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DECISIONS UNDER UNCERTAINTY: THE FUZZY COMPROMISE DECISION SUPPORT PROBLEM

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A compromise Decision Support Problem is used to improve an alternative through modification to achieve multiple objectives. However, the compromise DSP requires precise numerical information to yield rigorously accurate results. In the early stages of conceptual design such precise information is often unavailable. For example, a design should be reliable, manufacturable, maintainable and cost-efficient. Although inherently vague, each qualifier specifies an important attribute that the design must possess. Such vagueness may be modeled rigourously using the mathematics of fuzzy set theory. In this paper, we introduce a fuzzy formulation of the compromise DSP; a formulation which is particularly suitable for modeling the decisions required in the early stages of engineering design. We investigate the properties of the fuzzy compromise DSP in the context of designing a planar, four-bar linkage.

KEY WORDS: Uncertainty, fuzzy sets, decision support, compromise, four-bar linkage.

NOTATION

$\in$	Is a member of the set, or, is contained in.
$\wedge$	The intersection of.
U	The union of sets.
$\rightarrow$	Indicates a mapping from the set on the left to the set on the right.
$\gtrsim$	Is almost positive.
sup	The least upper bound.
inf	The greatest lower bound.
max	The largest of fuzzy sets.

min	The smallest of, or the intersection of fuzzy sets.
$A\{x, \mu_A(x)\}$	A represents the fuzzy set A at the value x whose grade of membership is determined by the membership function, $\mu_A(x)$.
$(\mathrm{Ac})_j$	The constants (parameters) associated with the capability of the system with respect to the jth constraint on the system.
$(\mathrm{Ad})_j$	The constants (parameters) associated with the demand on the system due to the jth constraint.
$(\mathrm{Ap})_k$	The constants (parameters) associated with the performance of the system on the kth target.
$(\mathrm{At})_k$	Constants (constants) associated with the designer's aspirations for the kth target.
$\mathrm{C}_j((\mathrm{Ac})_j, \mathbf{X})$	A linear or nonlinear capability associated with a constraint that is a function of the parameters $(\mathrm{Ac})_j$ and the system variables, $\mathbf{X}$. Italics are used to indicate fuzziness.
c	The extent of the cloud of fuzziness surrounding the main value of a fuzzy set. This is numerically equivalent to the range of the membership function.
c*	The fuzzifier associated with the grouped constraints.
$(\mathrm{cc})_j$	The fuzzifier associated with the parameters specifying the system's capability in meeting the jth constraint.
$(\mathrm{cd})_j$	The fuzzifier associated with the demand due to the jth constraint.
$(\mathrm{cp})_k$	The fuzzifier associated with the performance on the kth target.
$(\mathrm{ct})_k$	The fuzzifier associated with the kth target.
$\mathrm{D}_j((\mathrm{Ad})_j, \mathbf{X})$	Demand associated with the jth constraint. Demand is a function of the system variables, $\mathbf{X}$, and parameters $(\mathrm{Ad})_j$. Fuzzy demand is denoted by italics.
DC	A decision.
$\mathrm{d}_k^+, \mathrm{d}_k^-$	Deviation variables used in the crisp (non-fuzzy) compromise DSP formulation.
H	A possibility distribution.
$(\mathrm{Hc})_j$	The possibility distribution for the capability of meeting the jth constraint.
$(\mathrm{Hd})_j$	The possibility distribution for the degree of compatibility of the system associated with the demand from the jth constraint.
$(\mathrm{Hp})_k$	The possibility distribution for the performance on the kth target.
$(\mathrm{Ht})_k$	The possibility distribution for the kth target.
H*	The possibility distribution representing the degree of compatibility of the system with the constraints when the constraints are grouped. $\mathrm{H}^* = 1$ implies complete compatibility, $\mathrm{H}^* = 0$ implies total incompatibility.
J	Total number of constraints in a DSP.
K	Total number of goals in a DSP.
M	The number of system variables in a DSP.
m	The main value of a fuzzy set (the value which is surrounded by a cloud of fuzziness).

$(mc)_j$	The main value of the fuzzy set which represents the system's capability on the jth constraint.
$(md)_j$	The main value of the fuzzy set which represents the demand associated with the jth constraint.
$(mp)_k$	The main value of the fuzzy set which represents the performance associated with the kth target.
$(mt)_k$	The main value of the fuzzy set which represents the kth target.
$P_k((Ap)_k, \mathbf{X})$	Actual performance of a system characterized by the system variables $\mathbf{X}$ and the parameters $(Ap)_k$. A fuzzy performance function is denoted by italics.
Pl	Priority ranking factors for the achievement of the system goals used in both the crisp and the fuzzy formulation DSP.
$T_k((At)_k, \mathbf{X})$	Target or aspiration level for system performance at the point defined by the system variables $\mathbf{X}$ and the parameters $(At)_k$. A fuzzy aspiration level is denoted by italics.
$\mathbf{X}$	A crisp vector of system variables.
Z	An achievement function representing the difference between system performance and the designer's goals for the system.
$\mu_A(x)$	The membership function associated with the fuzzy set A.

1 DECISION SUPPORT IN THE VERY EARLY STAGES OF DESIGN

A comprehensive approach called the **D**ecision **S**upport **P**roblem (DSP) Technique[1–4] is being developed and implemented, at the University of Houston, to provide support for human judgment in designing an artifact that can be manufactured and maintained. Decision Support Problems provide a means of modeling decisions encountered in design, manufacture and maintenance. Formulation and solution of DSPs provide a means for making the following types of decisions:

- **Selection**—the indication of a preference, based on multiple attributes, for one among several feasible alternatives.
- **Compromise (trade-off)**—the improvement of an alternative through modification.
- **Hierarchical**—decisions that involve interaction between sub-decisions.
- **Conditional**—decisions in which the risk and uncertainty of the outcome are taken into account.

Compromise DSPs refer to a class of constrained, multiobjective optimization problems that are used in a wide variety of engineering applications. Both selection and compromise DSPs can be a part of the hierarchical representation of an engineering system, which involves an ordered and directed set of DSPs where the sequence of interactions among them is clearly defined. Applications of these DSPs include the design of ships, damage tolerant structural and mechanical systems, the design of aircraft, mechanisms, thermal energy systems, design using composite materials and data compression. A detailed set of references to these applications is presented in Ref. [5]. DSP's have been developed for hierarchical design; coupled selection-compromise, compromise-compromise and selection-selection[6]. These constructs have been used to study the interaction between design and manufacture[7] and between various events in the conceptual phase of the design process[8]. The compromise DSP is solved using a unique optimization scheme called Adaptive Linear Programming[9]. Other formulations of conditional decisions are described in Refs. [10–12].

For real-world, practical systems, all of the information for modeling systems comprehensively and correctly in the early stages of the project will not be available. In the preliminary stages of engineering design, there is great uncertainty about the nature of the object that is being designed. This uncertainty stems from *vagueness* or *imprecision* of knowledge about the object being designed rather than from errors in repeated measurements of the object being designed (there can be no measurements as the object does not exist yet). Hence, standard probabilistic approaches cannot form an accurate mathematical representation of the object being designed. However, both vagueness and imprecision can be modeled rigorously using fuzzy set theory[13]. Therefore, we are investigating the incorporation of the mathematics of fuzzy sets into methods being developed for use in the very early stages of design.

In this paper, we present a theoretical model for the fuzzy compromise DSPs followed by an example of their use, a non-linear kinematics problem involving the minimization of the structural error in a path-generating four bar linkage. The standard non-fuzzy (crisp) formulation of the compromise DSP is a specific case of the fuzzy compromise DSP. Also, the importance of being able to fuzzify constraints and goals independently is shown.

1.1 *The compromise Decision Support Problem*

A compromise DSP is defined using the following descriptors: system and deviation variables; system constraints and goals are defined by a set of constant parameters and system variables; bounds on the system variables; and a deviation function. The compromise DSP, its descriptors and its mathematical form have been described in several publications[3,9] and will therefore not be repeated here. The generalized formulation of the fuzzy compromise DSP that follows, however, has not been published elsewhere and it reads as follows:

Given

- An alternative defined by the vector of M independent system variables, $\mathbf{X}$.
- J system constraints which must be satisfied for an acceptable solution.
- $C_j((\mathrm{Ac})_j, \mathbf{X})$ is the capability associated with the jth system constraint. $(\mathrm{Ac})_j$ represents the constant parameters needed to characterize the capability associated with the jth constraint. The capability can be a linear or a nonlinear function.

• $D_j((Ad)_j, \mathbf{X})$ is the demand associated with the jth system constraint. $(Ad)_j)$ represents the constants needed to characterize the demand. These constants are some of the parameters characterizing the compromise DSP. $\mathbf{X}$ represents the system variables.

• K is the number of system goals which must be achieved to attain a specified target, $T_k((At)_k, \mathbf{X})$. $(At)_k$ represents the constants necessary to specify the kth target; these constants are some of the parameters associated with the compromise DSP.

• $P_k((Ap)_k, \mathbf{X})$ is a function specifying the performance associated with the kth system goal. $(Ap)_k$ represents the constants needed to characterize the system's performance on the kth target. These constants are some of the parameters associated with the compromise DSP.

Find

• The values of the independent system variables, X_i, $i = 1, \ldots, M$.

• The values of the non-negative deviation variables indicating the extent to which the target values are attained. d_k^- and d_k^+ represent under-achievement and over-achievement of the target, k, where $k = 1, \ldots, K$, such that $[(d_k^-$ and $d_k^+ \geq 0)]$ and $[(d_k^- \cdot d_k^+) = 0]$.

Satisfy

• *System Constraints* [Capability is Equal to or Exceeds Demand]:

$$C_j((Ac)_j, \mathbf{X}) \geq D_j((Ad)_j, \mathbf{X}); \qquad J = 1, \ldots, J. \tag{1}$$

With lower and upper bounds on the system variables:

$$X_i^{min} \leq X_i \leq X_i^{max} \qquad i = 1, \ldots, M.$$

• *System Goals* [Target is Equal to or Exceeds Performance]:

$$P_k((Ap)_k, \mathbf{X}) + d_k^- - d_k^+ = T_k((At)_k, \mathbf{X}) \qquad k = 1, \ldots, K. \tag{2}$$

Minimize

• A deviation function, Z, quantifies the deviation of the system performance, $P_k((Ap)_k, \mathbf{X})$, from the ideal as defined by the set of target values, $T_k((Ag)_k, \mathbf{X})$, and their associated priority levels. There are two ways of representing the deviation function.

Preemptive Deviation Function

In the preemptive formulation, the deviation function is:

$$Z = [f_1(d_k^-, d_k^+), \ldots, f_k(d_i^-, d_i^+)] \qquad k = 1, \ldots, K \tag{3}$$

where the functions of the deviation variables are ranked lexicographically[10].

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