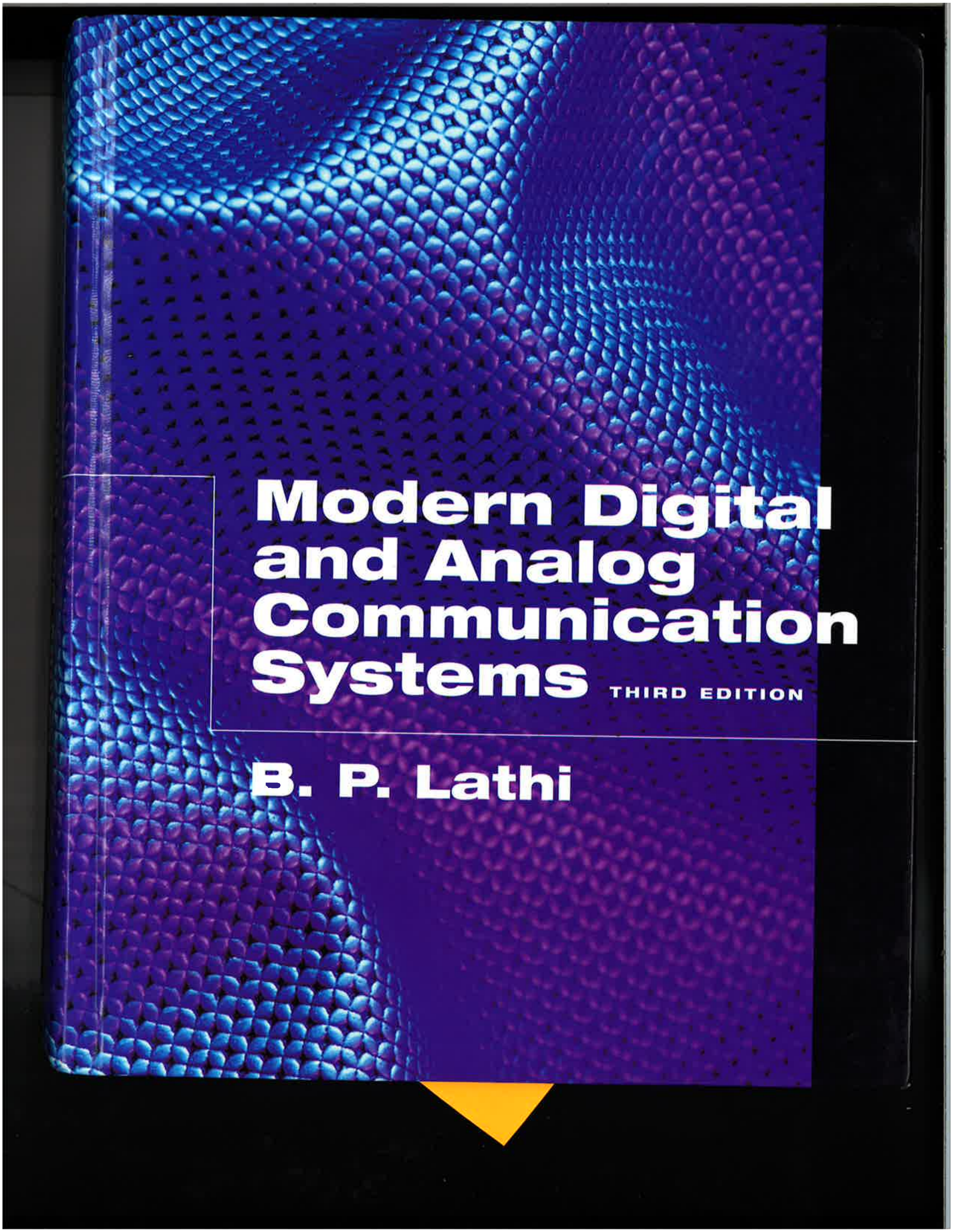
Modern Digital and Analog Communication Systems
THIRD EDITION
B. P. Lathi

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Although we have proved these results for a real $g(t)$, Eqs. (3.79), (3.80), (3.81), and (3.84) are equally valid for a complex $g(t)$.

The concept and relationships for signal power are parallel to those for signal energy. This is brought out in Table 3.3.

Signal Power Is Its Mean Square Value

A glance at Eq. (3.75) shows that the signal power is the time average or mean of its squared value. In other words P_g is the mean square value of $g(t)$. We must remember, however, that this is a time mean, not a statistical mean (to be discussed in later chapters). Statistical means are denoted by overbars. Thus, the (statistical) mean square of a variable x is denoted by $\overline{x^2}$. To distinguish from this kind of mean, we shall use a wavy overline to denote a time average. Thus, the time mean square value of $g(t)$ will be denoted by $\widetilde{g^2(t)}$. The time averages are conventionally denoted by pointed brackets, such as $< g^2(t) >$. We shall, however, use the wavy overline notation because it is much easier to associate means with a bar on top rather than the brackets. Using this notation, we see that

$$P_g = \widetilde{g^2(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)\, dt \tag{3.85a}$$

Note that the rms value of a signal is the square root of its mean square value. Therefore,

$$[g(t)]\text{rms} = \sqrt{P_g} \tag{3.85b}$$

From Eqs. (3.82), it is clear that for a real signal $g(t)$, the time autocorrelation function $\mathcal{R}_g(\tau)$ is the time mean of $g(t)g(t+\tau)$. Thus,

$$\mathcal{R}_g(\tau) = \widetilde{g(t)g(t \pm \tau)} \tag{3.86}$$

This discussion also explains why we have been using the term time autocorrelation rather than just autocorrelation. This is to distinguish clearly the present autocorrelation function (a time average) from the statistical autocorrelation function (a statistical average) to be introduced in a future chapter.

Interpretation of Power Spectral Density

Because the PSD is a time average of the ESD of $g(t)$, we can argue along the lines used in the interpretation of ESD. We can readily show that the PSD $S_g(\omega)$ represents the power per unit

Table 3.3

$E_g = \int_{-\infty}^{\infty} g^2(t)\, dt$	$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)\, dt = \lim_{T\to\infty} \frac{E_{g_T}}{T}$
$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)\, dt$	$\mathcal{R}_g(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)g(t+\tau)\, dt = \lim_{T\to\infty} \frac{\psi_{g_T}(\tau)}{T}$
$\Psi_g(\omega) = \lvert G(\omega)\rvert^2$	$S_g(\omega) = \lim_{T\to\infty} \frac{\lvert G_T(\omega)\rvert^2}{T} = \lim_{T\to\infty} \frac{\Psi_{g_T}(\omega)}{T}$
$\psi_g(\tau) \Longleftrightarrow \Psi_g(\omega)$	$\mathcal{R}_g(\tau) \Longleftrightarrow S_g(\omega)$
$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_g(\omega)\, d\omega$	$P_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_g(\omega)\, d\omega$

bandwidth (in hertz) of the spectral components at the frequency ω. The power contributed by the spectral components within the band ω_1 to ω_2 is given by

$$\Delta P_g = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} S_g(\omega)\, d\omega \tag{3.87}$$

Autocorrelation Method: A Powerful Tool

For a signal $g(t)$, the ESD, which is equal to $|G(\omega)|^2$, can also be found by taking the Fourier transform of its autocorrelation function. If the Fourier transform of a signal is enough to determine its ESD, then why do we needlessly complicate our lives by talking about autocorrelation functions? The reason for following this alternate route is to lay a foundation for dealing with power signals and random signals. The Fourier transform of a power signal generally does not exist. Moreover, the luxury of finding the Fourier transform is available only for deterministic signals, which can be described as functions of time. The random message signals that occur in communication problems (e.g., random binary pulse train) cannot be described as functions of time, and it is impossible to find their Fourier transforms. However, the autocorrelation function for such signals can be determined from their statistical information. This allows us to determine the PSD (the spectral information) of such a signal. Indeed, we may consider the autocorrelation approach as the generalization of Fourier techniques to power signals and random signals. The following example of a random binary pulse train dramatically illustrates the power of this technique.

EXAMPLE 3.23 Figure 3.42a shows a random binary pulse train $g(t)$. The pulse width is $T_b/2$, and one binary digit is transmitted every T_b seconds. A binary **1** is transmitted by the positive pulse, and a binary **0** is transmitted by the negative pulse. The two symbols are equally likely and occur randomly. We shall determine the autocorrelation function, the PSD, and the essential bandwidth of this signal.

We cannot describe this signal as a function of time because the precise waveform is not known due to its random nature. We do, however, know its behavior in terms of the averages (the statistical information). The autocorrelation function, being an average parameter (time average) of the signal, is determinable from the given statistical (average) information. We have [Eq. (3.82b)]

$$\mathcal{R}_g(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g(t)g(t-\tau)\, dt$$

Figure 3.42b shows $g(t)$ by solid lines and $g(t-\tau)$, which is $g(t)$ delayed by τ, by dashed lines. To determine the integrand on the right-hand side of the above equation, we multiply $g(t)$ with $g(t-\tau)$, find the area under the product $g(t)g(t-\tau)$, and divide it by the averaging interval T. Let there be N bits (pulses) during this interval T so that $T = NT_b$, and as $T\to\infty$, $N\to\infty$. Thus,

$$\mathcal{R}_g(\tau) = \lim_{N\to\infty} \frac{1}{NT_b}\int_{-NT_b/2}^{NT_b/2} g(t)g(t-\tau)\, dt$$

Let us first consider the case of $\tau < T_b/2$. In this case there is an overlap (shown by the shaded region) between each pulse of $g(t)$ and that of $g(t-\tau)$. The area under