# A COMPARISON OF PEAK POWER REDUCTION SCHEMES FOR OFDM

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Abstract — Two powerful and distortionless peak power reduction schemes for <u>O</u>rthogonal <u>Frequency Division Multi-</u> plexing (OFDM) are compared. One investigated technique is <u>selected mapping</u> (SLM) where the actual transmit signal is selected from a set of signals and the second scheme utilizes phase rotated <u>partial transmit <u>sequences</u> (PTS) to construct the transmit signal. Both approaches are very flexible as they do not impose any restriction on the modulation applied in the subcarriers or on their number. They both introduce some additional system complexity but nearly vanishing redundancy to achieve markedly improved statistics of the multicarrier transmit signal. The schemes are compared by simulation results with respect to the required system complexity and transmit signal redundancy.</u>

### 1. INTRODUCTION

Besides a lot of advantages, some drawbacks become apparent, when using OFDM in transmission systems. A major obstacle is that the multiplex signal exhibits a very high peak-to-average power ratio (PAR). Therefore, nonlinearities may get overloaded by high signal peaks, causing intermodulation among subcarriers and — more critical — undesired out-of-band radiation. If RF power amplifiers are operated without large power back-offs, it is impossible to keep the out-of-band power below specified limits. This leads to very inefficient amplification and expensive transmitters so that it is highly desirable to reduce the PAR. A variety of methods for that purpose is proposed in literature (e.g. [4, 10, 3]).

Here, we concentrate on two recently proposed flexible and distortionless methods for the reduction of the PAR by way of introducing little redundancy. The SLM method [1, 2](similar methods are described in [9, 5]) is compared to the PTS approach [8, 7]. In SLM the transmitter selects one favorable transmit signal from a set of sufficiently different signals which all represent the same information, while in PTS the transmitter constructs its transmit signal with low PAR by coordinated addition of appropriately phase rotated signal parts.

Section 2 recapitulates OFDM signaling. In Section 3 we report statistical characteristics of the OFDM transmit signal. The two investigated PAR reduction schemes are looked at again in Section 4. Simulation results to compare their performance are presented in Section 5. There, the PAR reduction capability of both schemes is set against the theoretical limit of achievable minimum PAR versus redundancy and we will find that they are considerably near this limit.

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#### 2. OFDM TRANSMISSION

The idea of OFDM is to use  $D_u$  separate subcarriers, having a uniform frequency spacing. The frequency multiplexing is implemented by using the inverse discrete Fourier transform (IDFT) for D-ary ( $D \ge D_u$ ) vectors in the modulator.

At first, binary data is mapped onto  $D_u$  carriers. Thereby, subcarrier  $\nu$  of OFDM symbol interval  $\mu$  is modulated with the complex coefficient  $A_{\mu,\nu}$ . Here, we assume that in all  $D_u$ active carriers the same complex-valued zero-mean signal set  $\mathcal{A}$  with variance  $\sigma_{\mathcal{A}}^2$  is used, but the results can easily be extended to mixed signal constellations. Inactive carriers are set to zero in order to shape the power density spectrum of the transmit signal appropriately.

The subcarrier vector  $\mathbf{A}_{\mu} = [A_{\mu,0}, \ldots, A_{\mu,D-1}]$  comprising all carrier amplitudes associated with OFDM symbol interval  $\mu$  is transformed into time domain, using a D-point IDFT. This results in the T-spaced discrete-time representation of the transmit signal in the  $\mu$ -th block, given by  $\mathbf{a}_{\mu} = [a_{\mu,0}, \ldots, a_{\mu,D-1}]$  with  $a_{\mu,\rho} = \frac{1}{\sqrt{D}} \sum_{\nu=0}^{D-1} A_{\mu,\nu} \cdot \mathrm{e}^{+\mathrm{j}\frac{2\pi}{D}\nu\rho}, \quad 0 \leq \rho < D$ . In the following  $\mathbf{a}_{\mu} = \mathrm{IDFT} \{\mathbf{A}_{\mu}\}$  denotes this transform relationship. Here, the modulation period T is related with the symbol period  $T_{\mathrm{s}}$  in each subcarrier by  $T_{\mathrm{s}} = D \cdot T$ .

Finally, the samples  $a_{\mu,\rho}$  are transmitted using ordinary T-spaced pulse amplitude modulation. The guard interval usually introduced before transmission consists of a partial repetition of some  $a_{\mu,\rho}$  and therefore does not affect the PAR. Thus, it is not considered here.

For what follows we coin the term *transmit sequence* for  $\mathbf{a}_{\mu}$ . The peak power optimized alternative transmit sequence will be denoted as  $\tilde{\mathbf{a}}_{\mu}$ .

# 3. TRANSMIT SEQUENCE STATISTICS

Clearly, the power amplifier has to deal with the continuoustime transmit signal after a specific impulse shaping. For simplicity, we will only focus on the PAR of the underlying T-spaced sampled representation  $\mathbf{a}_{\mu}$  of this signal. Under certain circumstances and depending on the steepness of the impulse-shaping filter's frequency response in the transition region (length of impulse response), special attention has to be dedicated to the continuous-time behaviour, too.

#### 3.1. STATISTICAL PROPERTIES

We define a discrete–time PAR associated with OFDM symbol interval  $\mu$  as

$$\chi_{\mu} \stackrel{\text{def}}{=} \max_{0 \le \rho < D} \left| a_{\mu,\rho} \right|^2 \Big/ \mathcal{E} \left\{ \left| a_{\mu,\rho} \right|^2 \right\}, \tag{1}$$

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where  $\mathcal{E}\left\{\cdot\right\}$  denotes expectation. Due to Parseval's theorem the average power of the transmit sequences is  $\sigma_a^2 \stackrel{\text{def}}{=} \mathcal{E}\left\{\left|a_{\mu,\rho}\right|^2\right\} = \frac{D_u}{D} \cdot \sigma_{\mathcal{A}}^2$ .

Applying the central limit theorem, while assuming that  $D_{\rm u}$  is sufficiently large ( $\geq 64$  is sufficient), the  $a_{\mu,\rho}$  are zeromean complex-valued near Gaussian distributed random variables with variance  $\sigma_a^2$ .

Introducing the OFDM transmit signal magnitude  $u = |a_{\mu,\rho}| \ge 0$ , we obtain (independent from D) the Rayleigh density

$$p_u(u) = \frac{2u}{\sigma_a^2} \cdot e^{-u^2/\sigma_a^2} \cdot \delta_{-1}(u)$$
(2)

for the probability density function (pdf) of u (cf. Fig. 4). Clearly,  $\overline{\delta}_{-1}(u)$  in Eq. (2) denotes the unit step function.

Following the exposition in [2, 5, 6], the probability that  $\chi_{\mu}$  of a randomly generated *D*-carrier OFDM symbol exceeds the PAR threshold  $\chi_0 = a_0^2/\sigma_a^2$  can be approximated by (cf. Fig. 3)

$$\Pr\{\chi_{\mu} > \chi_{0}\} = 1 - \left(1 - e^{-\chi_{0}}\right)^{D}.$$
 (3)

Note that the latter expression does not depend on the PAR of the signal set  $\mathcal{A}$  used in the subcarriers.

#### 3.2. THEORETICAL LIMIT FOR MINIMUM PAR

The ideal distortionless PAR reduction scheme introduces redundancy to exclude "bad" OFDM symbols from transmission. Ideally,  $R_{\rm ap}$  ("antipeak") bits per symbol allow to reject the larger fraction of  $(1 - 2^{-R_{\rm ap}})$  from the entire set of possible OFDM symbols [6]. If e.g.  $\Pr\{\chi_{\mu} > \chi_{0}\} = \frac{3}{4}$ , then  $\frac{1}{4}$  of the entire set of possibly generated OFDM symbols have a PAR lower than  $\chi_{0}$ . Clearly, only  $R_{\rm ap} = 2$  bits per symbol are required to distinguish these favorable OFDM symbols from the undesired rest. Therefore,  $1 - 2^{-R_{\rm ap}} = \Pr\{\chi_{\mu} > \chi_{0}\}$  gives the relation between redundancy  $R_{\rm ap}$  and the theoretically achievable minimum PAR  $\chi_{0}$ . Incorporating Eq. (3) and solving for  $\chi_{0}$  yields

$$\chi_0 = -\ln\left(1 - 2^{-\frac{R_{\rm ap}}{D}}\right),\tag{4}$$

which represents the lower bound for  $\chi_0$  when  $R_{\rm ap}$  bits redundancy are distributed on *D* carriers, no matter which modulation is used in them (cf. Fig. 5).

In the following section two generally related methods are recapitulated which both spread the redundancy appropriately over the entire OFDM symbol. These two schemes do not result in an inflexible joint coding and modulation scheme as in [4, 10] and furthermore they are effective with an arbitrarily large number of subcarriers.

# 4. REDUCING PEAK POWER IN OFDM

# 4.1. SELECTED MAPPING

In this most general approach [1, 2] it is assumed that U statistically independent alternative transmit sequences  $\mathbf{a}_{\mu}^{(u)}$  represent the same information. Then, that sequence  $\tilde{\mathbf{a}}_{\mu} =$ 

 $\mathbf{a}_{\mu}^{(\tilde{u}_{\mu})}$  with the lowest PAR, denoted as  $\tilde{\chi}_{\mu}$ , is selected for transmission. The probability that  $\tilde{\chi}_{\mu}$  exceeds  $\chi_0$  is approximated by [2, 5]

$$\Pr\{\tilde{\chi}_{\mu} > \chi_{0}\} = \left(1 - \left(1 - e^{-\chi_{0}}\right)^{D}\right)^{U}.$$
 (5)

Because of the selected assignment of binary data to the transmit signal, this principle is called selected mapping in [2, 6].

A set of U markedly different, distinct, pseudo-random but fixed vectors  $\mathbf{P}^{(u)} = [P_0^{(u)}, \ldots, P_{D-1}^{(u)}]$ , with  $P_{\nu}^{(u)} = e^{\pm j \varphi_{\nu}^{(u)}}, \ \varphi_{\nu}^{(u)} \in [0, 2\pi), \ 0 \leq \nu < D, \ 1 \leq u \leq U$  must be defined. The subcarrier vector  $\mathbf{A}_{\mu}$  is multiplied subcarrierwise with each one of the U vectors  $\mathbf{P}^{(u)}$ , resulting in a set of U different subcarrier vectors  $\mathbf{A}_{\mu}^{(u)}$  with components

$$A_{\mu,\nu}^{(u)} = A_{\mu,\nu} \cdot P_{\nu}^{(u)}, \quad 0 \le \nu < D, \quad 1 \le u \le U.$$
 (6)

Then, all U alternative subcarrier vectors are transformed into time domain to get  $\mathbf{a}_{\mu}^{(u)} = \text{IDFT}\left\{\mathbf{A}_{\mu}^{(u)}\right\}$  and finally that transmit sequence  $\tilde{\mathbf{a}}_{\mu} = \mathbf{a}_{\mu}^{(\tilde{u}_{\mu})}$  with the lowest PAR  $\tilde{\chi}_{\mu}$ is chosen. The SLM–OFDM transmitter is depicted in Fig. 1, where it is visualized that one of the alternative subcarrier vectors can be the unchanged original one.



Optionally, differentially encoded modulation may be applied before the IDFT and right after generating the alternative OFDM symbols. At the receiver, differential demodulation has to be implemented right after the DFT.

## 4.2. PARTIAL TRANSMIT SEQUENCES

In this scheme [8, 7] the subcarrier vector  $\mathbf{A}_{\mu}$  is partitioned into V pairwise disjoint subblocks  $\mathbf{A}_{\mu}^{(v)}$ ,  $1 \leq v \leq V$ . All subcarrier positions in  $\mathbf{A}_{\mu}^{(v)}$ , which are already represented in another subblock are set to zero, so that  $\mathbf{A}_{\mu} = \sum_{v=1}^{V} \mathbf{A}_{\mu}^{(v)}$ . We introduce complex-valued rotation factors  $b_{\mu}^{(v)} = e^{\pm j\varphi_{\mu}^{(v)}}$ ,  $\varphi_{\mu}^{(v)} \in [0, 2\pi), 1 \leq v \leq V, \forall \mu$ , enabling a modified subcarrier vector

$$\check{\mathbf{A}}_{\mu} = \sum_{\nu=1}^{V} b_{\mu}^{(\nu)} \cdot \mathbf{A}_{\mu}^{(\nu)}, \tag{7}$$

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which represents the same information as  $\mathbf{A}_{\mu}$ , if the set  $\left\{b_{\mu}^{(v)}, 1 \leq v \leq V\right\}$  (as side information) is known for each  $\mu$ . Clearly, simply a joint rotation of all subcarriers in subblock v by the same angle  $\varphi_{\mu}^{(v)} = \arg\left(b_{\mu}^{(v)}\right)$  is performed.

To calculate  $\check{\mathbf{a}}_{\mu} = \text{IDFT} \{\check{\mathbf{A}}_{\mu}\}$ , the linearity of the IDFT is exploited. Accordingly, the subblocks are transformed by V separate and parallel D-point IDFTs, yielding

$$\check{\mathbf{a}}_{\mu} = \sum_{\nu=1}^{V} b_{\mu}^{(\nu)} \cdot \text{IDFT} \left\{ \mathbf{A}_{\mu}^{(\nu)} \right\} = \sum_{\nu=1}^{V} b_{\mu}^{(\nu)} \cdot \mathbf{a}_{\mu}^{(\nu)}, \qquad (8)$$

where the V so-called partial transmit sequences  $\mathbf{a}_{\mu}^{(v)} = \text{IDFT}\left\{\mathbf{A}_{\mu}^{(v)}\right\}$  have been introduced. Based on them a peak value optimization is performed by suitably choosing the free parameters  $b_{\mu}^{(v)}$  such that the PAR is minimized for  $\tilde{b}_{\mu}^{(v)}$ . The  $b_{\mu}^{(v)}$  may be chosen with continuous-valued phase angle, but more appropriate in practical systems is a restriction on a finite set of W (e.g. 4) allowed phase angles.

The optimum transmit sequence then is

$$\tilde{\mathbf{a}}_{\mu} = \sum_{\nu=1}^{V} \tilde{b}_{\mu}^{(\nu)} \cdot \mathbf{a}_{\mu}^{(\nu)}. \tag{9}$$

The PTS-OFDM transmitter is depicted in Fig. 2 with the hint, that one PTS can always be left unrotated.



Figure 2: PAR reduction in PTS-OFDM.

We refer to [8, 6] for the discussion of an advantageous application of PTS employing differentially encoded modulation across subcarriers (i.e. in direction of frequency).

So far, no specific assignment of subcarriers to subblocks (subblock partitioning) has been given, but it has considerable influence on the PAR reduction capability of PTS. This topic is discussed in [7], where a pseudo-random (but still disjoint) subblock partitioning has been found to be the best choice for high PAR reduction.

It should be noted, that PTS can be interpreted as a structurally modified special case of SLM, if  $W^{V-1} = U$  and the  $\mathbf{P}^{(u)}$  are chosen in accordance with the PTS partitioning and all the allowed rotation angle combinations  $\{b_{\mu}^{(v)}\}$ . But with this construction rule, especially for a large number of vectors  $\mathbf{P}^{(u)}$ , their statistical independence is usually no longer satisfied, so that Eq. (5) does not hold any longer.

# 4.3. REDUNDANCY (SIDE INFORMATION)

Both schemes require, that the receiver has knowledge about the generation of the transmitted OFDM signal in symbol period  $\mu$ . Thus, in PTS the set with all rotation factors  $\tilde{b}_{\mu}^{(v)}$ and in SLM the number  $\tilde{u}_{\mu}$  of the selected  $\mathbf{P}^{(\tilde{u}_{\mu})}$  has to be transmitted to the receiver unambiguously so that this one can derotate the subcarriers appropriately. The number of bits required for canonical representation of this side information is the redundancy  $R_{\rm ap}$  introduced by the PAR reduction scheme with PTS and SLM. As this side information is of highest importance to recover the data, it should be carefully protected by channel coding, but the hereby introduced additional redundancy is not considered here.

In PTS the number of admitted combinations of rotation angles  $\{b_{\mu}^{(v)}\}$  should not be excessively high, to keep the explicitely transmitted side information within a reasonable limit. If in PTS each  $b_{\mu}^{(v)}$  is exclusively chosen from a set of W admitted angles, then  $R_{\rm ap} = (V-1)\log_2 W$  bits per OFDM symbol are needed for this purpose. In SLM  $R_{\rm ap} = \log_2 U$ bits are required for side information.

Both schemes use the introduced redundancy to synthesize alternative signal representations, which all have to be checked for PAR. Clearly, their number is given by  $2^{R_{ap}}$ . In SLM this value is U while in PTS we obtain  $W^{V-1}$  alternatives, a number which can get very high.

In PTS the choice  $b_{\mu}^{(v)} \in \{\pm 1, \pm j\}$  (W = 4) is very interesting for an efficient implementation, as actually no multiplication must be performed, when rotating and combining the PTSs  $\mathbf{a}_{\mu}^{(v)}$  to the peak-optimized transmit sequence  $\tilde{\mathbf{a}}_{\mu}$  in Eq. (9). For SLM, choosing  $P_{\nu}^{(u)}$  from the latter set has the same advantage, when generating the alternative subcarrier vectors by applying Eq. (6).

# 5. COMPARATIVE SIMULATION RESULTS

The presented simulations were performed with  $D_u = D =$  128 carriers modulated with 16QAM. The statistics of peak and instantaneous power in randomly generated OFDM symbols have been investigated.

In PTS, an optimum pseudo-random [7] disjoint assignment of  $\approx D/V$  subcarriers to each subblock is used. Here, the optimum  $\tilde{b}^{(v)}_{\mu}$  are found by an exhaustive search over all combinations of rotation angles. For SLM, U statistically independent rotation vectors  $\mathbf{P}^{(u)}$  are used. The rotation vectors are actually obtained from random binary sequences mapped on 4PSK symbols. In Fig. 3 simulation results for  $\Pr{\{\chi_{\mu} > \chi_0\}}$  achieved with V PTS-subblocks, where each  $b_{\mu}^{(v)}$  is chosen from a 4PSK-constellation (W = 4) are set against SLM-OFDM with U alternative subcarrier vectors. Note that V = U IDFTs are needed in either scheme but PTS will usually provide a greater multiplicity of signal representations to be checked for PAR. The simulated characteristic of original OFDM and the theoretical expression from Eq. (3)are plotted there as well and theory corresponds well with the simulation result. It follows from this diagram that PTS with W = 4 rotations and V = 2 IDFTs (and therefore 4 signal



Figure 3: Probability that the PAR of a randomly generated 128-carrier OFDM transmit sequence exceeds  $\chi_0$ for U IDFTs in SLM and V IDFTs in PTS with W = 4.

representations) achieves a slightly better performance than SLM with U = 3 IDFTs (3 signal representations). The gap would get even larger if W is further increased in PTS (W signal representations, if V = 2). Generally, PTS outperforms SLM in PAR reduction, if the number of IDFTs is fixed, but clearly with more alternative signals to be processed.

As already mentioned 16QAM modulation (PAR of A: 2.55 dB) was used in each of the 128 subcarriers, but theoretically the results do not differ, when using 4PSK modulation (PAR of A: 0 dB). In fact, simulations with 4PSK resulted in minor changes (< 0.1 dB) of *all* depicted PAR statistics.



Figure 4: The pdf of  $u = |a_{\mu,\rho}|$  for various numbers of IDFTs in SLM and PTS with W = 4 (D = 128).

Fig. 4 shows the pdf of the transmit signal magnitude  $u = |a_{\mu,\rho}|$ . This is the statistical characteristic, the power amplifier has to cope with. The theoretical expression from Eq. (2) is illustrated additionally and coincides with the simulative result for original OFDM. The benefit of PTS and SLM can be seen from the considerably reduced pdf for high values of the normalized signal magnitude. The slope of the pdf can be adjusted by variation of V and U, respectively.

The effect of PTS and SLM is that both shift probability mass from high amplitude values to lower ones. As PTS is more powerful with the same number of IDFTs, the increase of the pdf around 5 dB is more distinct for PTS.

Table 1 gives a compact overview of PAR reduction capability for SLM set against PTS with various allowed rotation angles, numbers of subblocks and, not considered so far, two different subblock partitionings. The entries provide information about the number of bits  $R_{\rm ap}$  for PAR reduction per symbol, and the number of possible signal representations  $(2^{R_{\rm ap}})$  enabled by this redundancy. They all have to be checked for PAR, if a selection by exhaustive search is performed. The PAR reduction gain  $G_{\chi} \stackrel{\text{def}}{=} \chi_{\rm original}/\chi_{\rm reduced}$  at  $\Pr \{\chi_{\mu} > \chi_0\} = 10^{-5}$  is given in the lower row. The  $G_{\chi}^a$  achieved by a PTS subblock partitioning with exclusively <u>a</u>djacent subcarriers [6] is compared to the optimum  $G_{\chi}^r$  realizable for pseudo-<u>r</u>andom subblock partitioning [7]. For PTS, each table entry has to be read like this:  $2^{R_{\rm ap}}$ 

2 <sup>10</sup> ap	$R_{\rm ap}$ [bit]			
$10\log_{10}G^{\mathrm{a}}_{\chi}$ [dB]	$10\log_{10}G_{\chi}^{\mathrm{r}}$ [dB]			

V,U	2		3		4		5	
PTS	2	1	4	2	8	3	16	4
W = 2	1.2	2.0	2.5	3.3	3.4	4.1	4.1	4.7
PTS	4	2	16	4	64	6	256	8
W = 4	2.1	3.0	3.6	4.4	4.5	5.2	5.1	5.8
PTS	8	3	64	6	512	9	4k	12
W = 8	2.9	3.4	4.2	4.9	5.1	5.8	?	?
PTS	16	4	256	8	4k	12	64k	16
W=16	3.0	3.6	4.2	5.1	?	?	?	?
SLM	2	1	3	1.6	4	2	5	2.3
	2.0		2.8		3.3		3.6	

Table 1: PAR reduction gain  $G_{\chi}$  at  $\Pr \{\chi_{\mu} > \chi_0\} = 10^{-5}$  for PTS-OFDM with V subblocks and W possible rotation angles compared to SLM-OFDM with U alternative subcarrier vectors (D = 128).

Obviously, the pseudo-random assignment of subcarriers to subblocks is 0.5 to 0.9 dB better than the one with exclusively adjacent subcarriers per PTS subblock. The latter is an example for highly structured subblock partitioning, resulting in considerable performance degradation [7].

Note that for some combinations of W and V in PTS an exhaustive optimum search is prohibitive. Table 1 is for 128 carriers and clearly  $G_{\chi}$  will be different for other carrier numbers but the tendencies recognizable therein are preserved, especially the fact that for PTS with fixed  $R_{\rm ap}$  it is more advantageous to increase V instead of W.

It follows from Table 1 that pseudo-random subblock partitioning in PTS (W = 2) with V = 2 and 3 performs equivalent to SLM with U = 2 and 4, respectively. This shows that for small numbers of  $W^{V-1}$  and pseudo-randomized subblock partitioning in PTS, the  $\mathbf{P}^{(u)}$  of the equivalent SLM scheme are still statistically independent. This implies that 4 alternative signal representations generated by PTS with 3 IDFTs plus some further vectorial additions achieve the same performance as SLM with 4 IDFTs. In Fig. 5 the theoretical limit of Eq. (4) is plotted dashdotted. An ideal method using  $R_{\rm ap}$  bits redundancy per symbol can guarantee that no single OFDM symbol ever exceeds  $\chi_0$ . A distortionless limitation to PAR lower than  $\chi_0$  is not possible and this is illustrated as hatched area. Note that the limit derived from a simulated histogram is slightly worse when compared to theory derived from the central limit theorem with all its idealized assumptions.



Figure 5: PAR reduction performance vs. redundancy with respect to the theoretical limit.

We concentrate on the statistical nature of  $\chi_{\mu}$  and define  $\Pr \{\chi_{\mu} > \chi_0\} = 10^{-5}$  as unlikely enough not to produce significant out-of-band power after the amplifier. So the "stochastic" PARs occuring with this probability (and  $10^{-3}$  for comparison) are plotted over the number of bits needed for an explicit transmission of side information for pseudo-random subblock partitioning in PTS with W = 4 and  $V = 1, \ldots, 5$ and SLM with  $U = 1, \ldots, 5$ . Here, SLM outperforms PTS but clearly on the cost of system complexity. Note that only the marked points on the curve for SLM are simulation results. The dashed curves are derived from Eq. (5), as at  $R_{\rm ap} = 6$  an inacceptable high number of 64 IDFTs would be required, compared to only 4 IDFTs plus time-domain optimization in PTS. Given this redundancy, PTS is only 0.8 dB (at  $10^{-5}$ ) worse than SLM. For lower  $R_{\rm ap}$ , the gap gets even smaller.

## 6. SUMMARY AND CONCLUSIONS

The paper compared two recently proposed techniques which allow powerful but nonetheless distortionless PAR reduction for OFDM transmission. Both related schemes utilize several IDFTs instead of one and choose (construct) one signal from a multiplicity of (partial) transmit sequences. PTS–OFDM and SLM–OFDM work with arbitrary numbers of subcarriers and types of modulation in them.

In PTS only 1.2% redundancy (cf. Fig. 3, V = 4) is needed to reduce the discrete–time PAR by 5.2 dB at Pr { $\chi_{\mu} > \chi_0$ } =  $10^{-5}$ , achieving a stochastic PAR of quite low 7.1 dB in a 128 carrier system. If system complexity is ignored, SLM would even reduce the stochastic discrete–time PAR by 6 dB to 6.3

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dB with the same redundancy. SLM outperforms PTS in terms of PAR reduction vs. redundancy, but PTS is considerably better with respect to PAR reduction vs. additional system complexity (e.g. number of IDFTs) as it is capable to provide a greater manifold of alternative signal representations by using the same number of IDFTs together with some further vectorial additions. Obviously, complexity will be the main point of view, if practical OFDM systems are considered and so PTS (in an efficient implementational structure) will be a strong candidate.

PTS and SLM are near-optimum when PAR reduction capability vs. redundancy is considered. Thus, they seem to be the most powerful and flexible methods known to reduce OFDM peak power without nonlinear distortion.

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### References

- R. Bäuml. Verringerung des Spitzenwertfaktors bei Mehrträgerübertragungsverfahren. Diplomarbeit am Lehrstuhl für Nachrichtentechnik, Universität Erlangen-Nürnberg (in German), August 1996.
- [2] R. Bäuml, R. Fischer, and J. Huber. Reducing the Peak-to-Average Power Ratio of Multicarrier Modulation by Selected Mapping. *Electronics Letters*, vol.32,no.22:pp.2056-2057, October 1996.
- [3] M. Friese. Multicarrier modulation with low peak-to-average power ratio. *Electronics Letters*, vol.32,no.8:pp.713-714, April 1996.
- [4] A. Jones, T. Wilkinson, and S. Barton. Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes. *Electronics Letters*, vol.30,no.25:pp.2098-2099, December 1994.
- [5] D. J. Mestdagh and P. M. Spruyt. A Method to Reduce the Probability of Clipping in DMT-Based Transceivers. *IEEE Trans. on Commun.*, vol.44,no.10:pp.1234-1238, October 1996.
- [6] S. Müller, R. Bäuml, R. Fischer, and J. Huber. OFDM with Reduced Peak-to-Average Power Ratio by Multiple Signal Representation. Annals of Telecommunications, vol.52,no.1-2:pp.58-67, February 1997.
- [7] S. Müller and J. Huber. A Novel Peak Power Reduction Scheme for OFDM. Accepted for presentation at IEEE PIMRC'97, Helsinki, Finland, September 1997.
- [8] S. Müller and J. Huber. OFDM with Reduced Peak-to-Average Power Ratio by Optimum Combination of Partial Transmit Sequences. *Electronics Letters*, vol.33,no.5:pp.368-369, February 1997.
- [9] P. Van Eetvelt, G. Wade, and M. Tomlinson. Peak to Average Power Reduction for OFDM Schemes by Selective Scrambling. *Electronics Letters*, vol.32,no.21:pp.1963–1964, October 1996.
- [10] D. Wulich. Reduction of peak to mean ratio of multicarrier modulation using cyclic coding. *Electronics Letters*, vol.32,no.5:pp.432-433, February 1996.