A Method to Reduce the Probability of Clipping in DMT-Based Transceivers

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***Abstract*—A new method allowing a reduction in the probability of clipping in discrete multitone (DMT)-based transceivers is described. The method does not use any kind of precoding and can easily be implemented within conventional DMT-transceivers. The main advantage of the proposed method is an improvement of system performance in terms of overall signal-to-noise ratios (SNR's): with the simplest implementation option of the proposed method, up to about 8 dB improvement in the SNR as compared with previously reported brute force clipping methods can be achieved.**

I. Introduction

THE *discrete multitone* (DMT) modulation technique is emerging as a very powerful technique for applications ranging from asymmetric digital subscriber line (ADSL), digital audio broadcast (DAB) to interactive video on demand (IVOD) over CATV networks [1]–[3].

A DMT signal is the sum of N independently quadrature amplitude modulated (QAM) signals each being carried over a distinct carrier frequency. The frequency separation of the N carriers is equal to $1/T$ where T is the time duration of a DMT symbol. The real part of the complex envelope of the generated DMT signal can be expressed as

$$A(t) = \mathrm{Re}\left\{ \sum_{m=-\infty}^{+\infty} \sum_{k=0}^{N-1} [r_m^k \cdot e^{j2\pi(k/T)t} \cdot u(t - mT)] \right\} \quad (1)$$

where r_m^k denotes the QAM-phasor of carrier k (at frequency k/T) of the m-th DMT symbol and $(u)t$ is a rectangular transmit pulse of duration T.

In a practical transceiver, the DMT symbol (1) is generated by means of an inverse fast Fourier transform (IFFT) on the complex phasors $\{r_m^k\}, k \in [0, N-1]$ [4].

Fig. 1 shows the instantaneous amplitude $A(t)$ of two DMT symbols generated with two distinct sets of QAM-phasors $\{r_m^k\}_1$ and $\{r_m^k\}_2$, and $N = 256$. For both symbols, 16-QAM carrier modulation is assumed. A noticeable feature of the symbol in Fig. 1(b) as compared with the one in Fig. 1(a) is that it exhibits large amplitude spikes which arise when several frequency components add in-phase. These spikes may have a serious impact on the design complexity and feasibility of the transceiver's analog front-end (i.e., high resolution of D/A-A/D convertors and line drivers with a linear behavior over a large dynamical range). In addition, regulations can limit the peak envelope power or the probability of clipping [5]. The effect of amplitude clipping in DMT transceivers has been analyzed in the literature [6], [7] and methods based on encoding the input data in order to reduce the peak-to-average power ratio of the DMT signal have been proposed [8], [9]. The coding methods, however, require an increase in data rate and hence a reduction of the energy per bit for the same transmit power, resulting in performance degradation in terms of information handling capacity of the communication system.

In this letter, an alternative method is proposed. Since N is usually large (say $N \geq 128$), $A(t)$ can be accurately modeled as a Gaussian random process (central-limit theorem) with a zero mean and a variance σ^2 equal to the total signal power. Its probability density function (pdf), denoted as $p(x)$, is given by [6]

$$p(x) \cong \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x^2/2\sigma^2)}. \quad (2)$$

Therefore, large amplitude spikes arise very rarely (thanks to statistical averaging) so that by applying a specific processing (no coding) only on DMT signals whose amplitudes exceed a given amplitude A_{Clip}, one can obtain a DMT symbol stream with almost no amplitudes exceeding A_{Clip}.

The paper is organized as follows. Section II presents the basics of the proposed method. In Section III, the resulting improvement of system performance in terms of signal-to-noise ratio (SNR) is derived. Conclusions are reported in Section IV.

II. The Proposed Method

The basic idea behind the proposed method can be described as follows. Assume that the maximum amplitude of the clipped DMT signal, A_{Clip}, is chosen so that the probability of amplitude clipping is lower than a specified value. In the DMT transmitter, the symbols generated by the IFFT are analyzed by a peak detector which provides an indication of the presence or absence of amplitude clipping. According to this indication, two distinct actions are taken:

Case a: If the amplitude of the DMT symbol never exceeds A_{Clip}, then the symbol is sent to the transmitter front-end without any change.

Case b: If the generated DMT symbol has at least one sample whose amplitude exceeds A_{Clip}, then it is not passed directly to the transmitter front-end. Instead, the phasor of each QAM-modulated carrier is changed by means of a fixed phasor-transformation and a new DMT symbol is generated by the IFFT. By careful selection of the phasor-transformation, the probability of clipping this new symbol (second pass) will be very low. (The resulting overall clipping probability will be calculated later on.)

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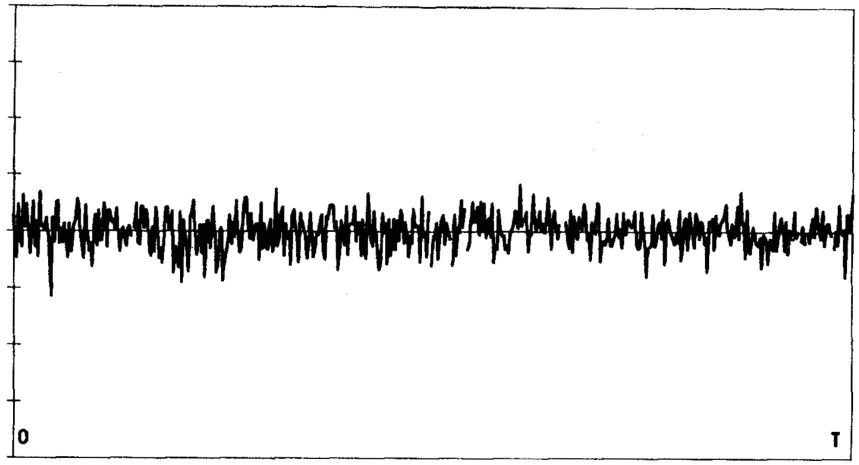
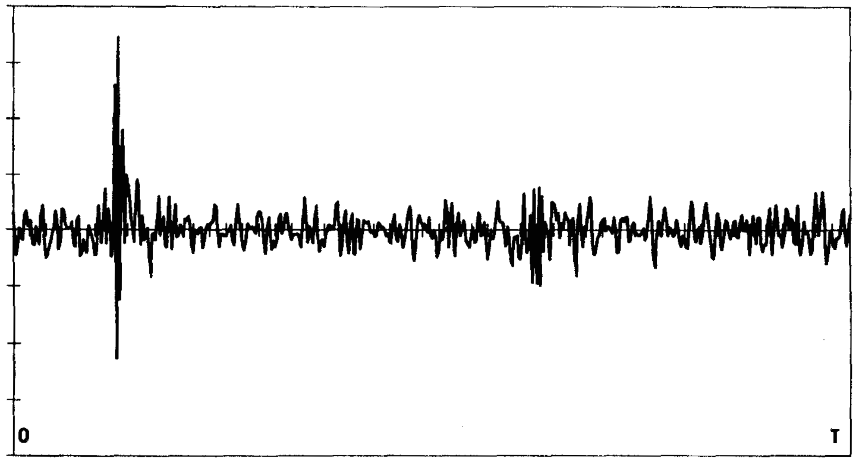


(a)



(b)

Fig. 1. Instantaneous amplitude $A(t)$ of two DMT symbols generated with two distinct sets of phasors $\{r_m^k\}_1$ and $\{r_m^k\}_2$, and $N = 256$.

The receiver at the far-end is informed about the application (or not) of the phasor-transformation at the transmitter, and applies the inverse transformation (Case b) or not (Case a) after demapping the QAM-modulated carriers. This extra information requires only one bit per DMT symbol. This bit could be provided by modulation of the pilot tone that otherwise carries no information and that is permanently used to maintain synchronization. Forward error correction coding and/or duplication of this information over another or several tones can be envisaged to improve the reliability of this data recovery.

Many phasor-transformations can be used. An easy-to-implement fixed random phasor transformation (known at the receiver) will be considered in what follows. Several other (more involved) transformations can be used as well without affecting the main results presented here.

The overall probability of clipping with the "two-pass" method described above can readily be obtained using (2). The probability that a given sample in the DMT symbol has an absolute amplitude larger than A_{Clip} ($A_{\mathrm{Clip}} > 0$) is simply given by

$$P = 2 \cdot \int_{A_{\mathrm{Clip}}}^{+\infty} p(x)\, dx = 1 - \mathrm{erf}\left(\frac{A_{\mathrm{Clip}}}{\sqrt{2} \cdot \sigma}\right) \tag{3}$$

where $\mathrm{erf}(t)$ is the error function defined by

$$\mathrm{erf}(t) = \frac{2}{\sqrt{\pi}} \cdot \int_0^t e^{-y^2}\, dy \cong 1 - \frac{e^{-t^2}}{l\sqrt{\pi}} \cdot \left[1 - \frac{1}{2t^2}\right].$$

Assuming $2N$ independent samples per DMT symbol, the probability that the symbol must be clipped after the first pass (i.e., at least one sample has an absolute amplitude larger than A_{Clip}) is given by

$$P_{\mathrm{Clip}/1} = 1 - (1 - P)^{2N}. \tag{4}$$

The validity of (4) has been confirmed with great accuracy (better than 1%) by extensive computer simulations.

We assume that due to the random phasor-transform (with large N), the probability that the symbol must be clipped after the second-pass, $P_{\mathrm{Clip}/2}$, is equal to $P_{\mathrm{Clip}/1}$. This is particularly the case if the transformation is a random bijection

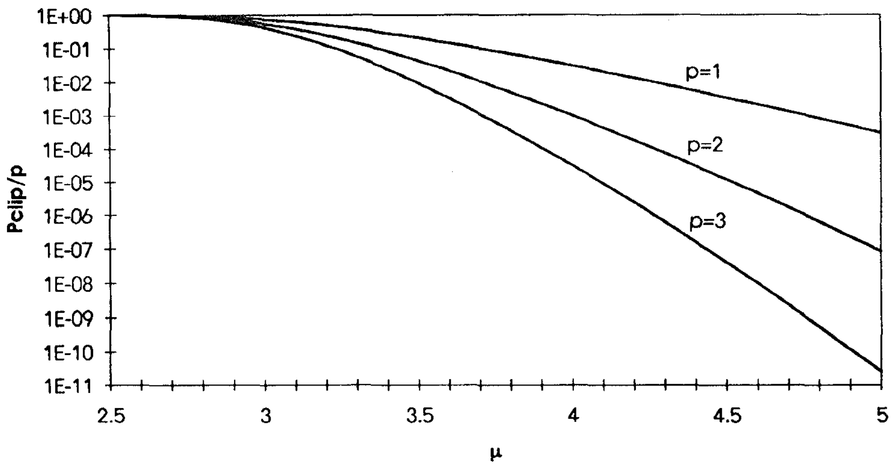


Fig. 2. Probability of clipping the transmitted DMT signal as a function of μ for $p = 1, 2, and 3$.

on the set of constellation points. This means that the sets of constellation points belonging to the sub-ensemble of clipped symbols are equiprobably transformed into the whole ensemble of DMT symbols. As the decision is taken after the first pass, only clipped symbols are submitted to a second pass, so that the overall clipping probability is given by

$$P_{\text{Clip/Total}} = P_{\text{Clip/1}} \cdot P_{\text{Clip/2}} = P^2_{\text{Clip/1}}. \tag{5}$$

Again, extensive computer simulations have been carried out to confirm the validity of (5).

Defining the dimensionless parameter $\mu = A_{\text{Clip}}/\sigma$ and making use of (3), (4), and (5), we obtain

$$P_{\text{Clip/Total}} = \left[1 - \text{erf}^{2N}\left(\frac{\mu}{\sqrt{2}}\right)\right]^2. \tag{6}$$

In general, for a p-pass method, the overall probability of clipping is given by $P_{\text{Clip/Total}} = P^p_{\text{Clip/1}}$, which tends rapidly to zero as p increases.

In practice, the number of passes (p) is limited by the speed of the IFFT at the transmitter. It is noteworthy, however, that e.g. the 2-pass method does not necessarily require a factor of 2 increase of the (I)FFT speed. Indeed, since the probability of clipping after the first pass is low, say less than 10^{-2}, the speed of the IFFT should only be increased by a few percent provided that a DMT symbol buffer is used to absorb the delay incurred by the second pass.

Note also that for $p > 2$, the method requires the transmission of $\log_2 p$ b to inform the far-end receiver about the number of passes that have been applied to generate the DMT symbol. In practice, this additional control information is negligible compared to the data rate.

Fig. 2 represents the clipping probability for the 1-pass, 2-pass, and 3-pass methods as a function of μ. For example, with $\mu = 4$ the 2-pass and the 3-pass methods reduce the probability of clipping down to 10^{-3} and less than 3.10^{-5}, respectively.

III. Performance Improvements

The system performance can be expressed in terms of the SNR for $p = 0$ (i.e., no clipping) and $\phi \geq 1$. In this section, we will restrict ourselves to the cases $p = 0, 1$, and 2 . Extension of the present analysis to the case where $p > 2$ is straightforward.

We consider an additive white Gaussian noise (AWGN) channel. Different noise sources will contribute to the overall SNR: a) at the transmitter: the clipping noise (only for $p = 1$ or 2) and the quantization noise of the D/A convertor, and b) at the receiver; the AWGN and the quantization noise of the A/D convertor.

We will assume a resolution of b b for the D/A and A/D convertors, and a quantization noise for $p = 0$ that is a factor α lower than the AWGN. Therefore, without clipping ($p = 0$), the quantization noise of the D/A and A/D convertors and the AWGN are, respectively, given by

$$Q_0 = \frac{(2A_{\max})^2}{12.2^{2b}} \tag{7}$$

$$\text{AWGN} = \alpha \cdot Q_0 \tag{8}$$

and where $A_{\max}$ is the maximum amplitude of the DMT symbols. $A_{\max}$ can be expressed as a function of the signal power σ^2 and the crest factor ν: $A_{\max} = \nu \cdot \sigma$. The crest factor is fixed by N and the QAM constellation size according to [6]

$$\nu = \frac{1 + \sqrt{2}}{2} \cdot \sqrt{3N} \cdot \sqrt{\frac{L-1}{L+1}} \tag{9}$$

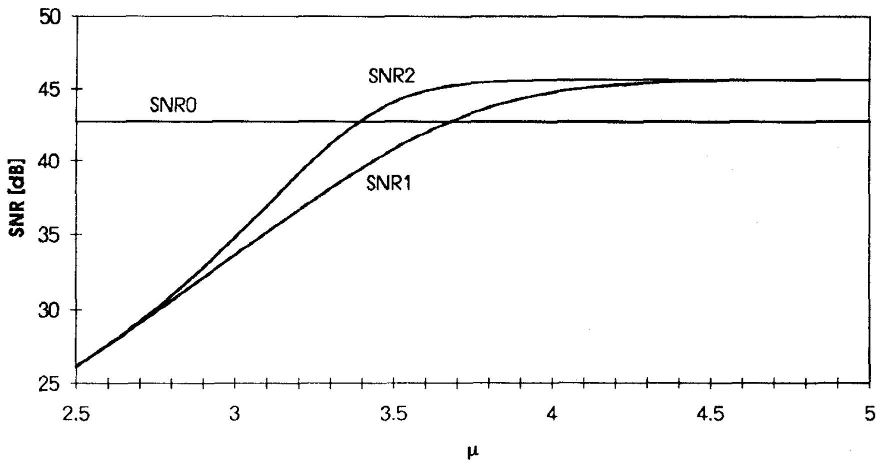
where L^2 is the QAM constellation size (e.g., $L = 4$ for 16-QAM).

When clipping is applied ($p = 1$ or 2), the quantization noise becomes

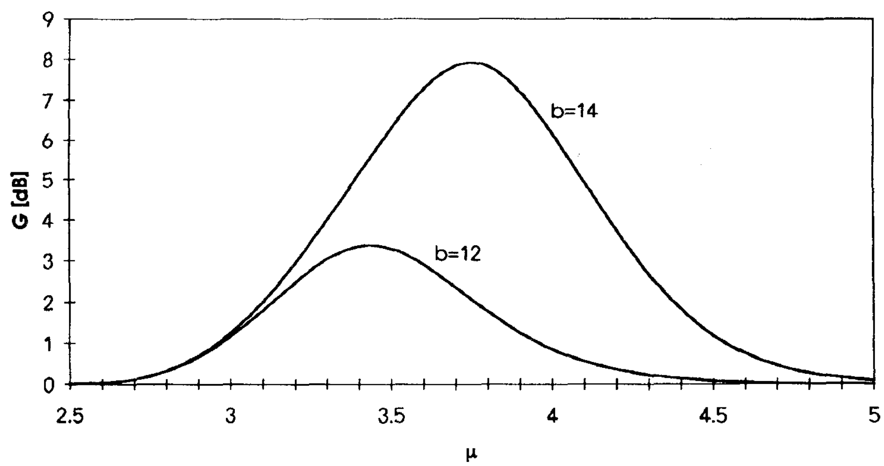
$$Q_{\text{Clip}} = \frac{(2A_{\text{Clip}})^2}{12.2^{2b}}. \tag{10}$$

The noise due to clipping has been derived in [6] for $p = 1$ and is rewritten here for convenience

$$N_{\text{Clip/1}} = 2 \cdot \int_{A_{\text{Clip}}}^{+\infty} (x - A_{\text{Clip}})^2 \cdot p(x)\, dx = \sigma^2$$



(a)



(b)

Fig. 3. (a) The SNR's (in dB) for $p = 0$ (no clipping), $p = 1$ and $p = 2$ as a function of μ for $a = 2, b = 12$ and (b) the gain G (in dB) as a function of μ for $a = 2, b = 12$ and $b = 14$.

$$\cdot \left\{ (1+\mu^2) \cdot \operatorname{erfc}\left(\frac{\mu}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} \cdot \mu \cdot e^{-(\mu^2/2)} \right\} \tag{11}$$

where $\operatorname{erfc}(t) = 1 - \operatorname{erf}(t)$.

In order to calculate the clipping noise for $p = 2$, we first need to determine the pdf of $A(t)$ for $|A| > A_{\text{Clip}}$, and then follow the same reasoning as in [6]. This pdf is readily obtained as

$$p'(x) = P_{\text{Clip}/1} \cdot p(x) \quad \text{for} \quad |x| > A_{\text{Clip}} \tag{12}$$

where $p(x)$ and $P_{\text{Clip}/1}$ are given by (2) and (4), respectively.

Therefore, the total power of the clipped portion of the DMT symbols for $p = 2$ is given by

$$N_{\text{Clip}/2} = 2 \cdot \int_{A_{\text{Clip}}}^{+\infty} (x - A_{\text{Clip}})^2 \cdot p'(x)\, dx = P_{\text{Clip}/1}.N_{\text{Clip}/1}. \tag{13}$$

Making use of $A_{\max}/A_{\text{Clip}} = \nu/\mu$, the overall noise for $p = 0$, 1 and 2 can be expressed as

$$N_{p=0} = 2 \cdot Q_0 + \text{AWGN} = (2+\alpha) \cdot Q_0 \tag{14.a}$$

$$\begin{aligned} N_{p=1} &= 2 \cdot Q_{\text{Clip}} + \text{AWGN} + N_{\text{Clip}/1} \\ &= \left[2\left(\frac{\mu}{\nu}\right)^2 + \alpha\right] \cdot Q_0 + N_{\text{Clip}/1} \end{aligned} \tag{14.b}$$

$$\begin{aligned} N_{p=2} &= 2 \cdot Q_{\text{Clip}} + \text{AWGN} + N_{\text{Clip}/2} \\ &= \left[2\left(\frac{\mu}{\nu}\right)^2 + \alpha\right] \cdot Q_0 + N_{\text{Clip}/2}. \end{aligned} \tag{14.c}$$

(The factor 2 in the right-hand side of (14) is due to the quantization noise of the D/A at the transmitter and the quantization noise of the A/D at the receiver.)

The associated SNR's are obtained using (3), (4), (7), (11), (13), and (14) and their closed-form expressions are given by

$$\text{SNR}_{p=0} = \frac{3.2^{2b}}{(2+\alpha) \cdot \nu^2} \tag{15.a}$$

$$\begin{aligned} \text{SNR}_{p=1} = \Bigg\{ &\left[2\left(\frac{\mu}{\nu}\right)^2 + \alpha\right] \cdot \left(\frac{\nu^2}{3.2^{2b}}\right) \\ &+ \Bigg[(1+\mu^2) \cdot \operatorname{erfc}\left(\frac{\mu}{\sqrt{2}}\right) \end{aligned}$$

$$- \sqrt{\frac{2}{\pi}} \cdot \mu \cdot e^{-(\mu^2/2)} \Bigg] \Bigg\}^{-1} \qquad (15.b)$$

$$\mathrm{SNR}_{p=2} = \Bigg\{ \left[2\left(\frac{\mu}{\nu}\right)^2 + \alpha \right] \cdot \left(\frac{\nu^2}{3.2^{2b}} \right) + \left(1 - \mathrm{erf}^{2N}\left(\frac{\mu}{\sqrt{2}} \right) \right) \cdot \Bigg[(1+\mu^2) \cdot \mathrm{erfc}\left(\frac{\mu}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \cdot \mu \cdot e^{-(\mu^2/2)} \Bigg] \Bigg\}^{-1}. \qquad (15.c)$$

The SNR's (in dB) for $p = 0, 1$, and 2 derived from (15) are plotted in Fig. 3(a) as a function of μ for $\alpha = 2$, $b = 12$ b and $\nu = 25.9$ [(9) with $L = 4$ and $N = 256$]. It is seen that, as compared with the $p = 0$ case, an improvement in SNR is obtained provided that $\mu \geq 3.7$ for $p = 1$ and $\mu \geq 3.4$ for $p = 2$. This improvement in SNR can be used at profit to reduce the required resolution of the D/A-A/D convertors as discussed in [6].

The system performance improvement provided by the proposed method $(p = 2)$ in comparison with $p = 1$ can be characterized by the gain G in SNR: $G = \mathrm{SNR}_{p=2}/\mathrm{SNR}_{p=1}$.

Fig. 3(b) shows the gain G (in dB) as a function of μ for $\alpha = 2$, $b = 12$ and $b = 14$. It is seen that G is bell-shaped and that its maximum value increases with b. The maximum gain is 3.4 dB for $b = 12$ and is as high as 7.9 dB for $b = 14$. Notice that for $p = 2$ the relevant values of G are only those associated with the values of μ that satisfy the condition $\mathrm{SNR}_{p=2} \geq \mathrm{SNR}_{p=0}$. Fortunately, for a given b, the value of μ that provides the maximum gain G is very close to the value of μ that satisfies the condition $\mathrm{SNR}_{p=2} = \mathrm{SNR}_{p=0}$.

IV. Conclusion

A method to decrease the probability of clipping DMT symbols by several orders of magnitude has been described. The method does not use any kind of pre-coding and hence, does not increase the actual transmission data rate. Significant improvements in SNR of about 3 dB up to 8 dB as compared to the case or brute force clipping can be achieved. This can be used at profit to reduce the required resolution of the D/A-A/D convertors, to decrease the maximum amplitude of the transmitted signal, or to provide an extra signal-to-noise margin of the communication system.

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