Dictionary of Mathematics Terms

Third Edition

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BARRON'S

Dedication
This book is for Lori.

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CONTENTS

V

VARIABLE A variable is a symbol that is used to represent a value from a particular set. For example, in algebra it is common to use letters to represent values from the set of real numbers. (See **algebra**.)

VARIANCE The variance of a random variable X is defined to be

$$\mathrm{Var}(X) = E[(X - E(X)) \times (X - E(X))]$$
$$= E[(X - E(X))^2]$$

where E stands for "expectation."

The variance can also be found from the formula:

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

The variance is often written as σ^2. (The Greek lowercase letter sigma (σ), is used to represent the square root of the variance, known as the *standard deviation*.)

The variance is a measure of how widespread the observations of X are likely to be. If you know for sure what the value of X will be, then $\mathrm{Var}(X) = 0$.

For example, if X is the number of heads that appear when a coin is tossed five times, then the probabilities are given in this table:

i	$\Pr(X = i)$	$i \times \Pr(X = i)$	$i^2 \times \Pr(X = i)$
0	1/32	0	0
1	5/32	5/32	5/32
2	10/32	20/32	40/32
3	10/32	30/32	90/32
4	5/32	20/32	80/32
5	1/32	5/32	25/32
sum:	1	2.5	7.5

The sum of column 3 $[i \times \Pr(X = i)]$ gives $E(X) = 2.5$; the sum of column 4 $[i^2 \times \Pr(X = i)]$ gives $E(X^2) = 7.5$. From this information we can find

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 7.5 - 2.5^2 = 1.25$$

Some properties of the variance are as follows.

If a and b are constants:

$$\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$$

If X and Y are independent random variables:

$$\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

In general:

$$\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X, Y)$$

where $\mathrm{Cov}(X, Y)$ is the covariance.

The variance of a list of numbers $x_1, x_2, \ldots x_n$ is given by either of these formulas:

$$\mathrm{Var}(x) = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}$$
$$= \overline{x^2} - (\bar{x})^2$$

where a bar over a quantity signifies average.

VECTOR A vector is a quantity that has both magnitude and direction. The quantity "60 miles per hour" is a regular number, or scalar. The quantity "60 miles per hour to the northwest" is a vector, because it has both size and direction. Vectors can be represented by drawing pictures of them. A vector is drawn as an arrow pointing in the direction of the vector, with length proportional to the size of the vector. (See figure 156.)

Vectors can also be represented by an ordered list of numbers, such as (3,4) or (1, 0, 3). Each number in this list is called a *component* of the vector. A vector in a plane (two dimensions) can be represented as an ordered pair.

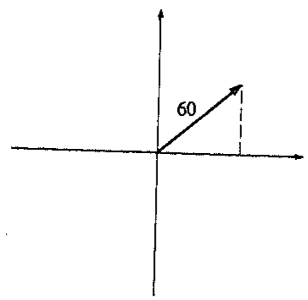

Figure 156 Vector

A vector in space (three dimensions) can be represented as an ordered triple.

Vectors are symbolized in print by boldface type, as in "vector **a**." A vector can also be symbolized by placing an arrow over it: $\vec{a}$.

The length, or magnitude or norm, of a vector **a** is written as $\|\mathbf{a}\|$

Addition of vectors is defined as follows: Move the tail of the second vector so that it touches the head of the first vector, and then the sum vector (called the *resultant*) stretches from the tail of the first vector to the head of the second vector. (See figure 157.) For vectors expressed by components, addition is easy: just add the components:

$$(3, 2) + (4, 1) = (7, 3)$$
$$(a, b) + (c, d) = (a + c, b + d)$$

To multiply a scalar by a vector, multiply each component by that scalar:

$$10(3, 2) = (30, 20)$$
$$n(a, b) = (na, nb)$$

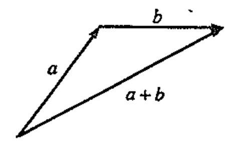

Figure 157 Adding vectors

To find two different ways of multiplying vectors, see **dot product** and **cross product**.

VECTOR FIELD A two-dimensional vector field **f** transforms a vector (x, y) into another vector $\mathbf{f}(x, y) = [f_x(x, y), f_y(x, y)]$. Here $f_x(x, y)$ and $f_y(x, y)$ are the two components of the vector field; each is a scalar function of two variables. An example of a vector field is:

$$\mathbf{f}(x,y) = \left[\frac{-y}{\sqrt{x^2 + y^2}}, \frac{x}{\sqrt{x^2 + y^2}}\right]$$

If we evaluate this vector field at (3,4) we find:

$$\mathbf{f}(3,4) = \left[\frac{-4}{5}, \frac{3}{5}\right]$$

In this particular case, the output of the vector field is perpendicular to the input vector.

The same concept can be generalized to higher-dimensional vector fields. For examples of calculus operations on vector fields, see **divergence; curl; line integral; surface integral; Stokes' theorem; Maxwell's equations.**

VECTOR PRODUCT This is a synonym for **cross product.**

VELOCITY The velocity vector represents the rate of change of position of an object. To specify a velocity, it is necessary to specify both a speed and a direction (for example, 50 miles per hour to the northwest).

If the motion is in one dimension, then the velocity is the derivative of the function that gives the position of the object as a function of time. The derivative of the velocity is called the **acceleration.**

If the vector $[x(t), y(t), z(t)]$ gives the position of the object in three dimensional space, where each component of the vector is given as a function of time, then the velocity vector is the vector of derivatives of each component:

$$\text{velocity} = \left[\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right]$$

VENN DIAGRAM A Venn diagram (see figure 158) is a picture that illustrates the relationships between sets. The universal set you are considering is represented by a rectangle, and sets are represented by circles or ellipses. The possible relationships between two sets A and B are as follows:

Set B is a subset of set A, or set A is a subset of set B.

Set A and set B are disjoint (they have no elements in common).

Set A and set B have some elements in common.

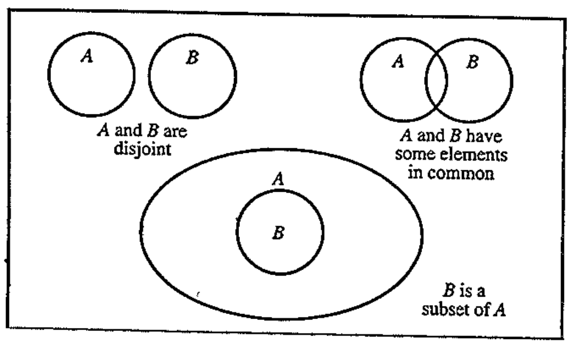

Figure 158

Figure 159 is a Venn diagram for the universal set of complex numbers.

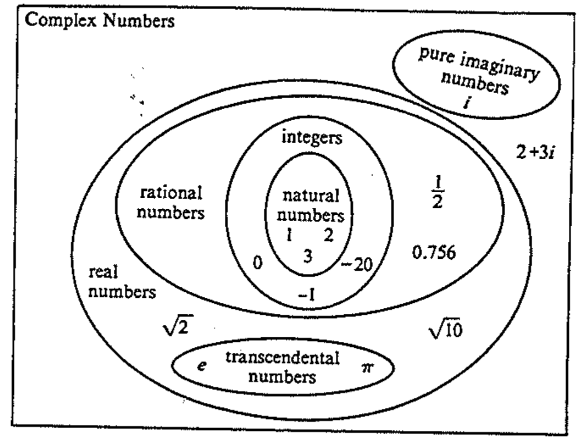

Figure 159 Venn diagram

VERTEX The vertex of an angle is the point where the two sides of the angle intersect.

VERTICAL ANGLES Two pairs of vertical angles are formed when two lines intersect. In figure 160, angle 1 and angle 2 are a pair of vertical angles. Angle 3 and angle 4 are another pair of vertical angles. The two angles in a pair of vertical angles are always equal in measure.

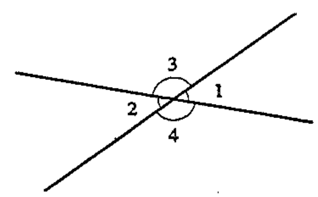

Figure 160 Vertical angles