Computer Vision
A Modern Approach

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different images of a surface in a fixed view illuminated by different sources. This method recovers the height of the surface at points corresponding to each pixel; in computer vision circles, the resulting representation is often known as a *height map*, *depth map*, or *dense depth map*.

Fix the camera and the surface in position and illuminate the surface using a point source that is far away compared with the size of the surface. We adopt a local shading model and assume that there is no ambient illumination (more about this later) so that the radiosity at a point P on the surface is

$$B(P) = \rho(P)\boldsymbol{N}(P) \cdot \boldsymbol{S}_1,$$

where $\boldsymbol{N}$ is the unit surface normal and $\boldsymbol{S}_1$ is the source vector. With our camera model, there is only one point P on the surface for each point (x, y) in the image, and we can write $B(x, y)$ for $B(P)$. Now we assume that the response of the camera is linear in the surface radiosity, so the value of a pixel at (x, y) is

$$\begin{aligned} I(x, y) &= kB(x, y) \\ &= k\rho(x, y)\boldsymbol{N}(x, y) \cdot \boldsymbol{S}_1 \\ &= \boldsymbol{g}(x, y) \cdot \boldsymbol{V}_1, \end{aligned}$$

where k is the constant connecting the camera response to the input radiance, $\boldsymbol{g}(x, y) = \rho(x, y)\boldsymbol{N}(x, y)$, and $\boldsymbol{V}_1 = k\boldsymbol{S}_1$.

In these equations, $\boldsymbol{g}(x, y)$ describes the surface and $\boldsymbol{V}_1$ is a property of the illumination and of the camera. We have a dot product between a vector field $\boldsymbol{g}(x, y)$ and a vector $\boldsymbol{V}_1$, which could be measured; with enough of these dot products, we could reconstruct $\boldsymbol{g}$ and so the surface.

5.4.1 Normal and Albedo from Many Views

Now if we have n sources, for each of which $\boldsymbol{V}_i$ is known, we stack each of these $\boldsymbol{V}_i$ into a known matrix $\mathcal{V}$, where

$$\mathcal{V} = \begin{pmatrix} \boldsymbol{V}_1^T \\ \boldsymbol{V}_2^T \\ \cdots \\ \boldsymbol{V}_n^T \end{pmatrix}.$$

For each image point, we stack the measurements into a vector

$$\boldsymbol{i}(x, y) = \{I_1(x, y), I_2(x, y), \ldots, I_n(x, y)\}^T.$$

Notice that we have one vector per image point; each vector contains all the image brightnesses observed at that point for different sources. Now we have

$$\boldsymbol{i}(x, y) = \mathcal{V}\boldsymbol{g}(x, y),$$

and $\boldsymbol{g}$ is obtained by solving this linear system—or rather, one linear system per point in the image. Typically, $n > 3$ so that a least squares solution is appropriate. This has the advantage that the residual error in the solution provides a check on our measurements.

The difficulty with this approach is that substantial regions of the surface may be in shadow for one or the other light (see Figure 5.10). There is a simple trick that deals with shadows. If there really is no ambient illumination, then we can form a matrix from the image vector and

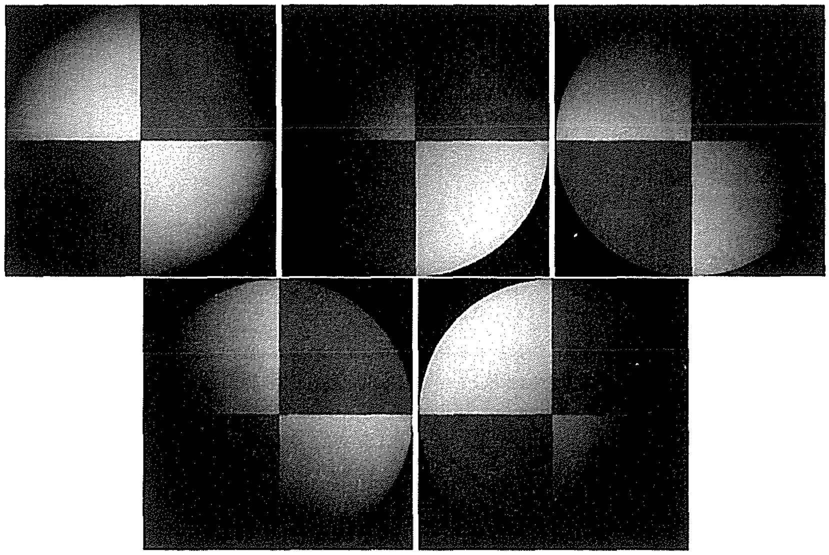


Figure 5.10 Five synthetic images of a sphere, all obtained in an orthographic view from the same viewing position. These images are shaded using a local shading model and a distant point source. This is a convex object, so the only view where there is no visible shadow occurs when the source direction is parallel to the viewing direction. The variations in brightness occuring under different sources code the shape of the surface.

We form

$$\mathcal{I}(x, y) = \begin{pmatrix} I_1(x, y) & \cdots & 0 & 0 \\ 0 & I_2(x, y) & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & I_n(x, y) \end{pmatrix}$$

and

$$\mathcal{I}\boldsymbol{i} = \mathcal{I}\mathcal{V}\boldsymbol{g}(x, y),$$

and $\mathcal{I}$ has the effect of zeroing the contributions from shadowed regions, because the relevant elements of the matrix are zero at points that are in shadow. Again, there is one linear system per point in the image; at each point, we solve this linear system to recover the $\boldsymbol{g}$ vector at that point.

Measuring Albedo We can extract the albedo from a measurement of $\boldsymbol{g}$ because $\boldsymbol{N}$ is the unit normal. This means that $|\boldsymbol{g}(x, y)| = \rho(x, y)$. This provides a check on our measurements as well. Because the albedo is in the range zero to one, any pixels where $|\boldsymbol{g}|$ is greater than one are suspect—either the pixel is not working or $\mathcal{V}$ is incorrect. Figure 5.11 shows albedo recovered using this method for the images of Figure 5.10.

Recovering Normals We can extract the surface normal from $\boldsymbol{g}$ because the normal is a unit vector

$$N(x, y) = \frac{1}{|\boldsymbol{g}(x, y)|}\boldsymbol{g}(x, y).$$

Figure 5.12 shows normal values recovered for the images of Figure 5.10.

Sec. 5.4

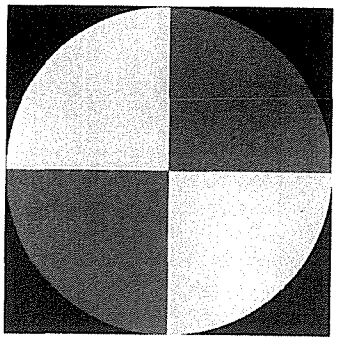


Figure 5.11 The magnitude of the vector field $g(x, y)$ recovered from the input data of Figure 5.10 represented as an image—this is the reflectance of the surface.

5.4.2 Shape from Normals

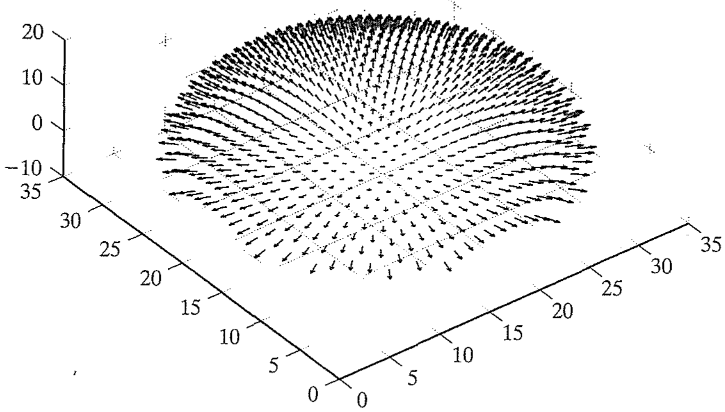
The surface is $(x, y, f(x, y))$, so the normal as a function of (x, y) is

$$N(x, y) = \frac{1}{\sqrt{1 + \frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial y}^2}} \left\{ -\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1 \right\}^T$$

To recover the depth map, we need to determine $f(x, y)$ from measured values of the unit normal.

Assume that the measured value of the unit normal at some point (x, y) is $(a(x, y), b(x, y), c(x, y))$. Then

$$\frac{\partial f}{\partial x} = \frac{a(x, y)}{c(x, y)} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{b(x, y)}{c(x, y)}.$$



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