase estimation and symbol timing estimation for carrier-type synchronous digital data signals, with tutorial objectives foremost. Carrier phase recovery for suppressed-carrier versions of double sideband (DSB), vestigial sideband (VSB), and quadrature amplitude modulation (QAM) signal formats is considered first. Then the problem of symbol timing recovery for a baseband pulse-amplitude modulation (PAM) signal is examined. Timing recovery circuits based on elementary statistical properties are discussed as well as timing recovery based on maximum-likelihood estimation theory. A relatively simple approach to evaluation of timing recovery circuit performance in terms of rms jitter of the timing parameters is presented.**

I. INTRODUCTION

IN digital data communication there is a hierarchy of synchronization problems to be considered. First, assuming that a carrier-type system is involved, there is the problem of *carrier synchronization* which concerns the generation of a reference carrier with a phase closely matching that of the data signal. This reference carrier is used at the data receiver to perform a coherent demodulation operation, creating a baseband data signal. Next comes the problem of synchronizing a receiver clock with the baseband data-symbol sequence. This is commonly called *bit synchronization*, even when the symbol alphabet happens not to be binary.

Depending on the type of system under consideration, problems of *word-*, *frame-*, and *packet-synchronization* will be encountered further down the hierarchy. A feature that distinguishes the latter problems from those of carrier and bit synchronization is that they are usually solved by means of special design of the message format, involving the repetitive insertion of bits or words into the data sequence solely for synchronization purposes. On the other hand, it is desirable that carrier and bit synchronization be effected without multiplexing special timing signals onto the data signal, which would use up a portion of the available channel capacity. Only timing recovery problems of this type are discussed in this paper. This excludes those systems wherein the transmitted signal contains an unmodulated component of sinusoidal carrier (such as with "on-off" keying). When an unmodulated component or pilot is present, the standard approach to carrier synchronization is to use a phase-locked loop (PLL) which locks onto the carrier component, and has a narrow enough loop bandwidth so as not to be excessively perturbed by the sideband components of the signal. There is a vast literature on the performance and design of the PLL and there are several textbooks dealing with synchronous communication systems which treat the PLL in great detail [1]-[5]. Although we consider only suppressed-carrier signal formats here, the PLL material is still relevant since these devices are often used as component parts of the overall phase recovery system.

For modulation formats which exhibit a high bandwidth efficiency, i.e., which have a large "bits per cycle" figure of merit, we find the accuracy requirements on carrier and bit synchronization increasingly severe. Unfortunately, it is also in these high-efficiency systems that we find it most difficult to extract accurate carrier phase and symbol timing information by means of simple operations performed on the received signal. The pressure to develop higher efficiency data transmission has led to a dramatically increased interest in timing recovery problems and, in particular, in the ultimate performance that can be achieved with optimal recovery schemes.

We begin our review of carrier synchronization problems with a brief discussion of the major types of modulation format. In each case (DSB, VSB, or QAM), we assume coherent demodulation whereby the received signal is multiplied by a locally generated reference carrier and the product is passed through a low-pass filter. We can get some idea of the phase accuracy, or degree of coherency, requirements for the various modulation formats by examining the expressions for the coherent detector output, assuming a noise-free input. Let us assume that the message signal, say, $a(t)$, is incorporated by the modulation scheme into the complex envelope $\beta(t)$ of the carrier signal.[1]

$$y(t) = \mathrm{Re}\,[\beta(t) \exp(j\theta) \exp(j2\pi f_0 t)] \tag{1}$$

and the reference carrier $r(t)$ is characterized by a constant complex envelope

$$r(t) = \mathrm{Re}\,[\exp(j\hat{\theta}) \exp(j2\pi f_0 t)]. \tag{2}$$

From (A-8), the output of the coherent detector is

$$z_1(t) = \tfrac{1}{2}\,\mathrm{Re}\,[\beta(t) \exp(j\theta - j\hat{\theta})]. \tag{3}$$

For the case of DSB modulation, we have $\beta(t) = a(t) + j0$, so $z_1(t)$ is simply proportional to $a(t)$. The phase error $\theta - \hat{\theta}$ in the reference carrier has only a second-order effect

Manuscript received June 28, 1979; revised March 26, 1980.
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[1] See the Appendix for definitions and basic relations concerning complex envelope representation of signals.

0090-6778/80/0800-1107$00.75 © 1980 IEEE

on detector performance. The only loss is that phase error causes a reduction, proportional to $\cos^2(\theta - \hat{\theta})$, in signal-to-noise ratio at the detector output when additive noise is present on the received signal.

For VSB modulation, however, phase error produces a more severe distortion. In this case $\beta(t) = a(t) + j\tilde{a}(t)$, where $\tilde{a}(t)$ is related to $a(t)$ by a time-invariant filtering operation which causes a cancellation of a major portion of one of the sidebands. In the limiting case of complete cancellation of a sideband (SSB), we have $\tilde{a}(t) = \hat{a}(t)$, the Hilbert transform of $a(t)$ [6]. The coherent detector output (3) for the VSB signal is

$$z_1(t) = \tfrac{1}{2}\, a(t) \cos(\theta - \hat{\theta}) - \tfrac{1}{2}\, \tilde{a}(t) \sin(\theta - \hat{\theta}) \tag{4}$$

and the second term in (4) introduces an interference called *quadrature distortion* when $\hat{\theta} \neq \theta$. As $\tilde{a}(t)$ has roughly the same power level as $a(t)$, a relatively small phase error must be maintained for low distortion, e.g., about 0.032 radian error for a 30 dB signal-to-distortion ratio.

In the QAM case, two superimposed DSB signals at the same carrier frequency are employed by making $\beta(t) = a(t) + jb(t)$, where $a(t)$ and $b(t)$ are two separate, possibly independent, message signals. A dual coherent detector, using a reference carrier and its $\pi/2$ phase-shifted version, separates the received signal into its in-phase (I) and quadrature (Q) components. Again considering only the noise-free case, these components are

$$\begin{aligned} c_I(t) &= \tfrac{1}{2}\, a(t) \cos(\theta - \hat{\theta}) - \tfrac{1}{2}\, b(t) \sin(\theta - \hat{\theta}) \\ c_Q(t) &= \tfrac{1}{2}\, b(t) \cos(\theta - \hat{\theta}) + \tfrac{1}{2}\, a(t) \sin(\theta - \hat{\theta}). \end{aligned} \tag{5}$$

From (5) it is clear that $\hat{\theta} \neq \theta$ introduces a *crosstalk* interference into the I and Q channels. As $a(t)$ and $b(t)$ can be expected to be at similar power levels, the phase accuracy requirements for QAM are high compared to straight DSB modulation.

From the previous discussion we see that the price for the approximate doubling of bandwidth efficiency in VSB or QAM, relative to DSB, is a greatly increased sensitivity to phase error. The problem is compounded by the fact that carrier phase recovery is much more difficult for VSB and QAM, compared to DSB.

II. CARRIER PHASE RECOVERY

Before examining specific carrier recovery circuits for the suppressed-carrier format, it is helpful to ask, "What properties must the carrier signal $y(t)$ possess in order that operations on $y(t)$ will produce a good estimate of the phase parameter θ?" A general answer to this question lies in the *cyclostationary* nature of the $y(t)$ process.[2] A cyclostationary process has statistical moments which are periodic in time, rather than constant as in the case of stationary processes [2], [6], [7]. To a large extent, synchronization capability can be characterized by the lowest-order moments of the process, such as the mean and autocorrelation. The $y(t)$ process is said to be cyclostationary in the wide sense if $E[y(t)]$ and $k_{yy}(t+\tau, t) = E[y(t+\tau)y(t)]$ are both periodic functions of t. A process modeled by (1) is typically cyclostationary with a period of $1/f_0$ or $1/2f_0$. The statistical moments of this process depend upon the value of the phase parameter θ and it is not surprising that efficient phase estimation procedures are similar to moment estimation procedures. It is important to note here that we are regarding θ as an unknown but nonrandom parameter. If instead we regarded θ as a random parameter uniformly distributed over a 2π interval, then the $y(t)$ process would typically be stationary, not cyclostationary.

A general property of cyclostationary processes is that there may be a correlation between components in different frequency bands, in contrast to the situation for stationary processes [8]. For carrier-type signals, the significance lies in the correlation between message components centered around the carrier frequency ($+f_0$) and the image components around ($-f_0$). This correlation is characterized by the cross-correlation function $k_{\beta\beta^*}(\tau) = E[\beta(t+\tau)\beta(t)]$ for a $y(t)$ process as in (1) when $\beta(t)$ is a stationary process.[3]

Considering first the DSB case with $\beta(t) = a(t) + j\theta$, and using (A-10) we have

$$\begin{aligned} k_{yy}(t+\tau, t) &= \tfrac{1}{2}\, \mathrm{Re}\,[k_{aa}(\tau) \exp(j2\pi f_0 \tau)] \\ &\quad + \tfrac{1}{2}\, \mathrm{Re}\,[k_{aa}(\tau) \exp(j4\pi f_0 t + j2\pi f_0 \tau + j2\theta)] \end{aligned} \tag{6}$$

where the second term in (6) exhibits the periodicity in t that makes $y(t)$ a cyclostationary process.

We are assuming that $y(t)$ contains no periodic components. Consider what happens, however, when $y(t)$ is passed through a square-law device. We see immediately from (6) that the output of the squarer has a periodic mean value, since

$$\begin{aligned} E[y^2(t)] &= k_{yy}(t, t) \\ &= \tfrac{1}{2}\, k_{aa}(0) + \tfrac{1}{2}\, k_{aa}(0)\, \mathrm{Re}\,[\exp(j2\theta + j4\pi f_0 t)]. \end{aligned} \tag{7}$$

If the squarer output is passed through a bandpass filter with transfer function $H(f)$ as shown in Fig. 1, and if $H(f)$ has a unity-gain passband in the vicinity of $f = 2f_0$, then the mean value of the filter output is a sinusoid with frequency $2f_0$, phase 2θ, and amplitude $\tfrac{1}{2}E[a^2(t)]$. In this sense, the squarer has produced a periodic component from the $y(t)$ signal.

It is often stated that the effect of the squarer is to produce a discrete component (a line at $2f_0$) in the spectrum of its output signal. This statement lacks precision and can lead to serious misinterpretations because $y^2(t)$ is not a stationary process, so the usual spectral density concept has no meaning. A stationary process can be derived from $y^2(t)$ by phase randomizing [6], but then the relevance to carrier phase recovery is lost because the discrete component has a completely indeterminate phase.

[2] In [2], these processes are called *periodic nonstationary*.

[3] Despite its appearance, this is not an autocorrelation function, due to the definition of autocorrelation for complex processes; see (A-11).

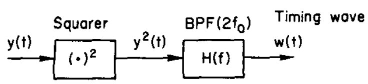

Fig. 1. Timing recovery circuit.

The output of the bandpass filter in Fig. 1 can be used directly to generate a reference carrier. Assuming that $H(f)$ completely suppresses the low-frequency terms [see (A-8)] the filter output is the *reference waveform*

$$w(t) = \tfrac{1}{2}\ \mathrm{Re}\ \{[\omega \otimes \beta^2](t) \exp(j2\theta) \exp(j4\pi f_0 t)\} \qquad (8)$$

where the convolution product $[\omega \otimes \beta^2]$ represents the filtering action of $H(f)$ in terms of its low-pass equivalent $\Omega(f)$ in (A-5). For the DSB case, $\beta^2(t) = a^2(t)$ is real and $\omega(t)$ is real[4] if $H(f)$ has a symmetric response about $2f_0$. Then the phase of the reference waveform is 2θ and the amplitude of the reference waveform fluctuates slowly [depending on the bandwidth of $H(f)$]. The reference carrier can be obtained by passing $w(t)$ through an infinite-gain clipper which removes the amplitude fluctuations. The square wave from the clipper can drive a frequency divider circuit which halves the frequency and phase. Alternatively, the bandpass filter output can be tracked by a PLL and the PLL oscillator output passed through the frequency-divider circuit.

There is another tracking loop arrangement, called the Costas loop, where the voltage-controlled oscillator (VCO) operates directly at f_0. We digress momentarily to describe the Costas loop and to point out that it is equivalent to the squarer followed by a PLL [1]-[3]. The equivalence is established by noting that the inputs to the loop filters in the two configurations shown in Fig. 2 are identical. In the PLL quiescent lock condition, the VCO output is in quadrature with the input signal so we introduce a $\pi/2$ phase shift into the VCO in the configurations of Fig. 2. Then using (A-8) to get the output of the multiplier/low-pass filter combinations, we see that the input to the loop filter is

$$v(t) = \tfrac{1}{8}\ \mathrm{Re}\ [A^2 \beta^2(t) \exp(j2\theta - j2\hat{\theta} - j\pi/2)] \qquad (9)$$

in both configurations if the amplitude of the VCO output is taken as $\frac{1}{2}A^2$ in the squarer/PLL configuration, and taken as A in the Costas loop.

Going back to (8), we see that phase recovery is perfect if $[\omega \otimes \beta^2]$ is real. Assuming $\omega(t)$ real, a phase error will result only if a quadrature component [relative to $\beta^2(t)$] appears at the output of the squarer. This points out the error, from a different viewpoint, of using the phase randomized spectrum of the squarer output to analyze the phase recovery performance because the spectrum approach obliterates the distinction between I and Q components. For the DSB case, a quadrature component will appear at the squarer output only if there is a quadrature component of interference added to the input signal $y(t)$. We can demonstrate this effect by considering the

[4] A real $\omega(t)$ corresponds to the case where the cross-coupling paths between input and output I and Q components in Fig. 10 are absent. If the bandpass function $H(f)$ does not exhibit the symmetrical amplitude response and antisymmetrical phase response about $2f_0$ for a real $\omega(t)$, then there simply is a fixed phase offset introduced by the bandpass filter.

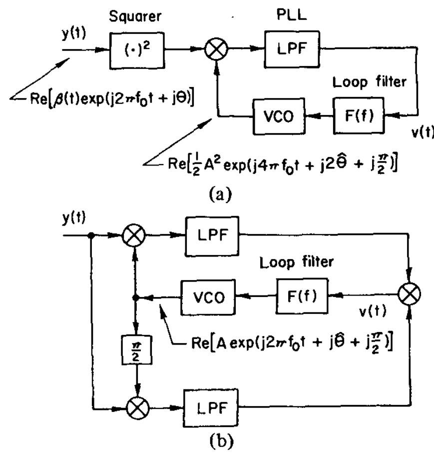

Fig. 2. Carrier phase tracking loops. (a) Squarer/PLL (b) Costas loop.

input signal to be $z(t) = y(t) + n(t)$ where $n(t)$ is white noise with a double-sided spectral density of N_0 W/Hz. We can represent $n(t)$ by the complex envelope, $[u_I(t) + ju_Q(t)] \exp(j\theta)$ where, from (A-15) the I and Q noise components relative to a phase θ are uncorrelated and have a spectral density of $2N_0$. The resulting phase of the reference waveform (8) is

$$2\hat{\theta} = 2\theta + \tan^{-1}\left[\frac{2\omega \otimes (\beta u_Q + u_I u_Q)}{\omega \otimes (\beta^2 + 2\beta u_I + u_I{}^2 - u_Q{}^2)}\right]. \qquad (10)$$

We can approximate the phase error $\phi = \hat{\theta} - \theta$ (also called *phase jitter* because $\hat{\theta}$ is a quantity that fluctuates with time) by neglecting the noise × noise term in the numerator and both signal × noise and noise × noise terms in the denominator in (10). Furthermore, we replace $\omega \otimes \beta^2$ by its expected value (averaging over the message process) and use the $\tan^{-1} x \cong x$ approximation. With all these simplifications, which are valid at sufficiently high signal-to-noise ratio and with sufficiently narrow-band $H(f)$, it is easy to derive an expression for the variance of the phase jitter.

$$\mathrm{var}\ \phi = (2N_0 B) S^{-1} \qquad (11a)$$

$$= \left(\frac{S}{N}\right)^{-1} \left(\frac{B}{W}\right) \qquad (11b)$$

where

$$B \triangleq \int_{-\infty}^{\infty} |\Omega(f)|^2\, df = \int_0^{\infty} |H(f)|^2\, df$$

is the *noise bandwidth* of the bandpass filter, recalling that we have set $\Omega(0) = 1$. The message signal power is $S = E[a^2(t)]$ and for the second version of the jitter formula (11b) we have assumed a signal bandwidth of W Hz and have defined a noise power over this band of $N = 2N_0 W$. This allows the satisfying physical interpretation of jitter variance being inversely proportional to signal-to-noise ratio and directly proportional to the bandwidth ratio of the phase recovery circuit and the message signal. For the smaller signal-to-noise ratios, the accuracy and convenience of the expression can be maintained by incorporating a correction factor known as the *squaring loss* [3].

When the signal itself carries a significant quadrature component, as in the case of the VSB signal, there will be a quadrature component at the squarer output that interferes with the phase recovery operation even at high signal-to-noise ratios. Let us suppose that the VSB signal is obtained by filtering a DSB signal with a bandpass filter with a real transfer function (no phase shift) and with a cutoff in the vicinity of f_0. The resulting quadrature component for the VSB signal is $\tilde{a}(t) = [p_Q \otimes a](t)$ and $p_Q(t)$ is derived from the low-pass equivalent transfer function for the bandpass filter in accordance with (A-7). The real transfer function condition makes $p_Q(t)$ an odd function of time, which also makes the cross-correlation function for $a(t)$ and $\tilde{a}(t)$ an odd function.

The result is that, for $\beta(t) = a(t) + j\tilde{a}(t)$, the autocorrelation for the VSB signal is

$$k_{yy}(t+\tau, t) = \tfrac{1}{2} \operatorname{Re} [\{k_{aa}(\tau) + k_{\tilde{a}\tilde{a}}(\tau) + j2k_{\tilde{a}a}(\tau)\} \cdot \exp(j2\pi f_0\tau)] + \tfrac{1}{2} \operatorname{Re} [\{k_{aa}(\tau) - k_{\tilde{a}\tilde{a}}(\tau)\} \cdot \exp(j4\pi f_0 t + j2\pi f_0\tau + j2\theta)]. \tag{12}$$

Comparing (12) with (6), we see that the second, cyclostationary, term is much smaller for the VSB case than the DSB case since the autocorrelation functions for $a(t)$ and $\tilde{a}(t)$ differ only to the extent that some of the low-frequency components in $\tilde{a}(t)$ are missing because of the VSB rolloff characteristic. Although the jitter performance will be poorer, the phase recovery circuit in Fig. 1 can still be used since the mean value of the reference waveform is a sinusoid exhibiting the desired phase, but with an amplitude which is proportional to the difference in power levels in $a(t)$ and $\tilde{a}(t)$.

$$E[w(t)] = \tfrac{1}{2} [k_{aa}(0) - k_{\tilde{a}\tilde{a}}(0)] \operatorname{Re} [\exp(j4\pi f_0 t + j2\theta)]. \tag{13}$$

However, it is not possible to get a very simple formula for the variance of phase jitter, as in (11), because the power spectral density of the quadrature component of $\beta^2(t)$, which is proportional to $a(t)\,\tilde{a}(t)$, vanishes at $f = 0$, unlike in the additive noise case. An accurate variance expression must take into account the particular shape of the $\Omega(f)$ filtering function as well as the shape of the VSB rolloff characteristic.

Our examination of phase recovery for DSB (with additive noise) and VSB modulation formats has indicated that rms phase jitter can be made as small as desired by making the width of $\Omega(f)$ sufficiently small. The corresponding parameter in the case of the tracking loop configuration is called the loop bandwidth [3]. These results, however, are for *steady-state* phase jitter since the signals at the receiver input were presumed to extend into the remote past. The difficulty with a very narrow phase recovery bandwidth is that excessive time is taken to get to the steady-state condition when a new signal process begins. This time interval is referred to as the *acquisition time* of the recovery circuit and in switched communication networks or polling systems it is usually very important to keep this interval small, even at the expense of the larger steady-state phase jitter. One way to accommodate the conflicting objectives in designing a carrier recovery circuit is to specify a minimum phase-recovery bandwidth and then adjust other parameters of the system to minimize the steady-state phase jitter.

Another problem with a very narrow-band bandpass filter is in the inherent mistuning sensitivity, where mistuning is a result of inaccuracies in filter element values or a result of small inaccuracies or drift in the carrier frequency. This problem is avoided with tracking loop configurations since they lock onto the carrier frequency. One the other hand, tracking loops have some problems also, one of the more serious being the "hang-up" problem [9] whereby the nonlinear nature of the loop can produce some greatly prolonged acquisition times.

Although we have modeled the phase recovery problem in terms of a constant unknown carrier phase, it may be important in some situations to consider the presence of fairly rapid fluctuations in carrier phase (independent of the message process). Such fluctuations are often called *phase noise* and if the spectral density of these fluctuations has a greater bandwidth than that of the phase recovery circuits, there is a phase error due to the inability to track the carrier phase. Phase error of this type, even in steady state, becomes *larger* as the bandwidth of the recovery circuits decreases.

Another practical consideration is a π-radian phase ambiguity in the phase recovery circuits we have been discussing. The result is a polarity ambiguity in the coherently demodulated signal. In many cases this polarity ambiguity is unimportant, but otherwise some *a priori* knowledge about the message signal will have to be used to resolve the ambiguity.

For a QAM signal with $\beta(t) = a(t) + jb(t)$, where $a(t)$ and $b(t)$ are independent zero-mean stationary processes, we get

$$k_{yy}(t+\tau, t) = \tfrac{1}{2} \operatorname{Re} [\{k_{aa}(\tau) + k_{bb}(\tau)\} \exp(j2\pi f_0\tau)] + \tfrac{1}{2} \operatorname{Re} [\{k_{aa}(\tau) - k_{bb}(\tau)\} \cdot \exp(j4\pi f_0 t + j2\pi f_0\tau + j2\theta)] \tag{14}$$

and the situation is very similar to the VSB case (12). In this case where $a(t)$ and $b(t)$ are uncorrelated, the mean reference waveform has the correct phase, but the amplitude vanishes if the power levels in the I and Q channels are the same.

$$E[w(t)] = \tfrac{1}{2} [k_{aa}(0) - k_{bb}(0)] \operatorname{Re} [\exp(j4\pi f_0 t + j2\theta)]. \tag{15}$$

Hence, unless the QAM format is intentionally unbalanced, the squaring approach in Fig. 1 does not work. We briefly examine what happens when the squarer is replaced by a fourth-power device in the recovery schemes we have been considering. From (1), we can obtain

$$y^4(t) = \tfrac{1}{8} \operatorname{Re} [\beta^4(t) \exp(j8\pi f_0 t + j4\theta)] + \tfrac{1}{2} \operatorname{Re} [\,|\beta(t)|^2 \beta^2(t) \exp(j4\pi f_0 t + j2\theta)] + \tfrac{3}{8} |\beta(t)|^4. \tag{16}$$

Now if we use a bandpass filter tuned to $4f_0$ which passes only the first term in (16), then the mean reference waveform at the filter output is

$$E[w(t)] = \tfrac{1}{4}\ \mathrm{Re}\ [\{\overline{a^4} - 3(\overline{a^2})^2\}\ \exp(j8\pi f_0 t + j4\theta)] \qquad (17)$$

still assuming independent $a(t)$ and $b(t)$ and a balanced QAM format, i.e., $k_{aa}(0) = k_{bb}(0) = \overline{a^2}$. Hence, a mean reference waveform exists even in the balanced QAM case if a fourth-power device is used.[5]

One very popular QAM format is quadriphase-shift keying (QPSK) where the standard carrier recovery technique is to use a fourth-power device followed by a PLL or to use an equivalent "double" Costas loop configuration [3]. The QPSK format, with independent data symbols, can be regarded as two independent binary phase-shift-keyed (BPSK) signals in phase quadrature. In a nonbandlimited situation each BPSK signal can be regarded as DSB-AM where the message waveform has a rectangular shape characterized by $a(t) = \pm 1$. In this case, the complex envelope of the QPSK signal is characterized by $\beta(t) = (\pm 1 \pm j)/\sqrt{2}$ or $\beta(t) = \exp\ (j(\pi/4) + j(\pi/2)k)$ with $k =$ 0, 1, 2, or 3. The result is that $\beta^4\ (t) = -1$ and the $4f_0$ component in (16) is a pure sinusoid with no fluctuations in either phase or amplitude. For PSK systems with a larger alphabet of phase positions, the result of (17) cannot generally be used as the I and Q components are no longer independent. Analysis of the larger alphabet cases shows that higher-order nonlinearities are required for successful phase recovery [3], [10]. For any balanced QAM format, such as QPSK, the phase recovery circuits discussed here give a $\pi/2$-radian phase ambiguity. This problem is often handled by use of a differential PSK scheme, whereby the information is transmitted as a sequence of phase changes rather than absolute values of phase.

III. PAM TIMING RECOVERY

The receiver synchronization problem in baseband PAM transmission is to find the correct sampling instants for extracting a sequence of numerical values from the received signal. For a synchronous pulse sequence with a pulse rate of $1/T$, the sampler operates synchronously at the same rate and the problem is to determine the correct sampling phase within a T-second interval. The model for the baseband PAM signal is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT - \tau) \qquad (18)$$

where $\{a_k\}$ is the message sequence and $g(t)$ is the signaling pulse. We want to make an accurate determination of τ, from operations performed on $x(t)$. We assume that $g(t)$ is so defined that the best sampling instants are at $t = kT + \tau$; $k = 0, \pm 1, \pm 2, \cdots$. The objective is to recover a close replica of the message sequence $\{a_k\}$ in terms of the sequence $\{\hat{a}_k = x(kT + \hat{\tau})\}$, assuming a normalization of $g(0) = 1$. In the noise-free case, the difference between a_k and $\hat{a}_k$ is due to intersymbol interference which can be minimized by proper shaping of the data pulse $g(t)$. With perfect timing ($\hat{\tau} = \tau$), the intersymbol interference is

$$\hat{a}_k - a_k = \sum_{n \neq k} a_n g(kT - nT) \qquad (19)$$

and this term can be made to vanish for pulses satisfying the Nyquist criterion, i.e., $g(nT) = 0$ for $n \neq 0$. For bandlimited Nyquist pulses, the intersymbol interference will not be zero when $\hat{\tau} \neq \tau$, and if the bandwidth is not significantly greater than the Nyquist bandwidth $(1/2T)$ the intersymbol interference can be quite severe even for small values of timing error. The problem is especially acute for multilevel (nonbinary) data sequences where timing accuracy of only a few percent of the symbol period is often required.

Symbol timing recovery is remarkably similar in most respects to carrier phase recovery and we find that similar signal processing will yield suitable estimates of the parameter τ. In the discussion to follow, we assume that $\{a_k\}$ is a zero-mean stationary sequence with independent elements. The resulting PAM signal (18) is a zero-mean cyclostationary process, although there are no periodic components present [6]. The square of the PAM signal does, however, possess a periodic mean value.

$$E[x^2(t)] = \overline{a^2} \sum_k g^2(t - kT - \tau). \qquad (20)$$

Using the Poisson Sum Formula [6], we can express (20) in the more convenient form of a Fourier series whose coefficients are given by the Fourier transform of $g^2(t)$.

$$E[x^2(t)] = \frac{\overline{a^2}}{T} \sum_{\ell} A_\ell \exp\left(\frac{j2\pi\ell}{T}(t - \tau)\right) \qquad (21)$$

where

$$A_\ell \triangleq \int_{-\infty}^{\infty} G\left(\frac{\ell}{T} - f\right) G(f)\, df.$$

For high bandwidth efficiency, we are often concerned with data pulses whose bandwidth is at most equal to twice the Nyquist bandwidth. Then $|G(f)| = 0$ for $|f| > 1/T$ and there are only three nonzero terms ($\ell = 0, \pm 1$) in (21).

This result suggests the use of a timing recovery circuit of the same form as shown in Fig. 1, where now the bandpass filter is tuned to the symbol rate, $1/T$. Alternate zero crossings of $w(t)$, a *timing wave* analogous to the reference waveform in Section II, are used as indications of the correct sampling instants. Letting $H(1/T) = 1$, the mean timing wave is a sinusoid with a phase of $-2\pi\tau/T$, for a real $G(f)$.

$$E[w(t)] = \frac{\overline{a^2}}{T}\ \mathrm{Re}\left[A_1 \exp\left(j\frac{2\pi t}{T} - j\frac{2\pi\tau}{T}\right)\right]. \qquad (22)$$

We see that the zero crossings of the mean timing wave are at a fixed time offset $(T/4)$ relative to the desired sampling instants.

[5] Unless $a(t)$ and $b(t)$ are Gaussian processes, for then $\overline{a^4} = 3(\overline{a^2})^2$.

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