Intuitive Guide to Principles of Communications
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Understanding Frequency Modulation (FM), Frequency Shift Keying (FSK), Sunde's FSK and MSK and some more

The process of modulation consists of mapping the information on to an electromagnetic medium (a carrier). This mapping can be digital or it can be analog. The modulation takes place by varying the three parameters of the sinusoid carrier.

1. Map the info into amplitude changes of the carrier
2. Map the info into changes in the phase of the carrier
3. Map the info into changes in the frequency of the carrier.

The first method is known as **amplitude modulation.** The second and third are both a from of **angle modulation**, with second known as phase and third as frequency modulation.

Let's start with a sinusoid carrier given by its general equation

$$c(t) = A_c \cos(2\pi f_c t + \phi_0)$$

This wave has an amplitude A_c a starting phase of fð$_0$ and the carrier frequency, f_c. The carrier in Figure 1 has amplitude of 1 v, with f_c of 4 Hz and starting phase of 45 degrees.

Generally when we refer to amplitude, we are talking about the maximum amplitude, but amplitude also means any instantaneous amplitude at any time t, and so it is really a variable quantity depending on where you specify it.

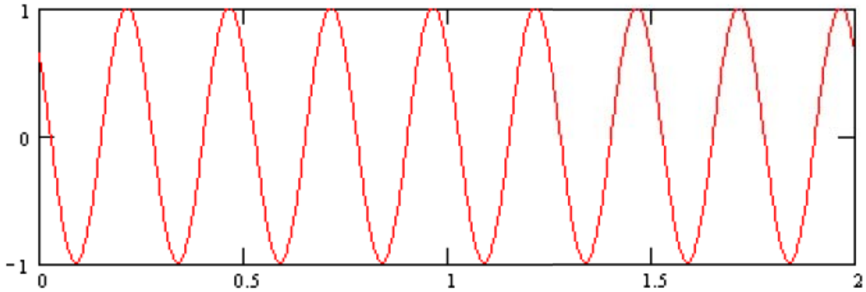
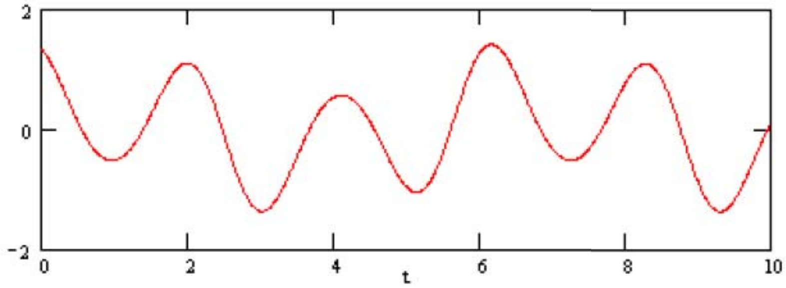
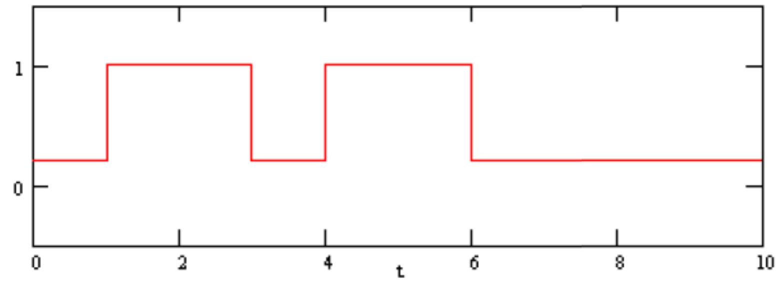


Figure 1 – A sinusoid carrier of frequency 4, starting phase of 45 degrees and amplitude of 1 volt.

The amplitude modulation changes the amplitude (instantaneous and maximum) in response to the information. Take the following two signals; one is analog and the other digital



a. Analog message signal, m(t)



b. Binary message signal, m(t)

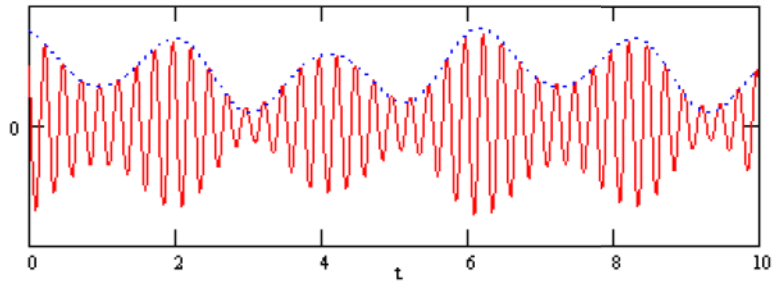
Fig 2 – Two arbitrary message signals, m(t)

About Amplitude Modulation (AM)

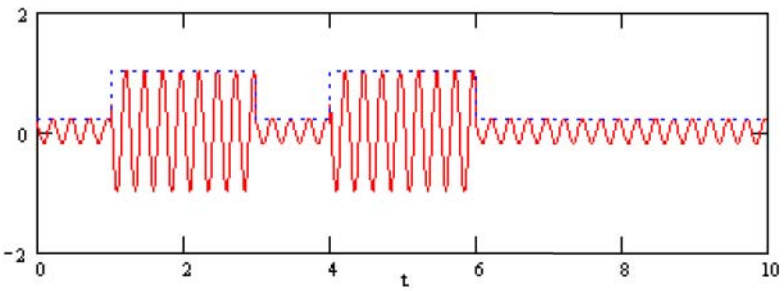
The amplitude modulated wave is created by multiplying the *amplitude* of a sinusoid carrier with the message signal.

$$\begin{aligned} s(t) &= m(t)\,c(t) \\ &= A_c m(t)\cos(2\pi f_c t + \phi_0) \end{aligned}$$

The modulated signal shown in Figure 3, is of carrier frequency f_c but now the amplitude changes in response to the information. We can see the analog information signal as the envelope of the modulated signal. Same is true for the digital signal.



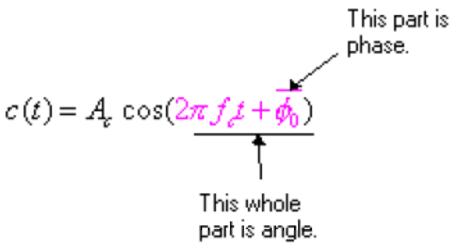
a. Amplitude modulated carrier shown with analog message signal as its envelope



c. Amplitude modulated carrier shown with binary message signal as its envelope

Figure 3 – Amplitude modulated carrier, a. analog, b. digital

Let’s look at the argument of the carrier. What is it?



The argument is an angle in radians. The argument of a cosine function is always an angle we know that from our first class in trigonometry. The second term is what is generally called the **phase**. In amplitude modulation only the amplitude of the carrier changes as we can see above for both binary and analog messages. Phase and frequency retain their initial values.

Any modulation method that changes the angle instead of the amplitude is called *angle modulation*. The angle consists of two parts, the phase and the frequency part. The modulation that changes the phase part is called phase modulation (PM) and one that changes the frequency part is called frequency modulation (FM).

How do you define frequency? Frequency is the number of 2 pð revolutions over a certain time period. Mathematically, we can write the expression for average frequency as

$$f_{\Delta t} = \frac{\phi_i(t+\Delta t) - \phi_i(t)}{2\pi\,\Delta t} \qquad 1$$

This equation says; *the average frequency is equal to the difference in the phase at time t + Dðt and time t, divided by 2pð* (or 360 degrees if we are dealing in Hz.)

Example: a signal changes phase from 45 to 2700 degrees over 0.1 second. What is its average frequency?

$$= \frac{2700-45}{360*0.1} = 73.75 \text{ Hz}$$

This is the average frequency over time period t = 0.1 secs. Perhaps it will be different over 0.2 secs or some other time period or maybe not, we don't know.

What is the instantaneous frequency of this signal at any particular moment in the 0.1 second period? We don't really know given this information.

The instantaneous frequency is defined as the limit of the average frequency as Dðt gets smaller and smaller and approaches 0. So we take limit of equation 1 to create an expression for instantaneous frequency, $f_i(t)$,.

The $f_i(t)$ is the limit of $f_{D ðt}(t)$ as Dðt goes to 0. The phase change over time Dðt is changed to a differential to indicate change from discrete to continuous.

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$
$$= \lim_{\Delta t \to 0} \frac{\phi_i(t+\Delta t) - \phi_i(t)}{2\pi\, \Delta t}$$
$$= \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$$

This last result is very important in developing understanding of both phase and frequency modulation! The 2pð factor has been moved up front. The remaining is just the differential of the phase.

Another way we can state this is by recognizing that radial frequency wð is equal to

$$\omega = 2\pi\, f_i(t)$$

It is also equal to the rate of change of phase,

$$\omega = \frac{d(\phi_i(t))}{dt}$$

so again we get,

$$f_t(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt} \qquad 2$$

Intuitively, it says; the frequency of a signal is equal to its phase change over time. When seen as a phasor, the signal phasor rotates in response to phase change. The faster it spins (phase change), the higher its frequency.

What does it mean, if I say: the phasor rotates for one cycle and then changes directions, goes the opposite way for one cycle and then changes direction again? This is a representation of a phase modulation. Changing directions means the signal has changed its phase by 180 deg.

We can do a simple minded phase modulation this way. Go N spins in clockwise directions in response to a 1 and N spins in counterclockwise directions in response to a 0. Here N represents frequency of the phasor.

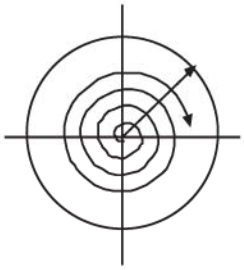


Figure 4 – The carrier as a phasor, the faster it spins, the higher the frequency.

If frequency is the rate of change of phase, then what is phase in terms of frequency? As we know from definition of frequency that it is number of full 2 pð rotations in a time period.

Given, a signal has traveled for 0.3 seconds, at a frequency of 10 Hz with a starting phase of 0, what is its phase now?

$$Phase\ now\ \phi = 2\pi f_i t = 10\,Hz \times 2\pi \times .3 = 20\ radians$$

This is an integration of the total number of radians covered by the signal in 0.3 secs. Now we write this as an integral,

$$\phi_i(t) = 2\pi \int_0^{.3} f_i(t)dt$$

and since this is average frequency, it is constant over this time period, we get

$$Phase\ now\ \phi = 2\pi f_i t = 10\,Hz \times 2\pi \times .3 = 20\ radians$$

We note the phase and frequency are related by

Phase a Integral of frequency

Frequency a Differential of phase

Phase modulation

Let the phase be variable. Going back to the original equation of the carrier, change the phase (the underlined term only) from a constant to a function of time.

$$c(t) = A_c \cos(2\pi f_c t + \underline{\phi(t)}\ )$$

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