

# Exploring Legal Patent Citations for Patent Valuation

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## ABSTRACT

Effective patent valuation is important for patent holders. Forward patent citations, widely used in assessing patent value, have been considered as reflecting knowledge flows, just like paper citations. However, patent citations also carry legal implication, which is important for patent valuation. We argue that patent citations can either be technological citations that indicate knowledge transfer or be legal citations that delimit the legal scope of citing patents. In this paper, we first develop citation-network based methods to infer patent quality measures at either the legal or technological dimension. Then we propose a probabilistic mixture approach to incorporate both the legal and technological dimensions in patent citations, and an iterative learning process that integrates a temporal decay function on legal citations, a probabilistic citation network based algorithm and a prediction model for patent valuation. We learn all the parameters together and use them for patent valuation. We demonstrate the effectiveness of our approach by using patent maintenance status as an indicator of patent value and discuss the insights we learned from this study.

## 1. INTRODUCTION

Patent valuation, i.e., assessing the value of patents, is an important but challenging task for firm technology and innovation management. Patent citations have been widely used in patent valuation [19, 8, 6, 7] on the ground that patent citations provide “paper trails” of knowledge flows among patents. The fact that a patent cites a large number of prior patents (hereafter, *backward citations*) suggests that the patented invention has built upon “the shoulders of giants”, i.e., a significant amount of prior knowledge. This implies that the invention has great **technological richness** (defined as the amount of prior knowledge a patent builds upon) and likely high technological quality and economic value. Similarly, when a patent is cited by a large number of subsequent patents (hereafter, *forward citations*), this indicates that the patented invention has led to a number of successful lines of innovation. Thus, the invention is likely to

be of high **technological influence** (defined as the technological impact that a patent has on subsequent inventions) and thus highly economically valuable.

From such technological aspects, one might think that patent citations are similar to paper citations. However, patent citations are actually quite different from paper citations in significant ways. In particular, patent citations could be interpreted in two dimensions: (i) a technological one that is related to knowledge flows, and (ii) a legal one that is related to delimitation of patent scope. Let’s consider the scenario where a patent application is under examination. The patent examiner needs to search for relevant prior art (prior inventions) to determine whether the invention is patentable based on its novelty and inventiveness in comparison to these prior inventions. Meanwhile, the examiner needs to determine the appropriate scope of the patent right by asking the applicant to modify, if necessary, the language of the claims. For example, if an inventor applies for a chemical compound that makes some novel structural modification to an existing drug. The examiner would grant a patent to the invention, but cite the prior patent on the existing drug to: (i) show the knowledge link between the two inventions and (ii) to narrow down the scope of the newly granted patent so that it would cover only the modification, not the original chemical structure. In fact, the scope of the newly granted patent could be so narrowed down by the prior patent on the existing drug that the firm owning the newly issued patent may have to get a license from the patentee of the existing drug patent in order to market the new drug. In this case, the cited prior patent acts as a blocking patent to the newly granted patent. Similarly, an applicant, under the U.S. Patent Law, has the obligation to disclose relevant prior art that she knows during her research, though she has no obligation to identify all possible relevant prior art when filing an application. These applicant-inserted citations could, on one hand, suggest knowledge flows, but on the other hand, be used to narrow down the patent scope.

Therefore, a citation made by patent *A* to patent *B* could suggest that there are knowledge flows from the cited patent to the citing patent (this aspect of patent citations defines the *technological citations*), or that the cited patent puts legal constraints on the scope of the citing patent (this aspect defines the *legal citations*), or both. The legal interpretation of patent citations has quite different implication in terms of what patent citations mean for patent valuation, compared to the technological interpretation of patent citations. From the legal aspect, when a patent cites a large number of backward citations, it could suggest that many prior patents

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might have been used to narrow down the scope of the citing patent or block the citing patent. Consequently the citing patent might have a very narrow **legal patent scope** (i.e., the scope of patent right claimed), and thus likely small commercial value. On the other hand, when a patent receives a large number of forward citations, it might be blocking or putting constraints on these subsequent patents. In this case, a patent with a large number of forward citations implies a high level of **legal blocking power**, and thus it is a highly valuable patent.

Furthermore, legal constraints in patent citations (from the legal aspect) are time sensitive. For a patent citation with two year lag between the citing and cited patents, the cited patent could block the citing patent for a long time. However, if a patent cites an expired prior patent, then the cited patent put no legal constraints on the citing patent.

Our study aims to explore the insights about the technological and legal dimensions of patent citations and propose corresponding measures for patent valuation. Specifically, based on the technological and legal interpretations of patent citations, we propose to capture four quality measures of patents, namely, *technological richness*, *technological influence*, *legal patent scope*, and *legal blocking power*.

More importantly, and quite intuitively, we propose that there exist mutual interdependence among these four measures of patents; and the two measures in the technological dimension are related to each other in a different way than the other two measures in the legal dimension are related. Consider the technological richness and influence associated with a patent *A*. If patent *A* cites prior patents that are of greater technological influence, other things being equal, the technological richness of the citing patent *A* is greater, as the invention builds on a lot of influential prior inventions. Meanwhile, if patent *A* is cited by subsequent patents that are of high technological richness, the patent *A*'s technological influence would be greater, as it leads to subsequent innovations of high quality.

By an interesting contrast, if patent *A* cites prior patents that are of greater legal blocking power, other things being equal, the legal patent scope of the citing patent *A* is smaller, as it is narrowed down by prior patents with large blocking power. However, if patent *A* is cited by subsequent patents that are of large legal patent scope, the patent *A*'s legal blocking power is greater as it put constraints on subsequent patents with broad patent right that are highly valuable.

We investigate different methods to quantify the four proposed measures. We first assume that a patent citation can be interpreted in the technological dimension or in the legal one (or both). We then consider the case where a patent citation represents a probabilistic mixture of both technological and legal citations, with the significance of legal citations decaying by time (i.e., the grant lag between a cited and citing patents). Accordingly, we capture their mutual interactions and iteratively learn the four measures using the patent data. Technically, we adopt a parameter learning process that integrates multiple models (including a temporal decay in legal citations, a probabilistic citation network based algorithm for quantifying the four proposed patent quality measures, and a prediction model for patent valuation).

To validate our idea of distinguishing legal citations from technological citations, we empirically apply the four proposed patent quality measures in patent valuation. We use

patent renewal status (patent maintenance) as an indicator of patent value in our experimentation. Our results show that separating technological and legal dimension in patent citations achieves better accuracy in experiments for patent value prediction. And our proposed patent quality measures based on legal citations show more important roles in predicting patent value than measures based on technological citations. Our study also confirms the mutual interdependence between technological influence and technological richness is different from that between blocking power and legal patent scope. Moreover, by applying a probabilistic model to quantify the proposed concepts, we validate that patent citation is a probabilistic mixture of technological and legal indications and the significance of a legal citation decays by time.

To the best of the authors' knowledge, this work represents the first attempt to explore the insight that a patent citation could be a mixture of technological and legal citations, to quantify the technological quality measures and legal quality measures corresponding to the two different dimensions in patent citations, and to apply them in patent valuation. In summary, our work has made the following major contributions:

- This study aims to exploit the technological and legal interpretation of patent citations and apply them to patent valuation.
- Four different patent quality measures, namely, technological richness, technological influence, legal patent scope and legal blocking power, and the interactions among them are proposed to capture the technological and legal information imbedded in patent citations.
- We propose a probabilistic model that considers a patent citation as a probabilistic mixture of technological and legal citations, with the relative weight on the legal dimension decays by time. We develop an algorithm that captures the interdependence among the four proposed patent quality measures to iteratively derive these measures and learn the model parameters, which are useful for analysis of patents in a firm or a field.
- Using patent renewals as an indicator of patent value, our experiments show that considering both the technological and legal dimensions of patent citations and applying these four patent measures can significantly improve patent evaluation, compared to the current practice that only involves the technological interpretation in patent evaluation.

The rest of this paper is organized as follows. We first assume a deterministic model of patent citations and introduce our algorithms to derive measures related to technological citations and legal citations in Section 2. Then we develop a probabilistic mixture model of patent citations to better capture those measures in Section 3. In Section 4, we introduce our evaluation methodology and conduct experiments on valuation of Drug&Medical patents in both firm-level and field-level. We finally review related works in Section 5 and draw conclusions in Section 6.

## 2. DETERMINISTIC MODEL AND ALGORITHMS

With the technological and legal interpretations for patent citations, an immediate question is how to model and quantify technological and legal citations. In this section, we first

discuss a heuristic and deterministic model for interpreting the technological and legal dimensions of patent citations. Then we propose algorithms to quantify the four proposed patent quality measures: technological influence, technological richness, blocking power, and legal patent scope, for a focal patent.

## 2.1 Modeling of Patent Citations

Here we assume that a patent citation always reflects knowledge flow from the cited patent to the citing patent. Therefore, the technological dimension of a patent citation *always* exists. However, the legal dimension of a patent citation only exists when the cited patent is not expired. In other words, if the cited patent is not expired when the citation is made, the citation is of both a legal and technological citation. Otherwise, the citation only reflects the technological (knowledge) flow. Consider an example where patent *A* is granted in year 2000 and was maintained (renewed) at its 4<sup>th</sup> year renewal but not at the 8<sup>th</sup> year. If a patent *B* cites patent *A* in year 2006, we consider the citation to have both legal and technological interpretations. However, for a patent *C* citing patent *A* in year 2009, the citation is only a technological citation. Accordingly, it is fairly easy to determine whether a patent citation is a legal citation or not, since we know when a patent is expired (based on the data on patent maintenance at *USPTO*).

**Citation Graphs.** Based on the discussion above, we derive two citation graphs. One represents the technological citation network, denoted as  $G_T = (V, E_T)$  where  $G_T$  is the same as the original patent citation network because here we assume that a patent citation always serves its technological functionality, i.e.,  $E_T(i, j) = 1$ , if  $p_i$  is cited by  $p_j$ .<sup>1</sup> On the other hand, the legal citation network, which captures the legal implication between patents, is denoted as  $G_L = (V, E_L)$  where  $E_L(i, j) = 1$  if  $p_i$  is not expired at the grant year of  $p_j$ ; and  $E_L(i, j) = 0$  if  $p_j$  is expired when  $p_j$  is granted.<sup>2</sup> Based on these two citation graphs, we propose to characterize a patent with four quality measures: technological influence score, technological richness score, blocking power score and legal patent scope score. In the following, we describe two basic approaches in quantifying the four features: one is the *CiteCount* algorithm and the other is the *CiteNet* algorithm.

## 2.2 CiteCount Algorithm

The CiteCount algorithm, similar to the conventional citation counting approach for assessing the quality of scientific literature, counts the number of citations of different types, based on the technological and legal citation graphs. Given the technological citation graph  $G_T$  and the legal citation graph  $G_L$ , we formally define the Technological Influence score (**TI**), Technological Richness score (**TR**), Blocking Power score (**BP**) and Legal Patent Scope score (**LS**) of a patent  $p_i$  as follow.

$$TI_i = \sum_j E_T(i, j) \quad (1)$$

$$TR_i = \sum_j E_T(j, i) \quad (2)$$

where  $E_T(i, j)$  refers to a technological citation to patent  $i$  made by patent  $j$ . Therefore, for patent  $p_i$ , its techno-

<sup>1</sup> $V$  is the set of patents and  $E_T$  is the set of edges in  $G_T$ .

<sup>2</sup> $E_L$  is the set of edges in  $G_L$ .

logical influence score is the number of forward citations it receives, while its technological richness score is the number of backward citations it makes.

$$BP_i = \sum_j E_L(i, j) \quad (3)$$

$$LS_i = \gamma - \sum_j E_L(j, i) \quad (4)$$

where  $E_L(i, j)$  refers to a legal citation to patent  $i$  made by patent  $j$  and  $\gamma$  is the global value of legal scope in all patents. Thus, for patent  $p_i$ , its blocking power score is the number of legal citations to  $p_i$  when it is not expired, while its legal scope score is dependent on the number of citations which it makes when the cited patents are not expired. With  $p_i$  making more legal citations, its legal scope is likely further narrowed. Therefore, the resulting legal scope is reduced from the original legal scope of  $p_i$  by its legal citations. Two issues arising here are (i) the setting of the initial (global) legal scope value and (ii) the amount of the legal scope to be deducted from this patent due to legal citations. We consider legal patent scope of patent  $i$ , as defined by Eq. (4), to be greater than 0. Thus,  $\gamma$  is inherently greater than the maximal number of legal citations/references made by patent  $p_i$ . While in this paper all patents are assumed to have the same initial legal patent scope, we may empirically set appropriate  $\gamma$  and use it to analyze the initial legal scopes in different domains or patent sets.

## 2.3 CiteNet Algorithm

As CiteCount only counts on one-hop neighbors in the graph, the potential influence of neighbor patents located in multi-hop neighborhood is not considered. As a result, the relationship and interdependence between technological influence and richness as well as between blocking power and legal influence are not well considered. To address this issue, we derive CiteNet, a patent citation network based algorithm, to capture these mutual independence. In the algorithm, we make the following intuitive assumptions:

1. The technological influence of a patent is determined by the number of technological citations it receives and the technological richness of these citing patent. The technological influence of a patent will be higher if it is cited by subsequent patents of higher technological richness, as it leads to follow-up innovations of high technological quality.
2. The technological richness of a patent depends on the number of technological citations it makes and the technological influence of these cited patents. The technological richness of a patent will be higher if it cites patents of higher technological influence because it is based on prior innovations of high technological impacts.
3. The blocking power of a patent depends on the number of legal citations it receives and the legal patent scope of these citing patents. The blocking power of patent will be greater, if it is cited by patents with larger legal patent scope because it blocks patents with broader scope.
4. The legal patent scope of a patent is determined by the number of the legal citations it makes and the blocking power of these cited patents in the legal citations.

Intuitively, the legal patent scope of a patent will be smaller if it cites a lot of prior patents with stronger blocking power, because these cited patents would narrow down its scope.

Therefore, given a citation network graph  $G = (V, E)$ , the CiteNet algorithm computes the four quality measures for each patent iteratively until the derived measures converge. In each iteration, the measures are derived as follows.

$$TI_i \leftarrow \sum_j TR_j \quad \text{where } E_T(i, j) = 1 \quad (5)$$

$$TR_i \leftarrow \sum_j TI_j \quad \text{where } E_T(j, i) = 1 \quad (6)$$

Note that Eq.(5) corresponds to the first assumption and Eq.(6) corresponds to the second assumption. Moreover, given the legal citation network  $G_L$  and a patent  $p_i$ , we have:

$$BP_i \leftarrow \sum_j LS_j \quad \text{where } E_L(i, j) = 1 \quad (7)$$

$$LS_i \leftarrow \gamma - \sum_j BP_j \quad \text{where } E_L(j, i) = 1 \quad (8)$$

Eq. (7) corresponds to the third assumption and Eq. (8) corresponds to the fourth assumption. Also, similar to the CiteNet method,  $\gamma$  is the initial value of legal scope and we subtract the blocking power of legal citations that  $p_i$  cites from its original patent legal scope. Moreover, after each iteration, we normalize the calculated scores for the four quality measures using  $\mathcal{L}$ -norm normalization to guarantee the convergence of the algorithm.

In summary, CiteNet captures the independence between technological influence and technological richness, and blocking power and legal scope in each iteration as shown in Eqs. (5)-(8). The proposed CiteNet algorithm derives the patent technological influence and richness in a way similar to the HITS algorithm that captures the mutually reinforcement between authoritative and hub web pages [13]. On the other hand, the derivation of the patent legal blocking power and patent scope is totally different. As Eqs. (7) and (8) show, they are based on different rules.

### 3. PROBABILISTIC MODELING

In the previous section, we assume that a citation is always a technological citation. Moreover, depending on whether the cited patent is expired at the time of being cited, the citation could have legal implication on the citing patent. Note that some potential issues may arise with such deterministic heuristics. For example, some patent citations are counted twice (as a technological citation and as a legal citation), whereas others are counted only once (only as a technological citation). This seems to be ad hoc. Is there a better way to model the two dimensions of a patent citation coherently? In particular, as explained earlier, a patent citation is likely to be a mixture of a technological citation and a legal citation, with different weights.

Additionally, the legal measures of a cited patent, corresponding to a patent citation, are assumed to remain constant, whether it is cited by a citing patent granted just a few years later or by another patent granted many years later. We argue that intuitively it may be more reasonable to assume that the legal power of a cited patent varies corresponding to different citing patents granted at different

years, given that they are all granted before the expiration of the cited patent. In other words, it is more realistic to assume that the legal implication of a patent citation decays as a function of the lagging years between the cited patent and the citing patent until the cited patent is expired.<sup>3</sup>

Therefore, we propose to adopt a probabilistic approach to model patent citations. We assume that the technological and legal interpretation of a patent citation takes some weights, i.e., their total weight equals one. Meanwhile, we assume that the weight for the legal dimension decays as a function of the lagging years between the cited and the citing patents. Moreover, it becomes zero when the cited patent expires.

With such a probabilistic/mixture model of patent citations, we propose, similar to Section 3, *probabilistic citation count (ProbCiteCount)* and *probabilistic citation network (ProbCiteNet)* to quantify the four quality measures of a patent. ProbCiteNet takes into consideration the citation network structure and the interdependence between the two technological measures and between the two legal measures, while ProbCiteCount does not.

Once the decay behavior of legal citations, the constraint over total weight of legal and technological citations, and the relationships among legal and technological quality measures are properly modeled, we shall be able to model the correlations between the quality measures and the patent value, e.g., by formulating it as a classification problem. Putting all these together allows us to not only to derive quality measures but also learn the model parameters (such as the time decay parameter) and analyze the importance of the patent quality measures in the classifiers for patent evaluation. Furthermore, learning the various model parameters enable us to study insights about valuation of patent citations in a field or a firm.

In the following, we first detail our approach to model the decay function for legal citations (see Section 3.1) as well as ProbCiteCount (see Section 3.2) and ProbCiteNet (see Section 3.3). Then, we introduce our prediction model for patent value and the learning process for deriving model parameters (see Section 3.4).

#### 3.1 Temporal Decay of Legal Citation Weights

Here we discuss the selection of a decay function to model the temporal decay of legal power. Two candidate functions are exponential decay or linear decay functions. An exponential decay function assumes that the rate of decay is proportional to its current value, while a linear decay function assumes that the rate of decay is constant over time, which is less applicable to our case. Consider a case where patent  $A$  (granted in 2000 and to expire in 2016) is cited by patent  $B$  in 2002 and by patent  $C$  in 2003. Since these inventions are very close in time, they are likely close substitutes to each other in the market. Thus, the one year difference between the citing patents  $B$  and  $C$  could mean significantly different market values. Consequently, the weights on the legal dimension of the two patent citations could be quite different. However, suppose that patent  $A$  is cited by patent  $D$  in 2014 and by patent  $E$  in 2015, which are very far away from patent  $A$  (which is about to expire). In this case, the weights for the legal dimension of the citations correspond-

<sup>3</sup>Here the lagging years refers to the number of years the grant date of the citing patent is lagging behind the grant date of the cited patent.



ing to  $A$  to  $D$  and  $A$  to  $E$  should be no much different, because inventions in patent  $A$  is fading out of the market when it is cited by patent  $D$  and  $E$ .

Moreover, the legal weight of a patent citation depends on the power of the cited patent in narrowing down and/or blocking the citing patent. Therefore, the rate at which the weight on a legal citation decays should be correlated with the turnover or product life cycle for the technology field of the cited patent. Hence, in this model, we assume that the technology domain of the cited patent, which reflected by the U.S. patent class of the cited patent, determines the temporal decay pattern, i.e., patents in the same U.S. class share the same temporal decay pattern.

Formally, let the parameter of the decay function for a given U.S. class  $u$  be  $\lambda_u$ . Given two patents  $p_i$  and  $p_j$  where  $p_i$  is cited by  $p_j$ , we define the weight for the legal dimension of this citation as follows.

$$P_L(p_i, p_j) = \begin{cases} \lambda_u e^{-\lambda_u |t_j - t_i|} & p_i \text{ is not expired when cited} \\ & \text{by } p_j \\ 0 & p_i \text{ is expired when cited by } p_j \end{cases}$$

where  $t_i$  and  $t_j$  are the grant dates of  $p_i$  and  $p_j$  respectively, and  $u$  denotes the U.S. Class of the cited patent  $p_i$ .

As we consider any citation to be a mixture of the technological and legal dimensions (with their total weight equals 1), the technological weight for patent  $p_j$  citing patent  $p_i$  is defined as follows.

$$P_T(p_i, p_j) = 1 - P_L(p_i, p_j) = 1 - \lambda_u e^{-\lambda_u |t_j - t_i|} \quad (10)$$

As such, Eq. (9) and Eq. (10) govern the weighted technological citation network and legal citation network, respectively. Next, we discuss the modeling of interdependency among the four quality measures with ProbCiteCount and ProbCiteNet.

### 3.2 Probabilistic Citation Count Algorithm

To present the probabilistic citation count (ProbCiteCount) algorithm, we first introduce an adjacency matrix defined for deriving the four quality measures, based on the weighted technological and legal citation networks.

**Adjacency Matrix for Technological and Legal Citations.** We use an adjacency matrix  $A_T$  to denote technological citations and  $A_L$  to denote the legal ones.  $A_T(i, j) = P_T(p_i, p_j)$  if  $p_i$  is cited by  $p_j$ , otherwise  $A_T(i, j) = 0$ . On the other hand,  $A_L(i, j) = P_L(p_i, p_j)$  if  $p_i$  is cited by  $p_j$ , otherwise  $A_L(i, j) = 0$ .

Based on  $A_T$  and  $A_L$ , we define technological influence (**TI**) and technological richness (**TR**) for a given patent  $p_i$  as follows.

$$TI_i \leftarrow \sum_j A_T(i, j) \quad (11)$$

$$TR_i \leftarrow \sum_j A_T^T(i, j) \quad (12)$$

where  $j$  is bounded to the set of patents in the corpus citing  $p_i$  and  $A_T^T$  is the transpose matrix of  $A_T$ . Next, we define blocking power (**BP**) and legal patent scope (**LS**) for a given patent  $p_i$ , based on citation count as follows.

$$BP_i \leftarrow \sum_j A_L(i, j) \quad (13)$$

$$LS_i \leftarrow \gamma - \sum_j A_L^T(i, j) \quad (14)$$

where  $A_L^T$  is the transpose of  $A_L$  and  $\gamma$  is the global legal patent scope for each patent, defined in Section 2.

### 3.3 Probabilistic Citation Network Algorithm

Given the weighted technological citation matrix  $A_T$  and legal citation matrix  $A_L$  defined above, the ProCiteNet algorithm iteratively derives the four quality measures, based on the mutual interdependence among them, as discussed in Section 2. Formally, we define technological influence (**TI**) and technological richness (**TR**) for the given patent  $p_i$  as follows.

$$TI_i \leftarrow \sum_j A_T \cdot TR_j \quad (15)$$

$$TR_i \leftarrow \sum_j A_T^T \cdot TI_j \quad (16)$$

In Eq. (15) and Eq. (16),  $j$  refers to the patents citing patent  $p_i$  and the patents cited by patent  $p_i$ , respectively.

Similarly, we define blocking power score (**BP**) and legal patent scope score (**LS**) for the given patent  $p_i$  as follows.

$$BP_i \leftarrow \sum_j A_L \cdot LS_j \quad (17)$$

$$LS_i \leftarrow (\gamma - \sum_j A_L^T \cdot BP_j) \quad (18)$$

where  $\gamma$  is the initial value of legal scope for each patent, and  $j$  refers to the patents citing patent  $p_i$  and the patents cited by patent  $p_i$ , respectively.

ProbCiteNet, similar to HITS algorithm [13], considers mutual reinforcement between the technological influence and richness, as well as the blocking power and legal patent scope. However, it is different from HITS in that it operates on weighted technological and legal citation graphs that take into account the two dimensions of patent citations and exponential decaying in weights. The weights on the citation networks are to be learned in an integrated learning process, which uses different rules for updating different features based on mutual interdependence among features.

### 3.4 Prediction Model for Patent Valuation

We argue that the four legal and technological measures can be used for patent valuation and thus derive a model to predict patent value using the proposed quality measures as features. In this section, we introduce a prediction model, which we combine with the probabilistic modeling of patent citations to learn model parameters and to derive classifiers for patent evaluation in an integrated learning process. We use logistic regression model for predicting patent value, and maximizing the object function with gradient ascent method to predict patent value. Accordingly, each patent  $p_i$  in the training sample can be represented as  $(x, y)$  where  $x$  is the feature set of  $p_i$ . i.e., the four features/quality measures, and  $y$  is the predictive label used in regression. For example, the patent maintenance status may serve as a label/indicator of patent value (as discussed in detail later in Section 4). For logistic regression, we define our hypothesis function  $h_\theta(x)$  as follows.

$$h_{\omega, \lambda}(x) = \frac{1}{1 + \exp(-\omega^T x(\lambda))} \quad (19)$$

where the model parameter  $\theta$  consists of (i)  $\omega$  – the weights of the features and (ii)  $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$  – the parameters of the exponential decay functions corresponding to US

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