INTERNATIONAL SEARCH REPORT

International Application No
PCT/US 01/24240

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 H04L12/18 H04L12/56

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7 H04L

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, WPI Data, PAJ, IBM-TDB, INSPEC, COMPENDEX

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 4 912 656 A (CAIN JOSEPH B ET AL) 27 March 1990 (1990-03-27)	1-3, 7-10,12, 19,20, 22,24-27
	column 5, line 60 -column 7, line 20	
A		13-18, 21,28-33

Y	US 5 056 085 A (VU THU V) 8 October 1991 (1991-10-08)	1-3, 7-10,12, 19,20, 22,24-27
	column 2, line 49 -column 3, line 12	

	-/--	

[X] Further documents are listed in the continuation of box C.

[X] Patent family members are listed in annex.

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
27 May 2002	05/06/2002

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL – 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Ströbeck, A.

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INTERNATIONAL SEARCH REPORT

International Application No
PCT/US 01/24240

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	ALAGAR S ET AL: "Reliable broadcast in mobile wireless networks" MILITARY COMMUNICATIONS CONFERENCE, 1995. MILCOM '95, CONFERENCE RECORD, IEEE SAN DIEGO, CA, USA 5-8 NOV. 1995, NEW YORK, NY, USA, IEEE, US, 5 November 1995 (1995-11-05), pages 236-240, XP010153965 ISBN: 0-7803-2489-7 page 237, left-hand column, line 43 -page 239, right-hand column, last line -----	1-33

1

INTERNATIONAL SEARCH REPORT

Information on patent family members

International Application No
PCT/US 01/24240

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US 4912656 A	27-03-1990	NONE	
US 5056085 A	08-10-1991	NONE	

Form PCT/ISA/210 (patent family annex) (July 1992)

UNITED STATES PATENT AND TRADEMARK OFFICE

UNITED STATES DEPARTMENT OF COMMERCE
United States Patent and Trademark Office
Address: COMMISSIONER FOR PATENTS
P.O. Box 1450
Alexandria, Virginia 22313-1450
www.uspto.gov

APPLICATION NO.	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO.	CONFIRMATION NO.
09/629,570	07/31/2000	Fred B. Holt	030048002US	5411

25096 7590 10/17/2003

PERKINS COIE LLP
PATENT-SEA
P.O. BOX 1247
SEATTLE, WA 98111-1247

EXAMINER	
WINTERS, MAREISHA N	
ART UNIT	**PAPER NUMBER**
2153	7

DATE MAILED: 10/17/2003

Please find below and/or attached an Office communication concerning this application or proceeding.

PTO-90C (Rev. 10/03)

PRG

Office Action Summary	Application No.	Applicant(s)	
	09/629,570	HOLT ET AL.	
	Examiner	**Art Unit**	
	Mareisha N. Winters	2153	

-- The MAILING DATE of this communication appears on the cover sheet with the correspond nce address --

Peri d for Reply

A SHORTENED STATUTORY PERIOD FOR REPLY IS SET TO EXPIRE 1 MONTH(S) FROM THE MAILING DATE OF THIS COMMUNICATION.

- Extensions of time may be available under the provisions of 37 CFR 1.136(a). In no event, however, may a reply be timely filed after SIX (6) MONTHS from the mailing date of this communication.
- If the period for reply specified above is less than thirty (30) days, a reply within the statutory minimum of thirty (30) days will be considered timely.
- If NO period for reply is specified above, the maximum statutory period will apply and will expire SIX (6) MONTHS from the mailing date of this communication.
- Failure to reply within the set or extended period for reply will, by statute, cause the application to become ABANDONED (35 U.S.C. § 133).
- Any reply received by the Office later than three months after the mailing date of this communication, even if timely filed, may reduce any earned patent term adjustment. See 37 CFR 1.704(b).

Status

1)☒ Responsive to communication(s) filed on *31 July 2000* .

2a)☐ This action is **FINAL**. 2b)☐ This action is non-final.

3)☐ Since this application is in condition for allowance except for formal matters, prosecution as to the merits is closed in accordance with the practice under *Ex parte Quayle*, 1935 C.D. 11, 453 O.G. 213.

Disposition of Claims

4)☒ Claim(s) *1-49* is/are pending in the application.

4a) Of the above claim(s) _____ is/are withdrawn from consideration.

5)☐ Claim(s) _____ is/are allowed.

6)☐ Claim(s) _____ is/are rejected.

7)☐ Claim(s) _____ is/are objected to.

8)☒ Claim(s) *1-49* are subject to restriction and/or election requirement.

Application Papers

9)☐ The specification is objected to by the Examiner.

10)☐ The drawing(s) filed on _____ is/are: a)☐ accepted or b)☐ objected to by the Examiner.

Applicant may not request that any objection to the drawing(s) be held in abeyance. See 37 CFR 1.85(a).

11)☐ The proposed drawing correction filed on _____ is: a)☐ approved b)☐ disapproved by the Examiner.

If approved, corrected drawings are required in reply to this Office action.

12)☐ The oath or declaration is objected to by the Examiner.

Priority under 35 U.S.C. §§ 119 and 120

13)☐ Acknowledgment is made of a claim for foreign priority under 35 U.S.C. § 119(a)-(d) or (f).

a)☐ All b)☐ Some * c)☐ None of:

1.☐ Certified copies of the priority documents have been received.

2.☐ Certified copies of the priority documents have been received in Application No. _____ .

3.☐ Copies of the certified copies of the priority documents have been received in this National Stage application from the International Bureau (PCT Rule 17.2(a)).

* See the attached detailed Office action for a list of the certified copies not received.

14)☐ Acknowledgment is made of a claim for domestic priority under 35 U.S.C. § 119(e) (to a provisional application).

a) ☐ The translation of the foreign language provisional application has been received.

15)☐ Acknowledgment is made of a claim for domestic priority under 35 U.S.C. §§ 120 and/or 121.

Attachment(s)

1) ☐ Notice of References Cited (PTO-892)
2) ☐ Notice of Draftsperson's Patent Drawing Review (PTO-948)
3) ☐ Information Disclosure Statement(s) (PTO-1449) Paper No(s) _____ .
4) ☐ Interview Summary (PTO-413) Paper No(s). _____ .
5) ☐ Notice of Informal Patent Application (PTO-152)
6) ☐ Other: .

DETAILED ACTION

1. Claims 1-49 are pending in the application.

Election/Restrictions

2. Restriction to one of the following inventions is required under 35 U.S.C. 121:

I. Claims 1-17 and 32-40, drawn to a method for adding a participant to a network, the method comprising identifying a pair, disconnecting the pair and reconnecting the pair, classified in class 709, subclasses 204, 205.

II. Claims 18-31, drawn to a method for sending a connection edge search message for adding a participant to a network, classified in class 709, subclass 225.

III. Claims 41-49, drawn to a method for detecting neighbors with empty ports in a network, classified in class 709, subclass 229.

3. The inventions are distinct from each other for the following reasons:

Inventions I, II and III are unrelated. Inventions are unrelated if it can be shown that they are not disclosed as capable of use together and they have different modes of operation, different functions, or different effects (MPEP § 806.04, MPEP § 808.01). In the instant case the different inventions are unrelated because:

- Invention I defines the function of adding a participant to a network by identifying a pair, disconnecting the pair and reconnecting the pair with the added participant that is not disclosed in Inventions II and III;
- Invention II defines the function of sending a connection edge search message for adding a participant to a network that is not disclosed in Inventions I and III; and

- Invention III defines the function of detecting neighbors with empty ports in a network that is not disclosed in Inventions I and II.

4. Because these inventions are distinct for the reasons given above and have acquired a separate status in the art as shown by their different classification, restriction for examination purposes as indicated is proper.

5. These inventions are distinct for the reason given above and the search required for each group is different and not co-extensive for examination purpose. For example, the searches for the two inventions would not be co-extensive because these groups would require different searches on PTO's classification class and subclass as following:

a. Group I search (claims 1-175 and 32-40) would require use of search class 709, subclasses 204 and 205 (not required for Groups II and III).

b. Group II search (claims 18-31) would require the search of class 709, subclass 225 (not required for Groups I and III).

c. Group III search (41-49) would require the search of class 709, subclass 229 (not required for Groups I and II).

6. Because these inventions are distinct for the reasons given above and they require different searches, restriction for examination purposes as indicated is proper.

7. Applicants are advised that the response to this requirement to be complete must include an election of the invention to be examined even though the requirement be traversed.

Conclusion

8. A shortened statutory period for response to this action is set to **expire 1 (one) month and 0 (zero)** days from the mail date of this letter. Fail to respond within the period for response

will result in **ABANDONMENT** of the application (see 35 U.S.C. 133, M.P.E.P 710.02, 710.02(b)).

Any inquiry concerning this communication or earlier communications from the examiner should be directed to Mareisha N. Winters whose telephone number is (703) 305-7838. The examiner can normally be reached on Monday-Friday, 8:00am-5:00pm.

If attempts to reach the examiner by telephone are unsuccessful, the examiner's supervisor, Glenton B. Burgess can be reached on (703) 305-4792. The official fax phone number for the organization where this application or proceeding is assigned is (703) 872-9306.

Any inquiry of a general nature or relating to the status of this application or proceeding should be directed to the receptionist whose telephone number is (703) 305-3900.

Mareisha N. Winters MNW
Patent Examiner
Art Unit 2153
October 15, 2003

ZARNI MAUNG
PRIMARY EXAMINER

10-30-03

2153

Attorney Docket No. 030048002US #8/6

Express Mail No. EV335523788US

PATENT 10-06-0

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

IN RE APPLICATION OF:	FRED B. HOLT *ET AL.*	EXAMINER: MAREISHA N. WINTERS
APPLICATION NO.:	09/629,570	ART UNIT: 2153
FILED:	JULY 31, 2000	CONF. NO: 5411
FOR:	**JOINING A BROADCAST CHANNEL**	

Amendment in Response to Restriction Requirement

Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

Sir:

In response to the Office Action dated October 17, 2003, please amend the application as reflected in the following listing of claims.

Amendment to the Claims

1. (Original) A computer-based method for adding a participant to a network of participants, each participant being connected to three or more other participants, the method comprising:

identifying pair of participants of the network that are connected;

disconnecting the participants of the identified pair from each other; and

connecting each participant of the identified pair of participants to the added participant.

A1

2. (Original) The method of claim 1 wherein each participant is connected to 4 participants.

3. (Original) The method of claim 1 wherein the identifying of a pair includes randomly selecting a pair of participants that are connected.

4. (Original) The method of claim 3 wherein the randomly selecting of a pair includes sending a message through the network on a randomly selected path.

5. (Original) The method of claim 4 wherein when a participant receives the message, the participant sends the message to a randomly selected participant to which it is connected.

6. (Original) The method of claim 4 wherein the randomly selected path is approximately proportional to the diameter of the network.

7. (Original) The method of claim 1 wherein the participant to be added requests a portal computer to initiate the identifying of the pair of participants.

8. (Original) The method of claim 7 wherein the initiating of the identifying of the pair of participants includes the portal computer sending a message to a connected participant requesting an edge connection.

9. (Original) The method of claim 8 wherein the portal computer indicates that the message is to travel a certain distance and wherein the participant that receives the message after the message has traveled that certain distance is one of the participants of the identified pair of participants.

10. (Original) The method of claim 9 wherein the certain distance is approximately twice the diameter of the network.

C1 cont

11. (Original) The method of claim 1 wherein the participants are connected via the Internet.

12. (Original) The method of claim 1 wherein the participants are connected via TCP/IP connections.

13. (Original) The method of claim 1 wherein the participants are computer processes.

14. (Original) A computer-based method for adding nodes to a graph that is m-regular and m-connected to maintain the graph as m-regular, where m is four or greater, the method comprising:

identifying p pairs of nodes of the graph that are connected, where p is one half of m;

disconnecting the nodes of each identified pair from each other; and

connecting each node of the identified pairs of nodes to the added node.

15. (Original) The method of claim 14 wherein identifying of the p pairs of nodes includes randomly selecting a pair of connected nodes.

16. (Original) The method of claim 14 wherein the nodes are computers and the connections are point-to-point communications connections.

a cont

17. (Original) The method of claim 14 wherein m is even.

18–31. (Cancelled)

32. (Original) A computer-readable medium containing instructions for controlling a computer system to connect a participant to a network of participants, each participant being connected to three or more other participants, the network representing a broadcast channel wherein each participant forwards broadcast messages that it receives to its neighbor participants, by a method comprising:

identifying a pair of participants of the network that are connected;

disconnecting the participants of the identified pair from each other; and

connecting each participant of the identified pair of participants to the added participant.

a2

33. (Original) The computer-readable medium of claim 32 wherein each participant is connected to 4 participants.

34. (Original) The computer-readable medium of claim 32 wherein the identifying of a pair includes randomly selecting a pair of participants that are connected.

35. (Original) The computer-readable medium of claim 34 wherein the randomly selecting of a pair includes sending a message through the network on a randomly selected path.

36. (Original) The computer-readable medium of claim 35 wherein when a participant receives the message, the participant sends the message to a randomly selected participant to which it is connected.

37. (Original) The computer-readable medium of claim 35 wherein the randomly selected path is approximately twice a diameter of the network.

38. (Original) The computer-readable medium of claim 32 wherein the participant to be added requests a portal computer to initiate the identifying of the pair of participants.

9/8 cont

39. (Original) The computer-readable medium of claim 38 wherein the initiating of the identifying of the pair of participants includes the portal computer sending a message to a connected participant requesting an edge connection.

40. (Original) The computer-readable medium of claim 38 wherein the portal computer indicates that the message is to travel a certain distance and wherein the participant that receives the message after the message has traveled that certain distance is one of the identified pair of participants.

41–49. (Cancelled)

REMARKS

In the above referenced Office Action, the Examiner divided the claims into the following groups:

I. Claims 1-17 and 32-40, drawn to a method for adding a participant to a network, the method comprising identifying a pair, disconnecting the pair and reconnecting the pair;

II. Claims 18-31, drawn to a method for sending a connection edge search message for adding a participant to a network; and

III. Claims 41-49 drawn to a method for detecting neighbors with empty ports in a network.

In response, the applicants elect Group I without traverse. Non-elected claims 18-31 and 41-49 have been canceled.

No fees are believed due with this communication. However, the Commissioner is hereby authorized and requested to charge any deficiency in fees herein to Deposit Account No. 50-0665.

Respectfully submitted,
Perkins Coie LLP

Date: 10/28/03

Chun M. Ng
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Edelman, Bradley

IEEE Search:

"flood routing"

new <near/3> node <near/5> (add or added or adding or join or joining or joins or connect or connected or connecting) <near/5> network

configur* <near/3> connection <near/5> network <near/5> (node or computer)

configur* <near/5> connection <near/5> network

(disconnect* or add* or connect* or join*) <near/4> (node) <near/4> network

(disconnect* or add* or connect* or join*) <near/4> (node) <near/4> network and (configur* or topology)

(configur* or reconfigur*) <near/3> (node or network) and graph

(configur* or reconfigur*) <near/3> (node or network) and graph and (add* or join*)

(configur* or reconfigur*) <near/5> (network) and (add* or join* or connect*) <near/5> node and graph

m-connected or k-connected or n-connected or m-regular or k-regular or n-regular

Google Terms used

graph topology network node connected
join add addition
internet regular disconnect configure incomplete
hypercube

L Numb r	Hits	S arch T xt	DB	Tim stamp
1	20	"5471623"	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:18
2	2	"6553020"	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:19
3	409	new near3 node near5 (add or added or adding or join or joining or joins or connect or connected or connecting) near5 network	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:27
4	380	new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting) near5 network	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:28
5	98	(new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting) near5 network).ti,ab,clm.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:33
6	56	((new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting) near5 network).ti,ab,clm.) and (@ad<20000731 or @prad<20000731)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:33
7	105	(new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting or enter or entering or entered) near5 network).ti,ab,clm.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:27
8	61	((new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting or enter or entering or entered) near5 network).ti,ab,clm.) and (@ad<20000731 or @prad<20000731)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:25
9	2310	configur$6 near5 connection near5 network	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:25
10	501	configur$6 near5 connection near5 network with (n de r c mputer)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:25

11	213	configur$6 n ar3 c nn cti n n ar5 n twork n ar5 (node r comput r)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:28
12	121	(configur$6 near3 c nn cti n n ar5 n tw rk near5 (node or computer)) and (@ad<20000731 or @prad<20000731)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:28
13	121	((configur$6 near3 connection near5 network near5 (node or computer)) and (@ad<20000731 or @prad<20000731)) not "5471623" not (((new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting or enter or entering or entered) near5 network).ti,ab,clm.) and (@ad<20000731 or @prad<20000731))	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:26
14	39	(((configur$6 near3 connection near5 network near5 (node or computer)) and (@ad<20000731 or @prad<20000731)) not "5471623" not (((new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting or enter or entering or entered) near5 network).ti,ab,clm.) and (@ad<20000731 or @prad<20000731))) and topology	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:26
15	12	(((configur$6 near3 connection near5 network near5 (node or computer)) and (@ad<20000731 or @prad<20000731)) not "5471623" not (((new near3 node near3 (add or added or adding or join or joining or joins or connect or connected or connecting or enter or entering or entered) near5 network).ti,ab,clm.) and (@ad<20000731 or @prad<20000731))) and (new near4 (node or computer or device) near (add or added or adding or join or joining or joins or connect or connected or connecting or enter or entering or entered))	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:28
16	404	configur$6 near3 connection near5 network near5 (node or computer or device)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:28
17	31	(configur$6 near3 connection near5 network near5 (node or computer or device)) and (new n ar4 (n de r computer or d vice) n ar (add r added r adding or join or joining r joins r connect or c nnect d or conn cting r nt r r ent ring r ent r d))	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:28

18	22	((configur$6 n ar3 c nn ction n ar5 n tw rk n ar5 (node or comput r r d vic)) and (n w n ar4 (node or comput r r d vic) n ar (add or added r adding or j in or j ining or j ins r conn ct or conn ct d r c nn cting or ent r or entering or nt red))) and (@ad<20000731 or @prad<20000731)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:34
19	2	"6490247"	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 17:34
20	32	m-connected or k-connected or n-connected or m-regular or k-regular or n-regular	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 18:24
21	263810	"32" and network	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 18:24
22	12	(m-connected or k-connected or n-connected or m-regular or k-regular or n-regular) and network	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 18:26
23	1377	(configur$6 or reconfigur$6) near3 (node or network) and graph and (add$3 or join$3) near4 (node or network)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 18:27
24	15	(configur$6 or reconfigur$6) near3 (node or network) same graph same (add$3 or join$3) near4 (node or network)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 18:27
-	36	"4912656"	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 12:31
-	0	m-regular	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 12:32
-	3	m-connected	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 12:48
-	12	5101480.URPN.	USPAT	2004/01/02 12:33

-	11	5099235.URPN.	USPAT	2004/01/02 12:45
-	6075	((disconn ct$4 or add$3 r c nn ct$3 r j in$3) n ar4 (node) n ar4 n twork).ti,ab,clm.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 12:48
-	232	(((disconnect$4 or add$3 or connect$3 or join$3) near4 (node) near4 network).ti,ab,clm.) and satellite	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 18:23
-	155	((((disconnect$4 or add$3 or connect$3 or join$3) near4 (node) near4 network).ti,ab,clm.) and satellite) and (@ad<20000731 or @prad<20000731)	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:28
-	152	(((((disconnect$4 or add$3 or connect$3 or join$3) near4 (node) near4 network).ti,ab,clm.) and satellite) and (@ad<20000731 or @prad<20000731)) not "4912656" not m-connected not 5101480.URPN. not 5099235.URPN.	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 13:20
-	4	k-regular or n-regular	USPAT; US-PGPUB; EPO; JPO; DERWENT; IBM_TDB	2004/01/02 16:16

PRY

UNITED STATES PATENT AND TRADEMARK OFFICE

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www.uspto.gov

APPLICATION NO.	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO.	CONFIRMATION NO.
09/629,570	07/31/2000	Fred B. Holt	030048002US	5411

25096 7590 01/12/2004

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EXAMINER	
EDELMAN, BRADLEY E	
ART UNIT	**PAPER NUMBER**
2153	9

DATE MAILED: 01/12/2004

Please find below and/or attached an Office communication concerning this application or proceeding.

PTO-90C (Rev. 10/03)

Office Action Summary	Application No.	Applicant(s)	
	09/629,570	HOLT ET AL.	
	Examiner	Art Unit	
	Bradley Edelman	2153	

-- The MAILING DATE of this communication appears on the cover sheet with the c rrespondence address --

Period for Reply

A SHORTENED STATUTORY PERIOD FOR REPLY IS SET TO EXPIRE 3 MONTH(S) FROM THE MAILING DATE OF THIS COMMUNICATION.

- Extensions of time may be available under the provisions of 37 CFR 1.136(a). In no event, however, may a reply be timely filed after SIX (6) MONTHS from the mailing date of this communication.
- If the period for reply specified above is less than thirty (30) days, a reply within the statutory minimum of thirty (30) days will be considered timely.
- If NO period for reply is specified above, the maximum statutory period will apply and will expire SIX (6) MONTHS from the mailing date of this communication.
- Failure to reply within the set or extended period for reply will, by statute, cause the application to become ABANDONED (35 U.S.C. § 133).
- Any reply received by the Office later than three months after the mailing date of this communication, even if timely filed, may reduce any earned patent term adjustment. See 37 CFR 1.704(b).

Status

1)☒ Responsive to communication(s) filed on 28 October 2003.

2a)☐ This action is **FINAL**. 2b)☒ This action is non-final.

3)☐ Since this application is in condition for allowance except for formal matters, prosecution as to the merits is closed in accordance with the practice under *Ex parte Quayle*, 1935 C.D. 11, 453 O.G. 213.

Disposition of Claims

4)☒ Claim(s) 1-17 and 32-40 is/are pending in the application.

4a) Of the above claim(s) _____ is/are withdrawn from consideration.

5)☐ Claim(s) _____ is/are allowed.

6)☒ Claim(s) 1-17 and 32-40 is/are rejected.

7)☐ Claim(s) _____ is/are objected to.

8)☐ Claim(s) _____ are subject to restriction and/or election requirement.

Application Papers

9)☒ The specification is objected to by the Examiner.

10)☐ The drawing(s) filed on _____ is/are: a)☐ accepted or b)☐ objected to by the Examiner.

Applicant may not request that any objection to the drawing(s) be held in abeyance. See 37 CFR 1.85(a).

Replacement drawing sheet(s) including the correction is required if the drawing(s) is objected to. See 37 CFR 1.121(d).

11)☐ The oath or declaration is objected to by the Examiner. Note the attached Office Action or form PTO-152.

Priority under 35 U.S.C. §§ 119 and 120

12)☐ Acknowledgment is made of a claim for foreign priority under 35 U.S.C. § 119(a)-(d) or (f).

a)☐ All b)☐ Some * c)☐ None of:

1.☐ Certified copies of the priority documents have been received.

2.☐ Certified copies of the priority documents have been received in Application No. _____.

3.☐ Copies of the certified copies of the priority documents have been received in this National Stage application from the International Bureau (PCT Rule 17.2(a)).

* See the attached detailed Office action for a list of the certified copies not received.

13)☐ Acknowledgment is made of a claim for domestic priority under 35 U.S.C. § 119(e) (to a provisional application) since a specific reference was included in the first sentence of the specification or in an Application Data Sheet. 37 CFR 1.78.

a) ☐ The translation of the foreign language provisional application has been received.

14)☐ Acknowledgment is made of a claim for domestic priority under 35 U.S.C. §§ 120 and/or 121 since a specific reference was included in the first sentence of the specification or in an Application Data Sheet. 37 CFR 1.78.

Attachment(s)

1) ☒ Notice of References Cited (PTO-892)
2) ☐ Notice of Draftsperson's Patent Drawing Review (PTO-948)
3) ☒ Information Disclosure Statement(s) (PTO-1449) Paper No(s) 5,6 .
4) ☐ Interview Summary (PTO-413) Paper No(s). _____ .
5) ☐ Notice of Informal Patent Application (PTO-152)
6) ☐ Other: .

DETAILED ACTION

This Office action is in response to Applicant's response to the restriction requirement and amendment filed on October 28, 2003. Claims 1-17 and 32-40 are presented for further examination.

Specification

1. This application does not contain an abstract of the disclosure as required by 37 CFR 1.72(b). An abstract on a separate sheet is required.

The disclosure is objected to because of the following informalities:

The status of the related cases listed on page 1 of the specification must be updated.

Appropriate correction is required.

Claim Rejections - 35 USC § 112

The following is a quotation of the second paragraph of 35 U.S.C. 112:

> The specification shall conclude with one or more claims particularly pointing out and distinctly claiming the subject matter which the applicant regards as his invention.

2. Claims 1-40 are rejected under 35 U.S.C. 112, second paragraph, as being indefinite for failing to particularly point out and distinctly claim the subject matter which applicant regards as the invention.

In considering claims 1, 14, and 32, these claims all contain the phrase "the added participant" or "the added node" in the last line of the claim. These phrases lack sufficient antecedent basis, as none of these claims mention a step of actually adding a node or participant to the network.

Claims 2-13, 15-31, and 33-40 depend from these claims, and are thus rejected as well.

In addition, claim 6 uses the term "approximately proportional," while claims 10 and 37 use the term "approximately twice the diameter." The term "approximately" is a relative term that renders the claim indefinite. The term is not defined by the claim, the specification does not provide a standard for ascertaining the requisite degree, and one of ordinary skill in the art would not be reasonably apprised of the scope of the invention.

Appropriate correction is required.

Claim Rejections - 35 USC § 102

The following is a quotation of the appropriate paragraphs of 35 U.S.C. 102 that form the basis for the rejections under this section made in this Office action:

> A person shall be entitled to a patent unless –
>
> (e) the invention was described in (1) an application for patent, published under section 122(b), by another filed in the United States before the invention by the applicant for patent or (2) a patent granted on an application for patent by another filed in the United States before the invention by the applicant for patent, except that an international application filed under the treaty defined in section 351(a) shall have the effects for purposes of this subsection of an application filed in the United States only if the international application designated the United States and was published under Article 21(2) of such treaty in the English language.

3. Claims 1, 2 and 13 are rejected under 35 U.S.C. 102(e) as being anticipated by Steele, Jr., et al. (U.S. Patent No. 6,603,742, hereinafter "Steele").

In considering claim 1, Steele discloses a computer-based method for adding a participant ("node") to a network of participants, each participant connected to three or more participants (see Fig. 6), the method comprising:

Identifying a pair of participants of the network that are connected, disconnecting the participants of the identified pair from each other, and connecting each participant of the identified pair of participants to the added participant (col. 12, lines 45-51; Figs. 5 and 6, wherein nodes 3 and 1 disconnect from each other, and each of them connects to the added node 7 – note that Fig. 6 of Steele appears incorrect and that the connection between nodes 5 and 2 in Fig. 6 should have been removed).

In considering claim 2, Steele further discloses that each participant is connected to 4 participants (See Figs. 5-6, wherein each participant is connected to at least 4 participants).

In considering claim 13, Steele further discloses that the participants are computer processes ("nodes").

Claim Rejections - 35 USC § 103

The following is a quotation of 35 U.S.C. 103(a) which forms the basis for all obviousness rejections set forth in this Office action:

> (a) A patent may not be obtained though the invention is not identically disclosed or described as set forth in section 102 of this title, if the differences between the subject matter sought to be patented and the prior art are such that the subject matter as a whole would have been obvious at the time the invention was made to a person having ordinary skill in the art to which said subject matter pertains. Patentability shall not be negatived by the manner in which the invention was made.

4. Claims 32-33 are rejected under 35 U.S.C. 103(a) as being unpatentable over Steele, in view of Cho et al. ("A Flood Routing Method for Data Networks," ICICS '97, hereinafter "Cho").

In considering claim 32, the claim contains a computer readable medium for performing the same steps as claim 1, and additionally requires that each network participant forwards broadcast messages that it receives to its neighbor participants. See the discussion of claim 1 for the description of those steps. Note, however, that Steele does not disclose that each network participant forwards broadcast messages that it receives to its neighbor participants. This is because Steele is only concerned with how nodes are added and/or subtracted to the network and how that affects network configuration. The system taught by Steele remains silent regarding the actual passing of data between nodes. Nonetheless, flood routing (i.e. broadcasting messages from each node to each neighboring node in a network) is well known, as evidenced by Cho. In a similar art, Cho discloses that flood routing is well known (p. 1418, Introduction, ¶ 1) and further describes a network system with multiple interconnected nodes (see Figs. 1, 3) that uses flood routing to pass information between nodes (p. 1418-1419, § 2, "Flood Routing Mechanism"). Given the teaching of Cho, a person having ordinary skill in the art would have readily recognized the desirability and advantages of using flood routing to send information between nodes in the system taught by Steele, because flood routing is a very reliable and robust method of data transmission (see Cho, p. 1418, Introduction, ¶ 1). Therefore, it would have been obvious to use flood routing to pass information in the network taught by Steele.

In considering claim 33, Steele further discloses that each participant is connected to 4 participants (See Figs. 5-6, wherein each participant is connected to at least 4 participants).

5. Claims 1-5, 7, 8, and 11-17 are rejected under 35 U.S.C. 103(a) as being unpatentable over Gilbert et al. (U.S. Patent No. 6,490,247, hereinafter "Gilbert") in view of Hughes et al. (U.S. Patent No. 6553,020, hereinafter "Hughes").

In considering claim 1, Gilbert discloses a computer-based method for adding a participant ("node") to a network of participants, the method comprising:

Identifying a pair of participants of the network that are connected (col. 6, lines 26-49, wherein the additional node contacts the two participants), disconnecting the participants of the identified pair from each other (col. 7, lines 7-8, "the two adjacent nodes drop connection to one another"), and connecting each participant of the identified pair of participants to the added participant (col. 7, lines 13-19, "the additional node connects with each of the adjacent nodes").

However, Gilbert does not disclose that each participant is connected to three or more other participants. Gilbert discloses instead, a ring-type network, wherein each node is connected to two other nodes (see col. 3, lines 25-36). Nonetheless, the use of other types of networks to connect participants, wherein each participant is connected to three or more participants, and wherein participants can be added to the network, is well known, as evidenced by Hughes. In a similar art, Hughes discloses a network for interconnecting nodes for communication across the network, wherein the nodes can be connected in a hypercube-type topology, or in some other type of topology such that each node is connected to 4 other nodes, wherein nodes can be added to the network (col. 14, lines 25-30, 67; col. 15, lines 1-5, 45-52; col. 4, lines 6-9, "additional users can be added later as demand grows"). Given the teaching of Hughes, a person having ordinary skill in the art would have readily recognized the desirability and advantages of using a similar technique as taught by Gilbert (i.e. disconnecting certain node connections and connecting the newly disconnected links to the added node) to connect additional participants in the system taught by Hughes, in order to maintain the network topology for added nodes, thereby maintaining the interconnectivity and reliability associated with hypercube and 4-connected networks. Therefore, it would have been obvious to use the technique disclosed by Gilbert for connecting new participants in a system such as the one taught by Hughes.

In considering claim 2, Hughes further discloses that each participant is connected to 4 participants (col. 14, lines 25-30, "hypercube"; col. 15, lines 45-52, "nodes 2 are connected in an arbitrary manner to up to a fixed number n of nearest nodes... where n=4..."; Fig. 9).

In considering claim 3, Gilbert further discloses that the pair of nodes selected for disconnection is selected arbitrarily (col. 6, lines 37-40, "the actual node that is contacted by the additional node does not matter," and can simply be "the first node on the list"). Although Gilbert does not explicitly state that selection is done randomly, the node is effectively being selected randomly, since any node can be first on the list. The same result would be achieved by selecting a node randomly from somewhere else on the list. Thus, the limitation of selecting the node randomly does not render the claimed invention patentably distinct over the method taught by Gilbert.

In considering claim 4, Gilbert further discloses that arbitrarily selecting the pair includes sending a message through the network on an arbitrarily selected path (col. 6, lines 30-31, 37-40, "an additional node contacts two adjacent nodes in the network," wherein "the actual node that is contacted by the additional node does not matter," such that the path selected will be the path to whichever node is arbitrarily and thus randomly selected).

In considering claim 5, Gilbert further discloses that when a participant ("primary node") receives the message, it sends the message to a selected participant to which it is connected ("adjacent node," col. 6, lines 50-59). However, Gilbert does not disclose that the message is sent to a randomly selected participant. Nonetheless, Gilbert discloses that the actual initial nodes contacted do not matter (see col. 6, lines 37-40). It follows then that the selection of the adjacent node also doesn't matter, so long as it is adjacent (note that Gilbert does not specify which adjacent node is selected). Selecting an adjacent node randomly, rather than, say, selecting one particular adjacent node over the other, is thus a matter of preference, and does not render the claimed invention patentably distinct over the method taught by Gilbert.

In considering claim 7, Gilbert further discloses that the participant to be added requests a portal computer to initiate the identifying of the pair of participants (col. 6, lines 45-47, "additional node 100 would contact node 10, and node 10 would provide additional node 100 information regarding node 16").

In considering claim 8, Gilbert further discloses that the initiating of the identifying of the pair of participants includes the portal computer sending a message to a connected participant requesting an edge connection (col. 6, lines 53-57, "primary node... receives all incoming calls from other nodes wishing to enter the network. The point of entry in the network for these other nodes is then between the primary node and an adjacent node to the primary node").

In considering claim 11, Hughes further discloses that the participants are connected via the Internet (col. 1, line 14, "Internet"; col. 14, lines 55-59, "Internet web-browsing"). It would have been obvious for the network in the participant adding system taught by Gilbert and Hughes to be the Internet, so that the participants could communicate with other users anywhere in the world. Therefore, it would have been obvious to use the participant adding system taught by Gilbert and Hughes on the Internet network.

In considering claim 12, although Hughes does not explicitly teach TCP/IP, Examiner takes official notice that TCP/IP is a standard well known protocol used for Internet communications. Therefore, it would have been obvious to connect the participants via TCP/IP for the same reasons as connecting participants via the Internet – i.e. to allow global communications on the existing Internet network.

In considering claim 13, Gilbert further discloses that the participants are computer processes ("nodes").

In considering claim 14, Gilbert discloses a computer-based method for adding nodes ("nodes") to a graph that is m-regular and m-connected (see Fig. 1, which is 2-regular and 2-connected) to maintain the graph as m-regular, the method comprising:

Identifying p pairs of nodes of the graph that are connected where p is half of m (p. is 1, see col. 6, lines 30-42, wherein a pair of adjacent nodes is identified);

Disconnecting the nodes of each identified pair from each other (col. 7, lines 7-8); and

Connecting each node of the identified pair of nodes to the added node (col. 7, lines 13-19).

However, Gilbert does not disclose that m is four or greater, and thus that the graph is at least 4-connected and 4-regular. Nonetheless, the use of 4-connected and 4-regular networks wherein nodes can be added to the network is well known, as evidenced by Hughes. In a similar art, Hughes discloses a network for interconnecting nodes for communication across the network, wherein the nodes can be connected in a hypercube-type topology, or in some other type of topology such that each node is connected to 4 other nodes, wherein nodes can be added to the network (col. 14, lines 25-30, 67; col. 15, lines 1-5, 45-52; col. 4, lines 6-9, "additional users can be added later as demand grows"). Given the teaching of Hughes, a person having ordinary skill in the art would have readily recognized the desirability and advantages of extending the node addition method taught by Gilbert (i.e. disconnecting p pairs of nodes node connections and connecting the newly disconnected links to the added node) to more highly connected (i.e. 4-connected) networks, in order to maintain the network topology for added nodes, thereby maintaining the interconnectivity and reliability associated with hypercube and 4-connected networks. Therefore, it would have been obvious to use the technique disclosed by Gilbert for connecting new participants to the 4-connected system taught by Hughes.

In considering claim 15, Gilbert further discloses that the pair of nodes selected for disconnection is selected arbitrarily (col. 6, lines 37-40, "the actual node that is contacted by the additional node does not matter," and can simply be "the first node on the list"). Although Gilbert does not explicitly state that selection is done randomly, the node effectively is being selected randomly, since any node can be first on the list. The same result would be achieved by selecting a node randomly from somewhere else on the list. Thus, the limitation of selecting the node randomly does not render the claimed invention patentably distinct over the method taught by Gilbert.

In considering claim 16, Hughes further discloses that the nodes are computers and the connections are point-to-point connections (abstract).

In considering claim 17, both Gilbert and Hughes further disclose that m is even (i.e. 2 or 4).

6. Claims 32-36, 38, and 39 are rejected under 35 U.S.C. 103(a) as being unpatentable over Gilbert in view of Hughes, and further in view of Cho et al. ("A Flood Routing Method for Data Networks," ICICS '97, hereinafter "Cho").

In considering claim 32, the claim contains a computer readable medium for performing the same steps as claim 1, and additionally requires that each network participant forwards broadcast messages that it receives to its neighbor participants. See the discussion of claim 1 for the description of those steps. Note, however, that neither Gilbert nor Hughes disclose that each network participant forwards broadcast messages that it receives to its neighbor participants. Nonetheless, flood routing (i.e. broadcasting messages from each node to each neighboring node in a network) is well known, as evidenced by Cho. In a similar art, Cho discloses that flood routing is well known (p. 1418, Introduction, ¶ 1) and further describes a network system with multiple interconnected nodes (see Figs. 1, 3) that uses flood routing to pass information between nodes (p. 1418-1419, § 2, "Flood Routing Mechanism"). Given the teaching of Cho, a person having ordinary skill in the art would have readily recognized the desirability and advantages of using flood routing to send information between nodes in the system taught by Gilbert and Hughes, because flood routing is a very reliable and robust method of data transmission (see Cho, p. 1418, Introduction, ¶ 1). Therefore, it would have been obvious to use flood routing to pass information in the network taught by Gilbert and Hughes.

In considering claim 33, Hughes further discloses that each participant is connected to 4 participants (col. 14, lines 25-30, "hypercube"; col. 15, lines 45-52, "nodes 2 are connected in an arbitrary manner to up to a fixed number n of nearest nodes... where n=4..."; Fig. 9).

In considering claim 34, Gilbert further discloses that the pair of nodes selected for disconnection is selected arbitrarily (col. 6, lines 37-40, "the actual node that is contacted by the additional node does not matter," and can simply be "the first node on the list"). Although Gilbert does not explicitly state that selection is done randomly, the node effectively is being selected randomly, since any node can be first on the list. The same result would be achieved by selecting a node randomly from somewhere else on the list. Thus, the limitation of selecting the node randomly does not render the claimed invention patentably distinct over the method taught by Gilbert.

In considering claim 35, Gilbert further discloses that arbitrarily selecting the pair includes sending a message through the network on an arbitrarily selected path (col. 6, lines 30-31, 37-40, "an additional node contacts two adjacent nodes in the network," wherein "the actual node that is contacted by the additional node does not matter," such that the path selected will be the path to whichever node is arbitrarily and thus randomly selected).

In considering claim 36, Gilbert further discloses that when a participant ("primary node") receives the message, it sends the message to a selected participant to which it is connected ("adjacent node," col. 6, lines 50-59). However, Gilbert does not disclose that the message is sent to a randomly selected participant. Nonetheless, Gilbert discloses that the actual initial nodes contacted do not matter (see col. 6, lines 37-40). It follows then that the selection of the adjacent node also doesn't matter, so long as it is adjacent (note that Gilbert does not specify which adjacent node is selected). Selecting an adjacent node randomly, rather than, say, selecting one particular adjacent node over the other, is thus a matter of preference, and does not render the claimed invention patentably distinct over the method taught by Gilbert.

In considering claim 38, Gilbert further discloses that the participant to be added requests a portal computer to initiate the identifying of the pair of participants (col. 6, lines 45-47, "additional node 100 would contact node 10, and node 10 would provide additional node 100 information regarding node 16").

In considering claim 39, Gilbert further discloses that the initiating of the identifying of the pair of participants includes the portal computer sending a message to a connected participant requesting an edge connection (col. 6, lines 53-57, "primary node... receives all incoming calls from other nodes wishing to enter the network. The point of entry in the network for these other nodes is then between the primary node and an adjacent node to the primary node").

Allowable Subject Matter

7. As allowable subject matter has been indicated, applicant's reply must either comply with all formal requirements or specifically traverse each requirement not complied with. See 37 CFR 1.111(b) and MPEP § 707.07(a).

Claims 9 and 40 would be allowable if rewritten to include all of the limitations of the base claim and any intervening claims, and if the base claims were rewritten to overcome the rejection(s) under 35 U.S.C. 112, second paragraph, set forth in this Office action.

The following is a statement of reasons for the indication of allowable subject matter: the prior art of record fails to disclose or render obvious all of the limitations of the claims, including the claimed distance-related selection steps described in claims 9, and 40.

Conclusion

The prior art made of record and not relied upon is considered pertinent to applicant's disclosure.

Any inquiry concerning this communication or earlier communications from the examiner should be directed to Bradley Edelman whose telephone number is (703) 306-3041. The examiner can normally be reached on Monday to Friday from 8:30 AM to 5:00 PM.

If attempts to reach the examiner by telephone are unsuccessful, the examiner's supervisor, Glen Burgess can be reached on (703) 305-4792. The fax phone numbers for the organization where this application or proceeding is assigned are as follows:

For all correspondences: (703) 872-9306.

Any inquiry of a general nature or relating to the status of this application or proceeding should be directed to the receptionist whose telephone number is (703) 305-3900.

Bradley Edelman

BE
January 6, 2004

Notice of References Cited	Application/Control No. 09/629,570	Applicant(s)/Patent Under Reexamination HOLT ET AL.	
	Examiner Bradley Edelman	Art Unit 2153	Page 1 of 2

U.S. PATENT DOCUMENTS

*		Document Number Country Code-Number-Kind Code	Date MM-YYYY	Name	Classification
	A	US-6,490,247 B1	12-2002	Gilbert et al.	370/222
	B	US-6,553,020 B1	04-2003	Hughes et al.	370/347
	C	US-6,603,742 B1	08-2003	Steele et al.	370/254
	D	US-5,471,623 A	11-1995	Napolitano, Jr., Leonard M.	709/243
	E	US-6,065,063 A	05-2000	Abali, Bulent	709/242
	F	US-6,505,289 B1	01-2003	Han et al.	712/11
	G	US-5,099,235 A	03-1992	Crookshanks, Rex J.	455/13.1
	H	US-5,732,086	03-1998	Liang et al.	370/410
	I	US-5,117,422 A	05-1992	Hauptschein et al.	370/255
	J	US-5,101,480 A	03-1992	Shin et al.	710/317
	K	US-			
	L	US-			
	M	US-			

FOREIGN PATENT DOCUMENTS

*		Document Number Country Code-Number-Kind Code	Date MM-YYYY	Country	Name	Classification
	N					
	O					
	P					
	Q					
	R					
	S					
	T					

NON-PATENT DOCUMENTS

*		Include as applicable: Author, Title Date, Publisher, Edition or Volume, Pertinent Pages)
	U	Cho et al., "A Flood Routing Method for Data Networks," September 1997, Proceedings of 1997 International Conference on Information, Communications and Signal Processing, Vol. 3, pp. 1418-1422.
	V	Bandyopadhyay et al., "A Flexible Architecture for Multi-Hop Optical Networks," October 1998, 7th International Conference on Computer Communications and Networks, 1998, pp. 472-478.
	W	Hsu, "On Four-Connecting a Triconnected Graph," October 1992, Annual Symposium on Foundations of Computer Science, 1992, pp. 70-79.
	X	Shiokawa et al., "Performance Analysis of Network Connective Probability of Multihop Network under Correlated Breakage," June 1996, 1996 IEEE International Conference on Communications, Vol. 3, pp. 1581-1585.

*A copy of this reference is not being furnished with this Office action. (See MPEP § 707.05(a).)
Dates in MM-YYYY format are publication dates. Classifications may be US or foreign.

Notice of References Cited	Application/Control No. 09/629,570	Applicant(s)/Patent Under Reexamination HOLT ET AL.	
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U.S. PATENT DOCUMENTS

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	A	US-			
	B	US-			
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	J	US-			
	K	US-			
	L	US-			
	M	US-			

FOREIGN PATENT DOCUMENTS

*		Document Number Country Code-Number-Kind Code	Date MM-YYYY	Country	Name	Classification
	N					
	O					
	P					
	Q					
	R					
	S					
	T					

NON-PATENT DOCUMENTS

*		Include as applicable: Author, Title Date, Publisher, Edition or Volume, Pertinent Pages)
	U	Komine et al., "A Distributed Restoration Algorithm for Multiple-Link and Node Failures of Transport Networks," December 199 IEEE GLOBECOM '90, 'Communications: Connecting the Future,' Vol. 1, pp. 459-463.
	V	
	W	
	X	

*A copy of this reference is not being furnished with this Office action. (See MPEP § 707.05(a).)
Dates in MM-YYYY format are publication dates. Classifications may be US or foreign.

International Conference on
Information, Communications and Signal Processing
ICICS '97
Singapore, 9-12 September 1997

3F1.4

A Flood Routing Method for Data Networks

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Abstract

In this paper, a new routing algorithm based on a flooding method is introduced. Flooding techniques have been used previously, e.g. for broadcasting the routing table in the ARPAnet [1] and other special purpose networks [3][4][5]. However, sending data using flooding can often saturate the network [2] and it is usually regarded as an inefficient broadcast mechanism. Our approach is to flood a very short packet to explore an optimal route without relying on a pre-established routing table, and an efficient flood control algorithm to reduce the signalling traffic overhead. This is an inherently robust mechanism in the face of a network configuration change, achieves automatic load sharing across alternative routes, and has potential to solve many contemporary routing problems. An earlier version of this mechanism was originally developed for virtual circuit establishment in the experimental Caroline ATM LAN [6][7] at Monash University.

1. Introduction

Flooding is a data broadcast technique which sends the duplicates of a packet to all neighboring nodes in a network. It is a very reliable method of data transmission because many copies of the original data are generated during the flooding phase, and the destination user can double check the correct reception of the original data. It is also a robust method because no matter how severely the network is damaged, flooding can guarantee at least one copy of the data will be transmitted to the destination, provided a path is available.

While the duplication of packets makes flooding a generally inappropriate method for data transmission, our approach is to take advantage of the simplicity and robustness of flooding for routing purposes. Very short packets are sent over all possible routes to search for the optimal route of the requested QoS and the data path is established via the selected route. Since the Flood Routing algorithm strictly controls the unnecessary packet duplication, the traffic overhead caused from the flooding traffic is minimal.

Use of flooding for routing purposes has been suggested before [3][4][5], and it has been noted that it can be guaranteed to form a shortest path route[10]. And an earlier protocol was proposed and implemented for the experimental local area ATM network (Caroline [6][7]). However the earlier protocol had problems with scaling timer values, and also required complex mechanism to solve potential race and deadlock problem. Our proposal greatly simplifies the previous mechanism and reduces the earlier problems.

Chapter 2 explains the procedure for route establishment and the simulation results are presented in chapter 3. The advantages of the Flood Routing are reviewed specifically in chapter 4. Chapter 5 concludes this paper with suggesting some possible application area and the future study issues.

2. Flood Routing Mechanism

Figure 1, 3, 4 show the stepwise procedure of the route establishment.

In the Figure 1, the host A is requesting a connection set up to the target host B. In the initial stage, a short connection request packet (CREQ) is delivered to the first hop router 1 and router 1 starts the flood of the CREQ packets.

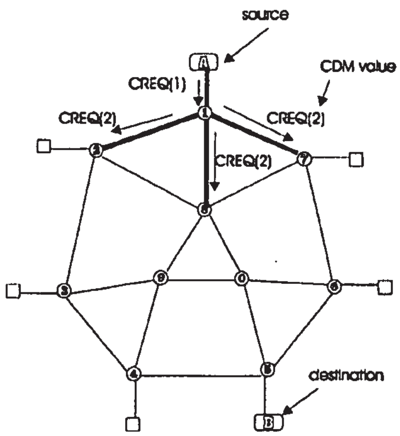

Figure 1

VC number (1byte=0)
Packet Type (1byte="CREQ")
CDM (1byte)
Source Address
Connection No (1byte)
Destination Address
QoS

Figure 2 CREQ Packet Format

Figure 2 shows the format of the CREQ packet. The CREQ packet contains a connection difficulty metric (CDM) field, QoS parameters and the source & destination addresses and connection number. The metric can be any accumulative measure representing the route difficulty, such as hop count, delay, buffer length, etc. The connection number is chosen by the source host to distinguish the different packet floods of the same source and destination.

When a router receives the CREQ packet, the router matches the packet information with the internal Flood Queue to see if the same packet has been received before. If the CREQ packet is new, it records the information in the Flood Queue, increases the CDM value, and forwards the packet to all output links with adequate capacity to meet the QoS except the received one. Thus the flood of CREQ packets propagate through the entire network.

The Flood Queue is a FIFO list which contains the information relating to the best CREQ packet the router has received for each recent flood. As the flood packet of a new connection arrives and the information is pushed into the Flood Queue, the old information gradually moves to the rear and eventually is removed. The queueing delay from the insertion to the deletion depends on the queue size and the call frequency, and provided this delay is enough to cover the time for network wide flood propagation and reply, there is no need for a timer to wait to the completion of the flood.

Since the CDM value is increased as the CREQ packet passes the routers, the metric value represents the route difficulty that the CREQ packet has experienced. Because of the repeated duplication of the packet, a router may receive another copy of the CREQ packet. In this case, the router compares the metric values of the two packets and if the most recently arrived packet has the better metric value, it updates the information in the Flood Queue and repeats the flood action. Otherwise the packet is discarded. As a consequence, all the routers keep the record of the best partial route and the output link to use for setting up the virtual circuit.

Figure 3 shows the intermediate routers 2, 7, 8 have chosen the links toward the router 1 as the best candidate link. If one of them is requested for the path to the source node A, the router will use this link for the virtual circuit set up.

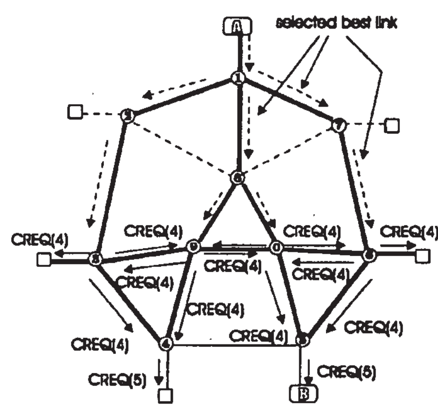

Figure 3

When the destination host receives a CREQ packet, it opens a short time-window to absorb possible further arriving CREQ packets. The expiration of the timer triggers the sending of the connection acceptance (CACC) packet along the best links indicated by the CREQ packet with the lowest CDM. The CACC packet is relayed back to the source host by the routers which at the same time install the virtual circuit via the optimal route. Finally, when the source host receives the CACC packet, the host may initiate data transmission.

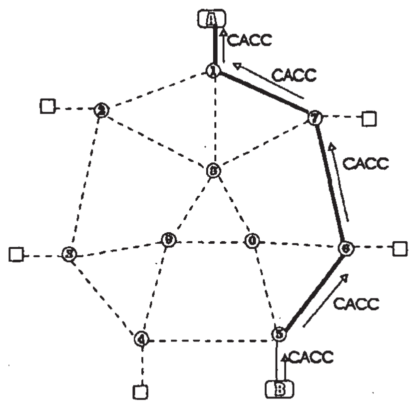

Figure 4

Note that bandwidth reservation occurs during the relay of the CACC packet. It is possible that the available QoS will have dropped below the requested level in one or more links. In this case, the source may either accept the lower QoS, or close the connection and try again.

More implementation details of the flooding protocol can be found in [9].

3. Simulation Result

One concern of Flood Routing is whether it will lead to congestion of the network by the signalling traffic. A simulation was carried out using various network conditions. Figure 5 shows the number of flooding packets produced in a connection trial in a normal traffic condition on a network consisting of 5 switching nodes, 9 hosts and 16 links. The simulation tested the event of 2000 seconds.

The graph shows that the total number of flooding packets per connection converges on the lower bound 18 with some exceptions. This is slightly higher than the number of the network links (16). This shows how the flood control mechanism is efficient in that the routers usually generate only one flooding packet per output link and this duplication process is rarely repeated again. As a result, the total number of flooding packets per connection is nearly same as the number of network links.

Considering the small size of the flooding packet, the bandwidth consumed by the signalling traffic is small. Suppose an ATM network using the Flood Routing generates 1000 calls per seconds, the bandwidth consumption by the signalling traffic will only be about 424 Kbps (= 1 K * 53 byte) per link and this does not include any additional route management traffic such as the routing table update.

From the simulation, it is observed that the average number and the maximum number of the flooding packets depends on the network topology and the traffic condition. If the network is simple topology such as a tree or a star shape, the average number of the flooding packets is nearly identical to the number of the network links. If the network is a complex topology such as a complete mesh topology, and there is a high traffic load, the routers tend to generate more packets because of the racing of the flooding packets.

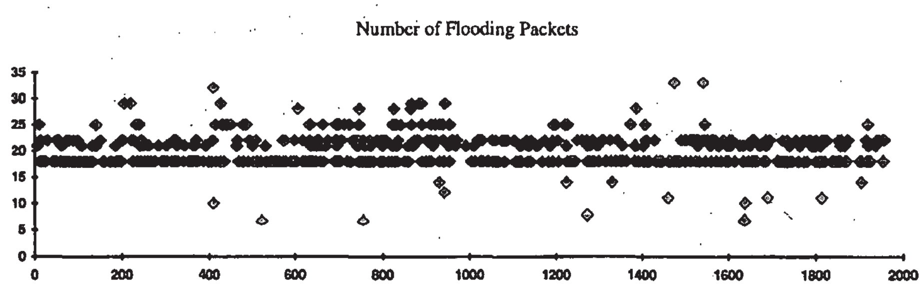

Figure 5

The connections established by Flood Routing successfully avoid busy links and disperse the communication paths to all possible routes. This reduced the chance of congestion and utilizes all network resources efficiently.

4. Advantages of the Flood Routing

The distinctive features of the Flood Routing method are :

(a) It facilitates the load sharing of available network resources. If many possible routes exist between two end points in a network, the Flood Routing can disperse different connections over different routes to share the network load. Figure 6 shows this example.

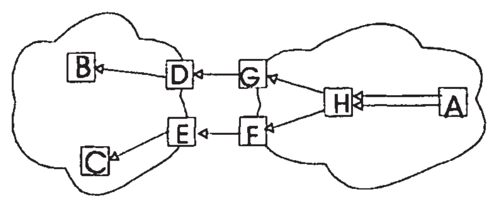

Figure 6 Example of Multipath Connection

In the sample network, there are more than two links exist between node A and H, and the node A used all links for different connections with balancing the load. More than two exterior routers are connecting the subnet 1 and the subnet 2, and the node H distributed the connections to all exterior routers. Therefore, all the network resources are utilized fully in Flood Routing network. This load sharing capability has been considered to be a difficult problem in table based routing algorithms.

(b) It automatically adapts to changes in the network configuration. For example, if the overall traffic between two end points has been increased, the network bandwidth can simply be expanded by adding more links between routers. The Flood Routing algorithm can recognize the additional links and use them for sharing the load in new connections.

(c) The method is robust. The Flood routing can achieve a successful connection even when the network is severely damaged, provided flooding packets can reach the destination. Once a flooding packet reaches the destination, the connection can be established via the un-damaged part of the network which was searched by the packet. This is very useful property in networks which are vulnerable but which require high reliability, such as military networks.

(d) The method is simple to manage, as it makes no use of routing tables. This table-less routing method does not have the problem like "Convergence time" of the Distance Vector routing [8].

(e) It is possible to find the optimal route of the requested bandwidth or the quality of service. While the packet flood is progressing, bandwidth requirement and QoS constraints specified in the flooding packets are examined by the routers and the links that does not meet the requirements are excluded from the routing decision. As a result, the route constructed with the qualified links can meet the bandwidth and the QoS requirements, usually in the first attempt.

(f) It is a loop-free routing algorithm. The only possible case that the route may consist a loop can be caused from the corrupted metric information. However this can be detected by a check sum.

(g) Since the flooding method is basically a broadcast mechanism, it can be used for locating resources in network. Many network applications are best served by a broadcast facility, such as distributed data bases, address resolution, or mobile communications. Implementing broadcast in point-to-point networks is not straight forward. The flooding technique provides a means to solve this problem. In particular, locating a mobile user by Flood Routing, and establishing a dynamic route is an interesting issue. Application to a movable network in which entire network units including both the mobile users as well as the switching nodes and the wireless links is another potential research area.

5. Future Study and Conclusion

In this paper, we introduced a revised Flood Routing technique. Flood Routing is a novel approach to network routing which has the potential to solve many of the routing problems in contemporary networks. The basic Flood Routing presented in this paper has been developed to be used in an ATM style network, however we believe a similar technique can also be applied to IP routing. Another promising area of application of this method would be military or mobile networks which require high mobility and reliability. Research to extend the point-to-point Flood Routing to optimal multi-point routing is now progressing. Further analysis of performance, and application to large scale networks are the future issues.

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A flood routing method for data networks

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Abstract:
A new routing algorithm based on a flooding method is introduced. Flooding techniqu have been used previously, e.g. for broadcasting the routing table in the ARPAnet an other special purpose networks. However, sending data using flooding can often satu the network and it is usually regarded as an inefficient broadcast mechanism. Our approach is to flood a very short packet to explore an optimal route without relying c preestablished routing table, and an efficient flood control algorithm to reduce the signalling traffic overhead. This is an inherently robust mechanism in the face of a network configuration change, it achieves automatic load sharing across alternative routes, and has the potential to solve contemporary routing problems. An version of mechanism was originally developed for virtual circuit establishment in the experime Caroline ATM LAN at Monash University

Index Terms:
data communication packet switching telecommunication congestion control telecommunica network routing telecommunication signalling telecommunication traffic ARPAnet ATM network Caroline ATM LAN Monash University automatic load sharing data networks flo control algorithm **flood routing method** network configuration optimal route routing algori routing table broadcasting signalling traffic overhead reduction virtual circuit establishment

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BE	AP	Alagar, S. and Venkatesan, S., "Reliable Broadcast in Mobile Wireless Networks," Department of Computer Science, University of Texas at Dallas, Military Communications Conference, 1995, MILCOM '95 Conference Record, IEEE San Diego, California, November 5-8, 1995 (pages 236-240)	
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A DISTRIBUTED RESTORATION ALGORITHM FOR MULTIPLE-LINK AND NODE FAILURES OF TRANSPORT NETWORKS

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Abstract

Broadband optical fiber networks will require fast restoration from multiple-link and node failures as well as single-link failures. This paper describes a new distributed restoration algorithm based on message flooding. The algorithm is an extension of our previously proposed algorithm for single-link failure. It restores the network from multiple-link and node failures, using multi-destination flooding and path route monitoring. We evaluated the algorithm by computer simulation, and verified that it can find alternate paths within 0.5s whenever the message processing delay at a node is 5ms.

1. Introduction

There is an increasing dependency on today's communication networks to implement strategic corporate functions. User demands for high-speed and economical communications services lead to the rapid deployment of high-capacity optical fibers in the transport networks. At the same time, the demands for high-reliability services raise a network survivability problem. For example, if the network is disabled for one hour, up to $6,000,000 loss of revenue can occur in the trading and investment banking industries [1]. As the capacity of the transmission link grows, a link cut results in more loss of services. Therefore, rapid restoration from failures is becoming more critical for network operations and management.

There have been many algorithms developed to restore networks, including centralized control [1] and distributed algorithms [2-4]. In centralized control, the network is controlled and managed from a central office. In distributed control, the processing load is distributed among the nodes and restoration is thus faster. However, more computation capability and high speed control data channels are required. Recently it has been possible to provide high performance microprocessors for digital cross-connect system (DCS). High capacity optical fibers enable high speed data transmission for OAM through overhead bytes, which is under study by CCITT.

The distributed algorithms proposed so far [2-4] are based on simple flooding [5]. When a node detects failure, it broadcasts a restoration message to adjacent nodes to find an alternate route. In the algorithm [2], a restoration message requests a spare DS-3 or STS-1 path and is sent through the path overhead of each spare path. To avoid congestion of the messages in this algorithm, a message in both the algorithms [3,4] requests a bundle of spare paths and is sent through the section overhead of each link. Algorithm [3] finds the maximum capacity along an alternate route, and our algorithm [4] finds the shortest alternate route. As described in [4], our algorithm was faster. However these algorithms are designed to handle single-link failures, they cannot handle multiple-link or node failures.

In this paper, we first discuss the major issues that must be addressed in order to handle multiple-link and node failures in Section 2. Based on these consideration, we propose a new restoration algorithm using multi-destination flooding and path route monitoring. These are described in Section 3. For a node failure, the node which detected the failure sends a restoration message to the last N-consecutive nodes each logical path passed through. An alternate path is made between the message sender node and one of the multiple nodes specified in the message. Each node collects the identifier of these nodes, using a path route monitoring technique. The algorithm was evaluated by computer simulation for multiple-link failure as well as for node failure. The results will be described in Section 4.

2. Limitations of simple flooding

In this section, we review simple flooding and discuss its limitations to handle multiple-link and node failures. In principle, the distributed algorithms [2-4] based on simple flooding work as follows. When a link fails, the two nodes connected to the link detect the failure and try to restore the path. One node becomes the sender and the other becomes the chooser (Fig. 1). The sender broadcasts restoration messages to all links with spare capacity. Every node except the sender and the chooser respond by re-broadcasting the message. When the restoration message reaches the chooser, the chooser returns an acknowledgement to the sender. In this way, alternate paths are found. Message congestion caused by routing messages far away is avoided by limiting the number of hops.

These algorithms based on simple flooding [2-4] usually assume a single-link failure, but in reality, some links which go different nodes may be in the same conduit. Therefore, if the conduit is cut, many links fail at the same time [3]. This is the case of multiple-link failure. Fire or earthquakes can also damage a large number of nodes, so the restoration algorithm must be able to handle these situations.

Simple flooding can not handle multiple-link or node failures because of following problems.

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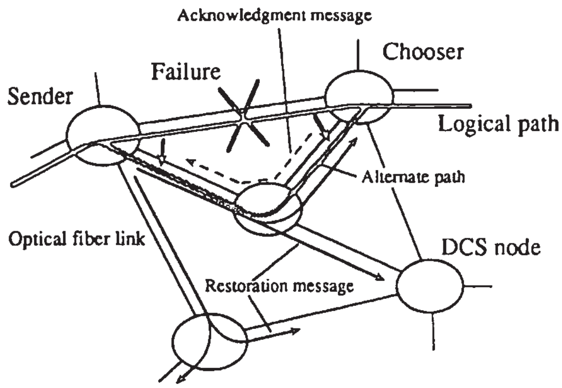

Fig. 1 Distributed restoration based on simple flooding

- *Contention of spare capacity*

In case of multiple-link failure, restoration messages coming from different nodes might contend for spare capacity on the same link. For example, if capacity is assigned to arriving messages in turn, the first message reserves the capacity. Whether or not the reserved capacity is later used for an alternate path, the reserved capacity is not released and therefore can not be assigned to another restoration message. Thus, the restoration ratio decreases.

- *Fault location*

Because the algorithms assume link failure, one of the two nodes connected to the failed link becomes the sender and the other becomes the chooser. However, for a node failure, there is a chooser and sender for each affected path. They are neighbors of the failed node and depend on the route of the paths. Each node detects failure by the loss of the signal on the link, and cannot distinguish between link or node failure.

The first problem could be alleviated by simple message cancelling. Spare capacity is assigned to restoration messages on a first-come, first-served basis. Assignment is cancelled when the message can not go forward due to hop limits or lack of capacity. During message flooding, cancel messages are sent to inform a node that a restoration message, which reserves spare capacity on a specific link, did not reach its destination and the served capacity of this link can be released for other restoration messages. Restoration messages are canceled immediately after reception if they are identical to messages already received, if the hop limit is reached, or if there is no more capacity at the node. In these cases, the unused capacity can be assigned to another restoration message.

Solving the second problem requires more sophisticated techniques and we propose a new distributed restoration algorithm in the following section.

3. Multi-destination flooding

To solve the fault location problem described above, we propose a new multi-destination flooding technique. We also propose path route monitoring which is essential to achieve multi-destination flooding.

3.1 Principle of multi-destination flooding

Simple flooding methods assume just one chooser. We extended this to allow multiple choosers as message destinations. When a node detects the loss of a signal from a link, the node can not tell whether the link or the node at the other end has failed. It sends a restoration message directed to the node which is the chooser in a link failure as well those that are choosers in a node failure. In Fig.2, for example, the link between nodes B and C fails, node B is the chooser for all affected paths, and nodes A and D are possible choosers for paths P1 and P2. If node B fails, nodes A and D become choosers for paths P1 and P2. The restoration message contains all choosers and the required capacity for each sender-chooser pair. The node which received the restoration message checks the destination field of the message, and if it is a chooser candidate, it returns an acknowledgment to the sender.

Thus, by extending simple flooding into multi-destination flooding, link or node failures do not have to be distinguished because there is always at least one chooser. Different messages are sent to the chooser candidates, but the same restoration message listing all candidates is sent towards all candidates. The number of restoration messages decreases and congestion is reduced.

Restoration processing consists of a broadcast phase, an acknowledgment phase, and a confirmation phase. To handle multiple failures, cancel processing is performed during the broadcast and acknowledgment phases.

The node states are sender, chooser, reserved tandem, and fixed tandem. The sender is the node which detected the failure. The chooser is the destination node of a restoration message. Chooser candidates set by the sender become choosers when they receive

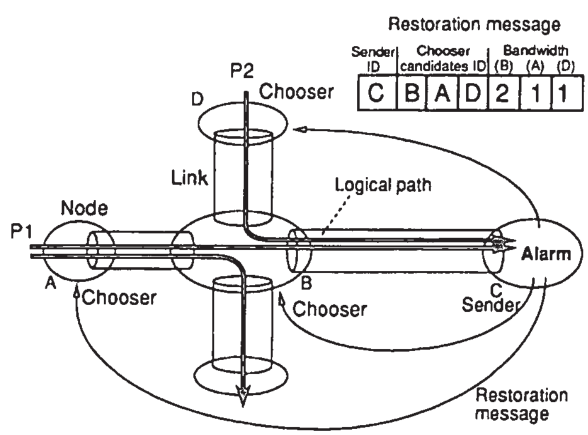

Fig. 2 Multi-destination flooding

a restoration message. The reserved tandem is a candidate node for alternate paths reserved by the restoration message. A received confirmation message of the sender turns a reserved tandem node into a fixed tandem node.

a) Broadcast phase

In the broadcast phase, the sender broadcasts restoration messages which reserve spare capacity in the network toward chooser candidates. A failure occurring on a link or node is detected by the next node on the path below the failure. This node becomes the sender. The sender looks up the chooser candidates and their capacities for the failed paths which were determined before by the path route monitoring described in the following section. The restoration message is then broadcast.

The restoration message contains the following information.

1) Message type : restoration, acknowledgment, confirmation, cancel
2) Message index
3) Sender ID
4) Chooser IDs (Multiple destination)
5) Required capacity of each sender-chooser pair
6) Reserved capacity
7) Hop count

The message index is set by the sender. It represents the number of flooding waves broadcast. The combination of the message index, the sender ID and chooser IDs is the Message ID. The required capacity is the capacity required between the sender and the various choosers. The reserved capacity is the capacity of the route taken by the restoration message.

The sender broadcasts the restoration message to all connected links except failed links and then waits for an acknowledgment from one of the choosers. Each node in the network except the sender and chooser receives a restoration message, and examines the hop count and the Message ID. If the hop count reaches the limit set by the sender, or a message with the same ID has arrived before, the node returns a cancel message to the link originating the restoration message. Otherwise, the state of the node is set to reserved tandem. If spare capacity is available, a restoration message is broadcast. If the spare capacity of a link is insufficient, the reserved capacity is set to the spare capacity of the link. A node that finds its own node ID among the chooser IDs in the restoration message becomes the chooser. Figure 3 shows the broadcast phase when a failure has occurred at node B.

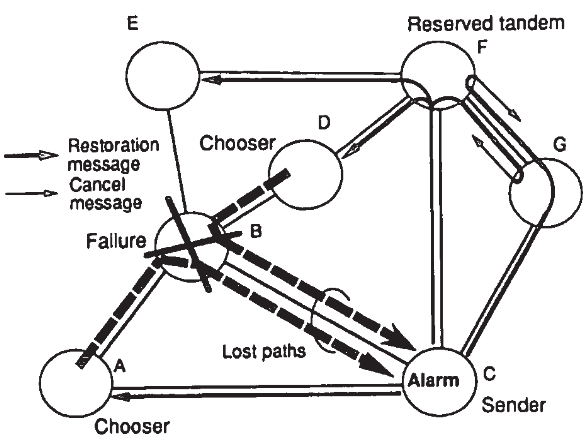

Fig. 3 Broadcast phase

b) Acknowledgment phase

In the acknowledgment phase, the chooser sends an acknowledgment message to the sender. By the entries in the acknowledgment message, the sender is informed which chooser the acknowledgement message is from. If another restoration message with the same message ID arrives at the chooser, it is canceled.

A reserved tandem node which receives an acknowledgment message passes it back to the source of the corresponding restoration message. All other reserved spare capacity of this restoration message is canceled. Message flow during an acknowledgment phase is shown in Fig. 4.

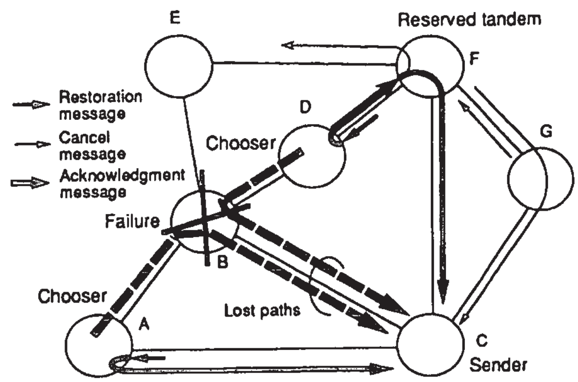

Fig. 4 Acknowledgment phase

c) Confirmation phase

When the acknowledgment message reaches the sender, a confirmation message is sent to the chooser. The reserved spares are switched over to alternate paths. If the sender received acknowledgment or canceled messages from all links it sent restoration messages to, and if the restoration of the failure is not completed, the sender increments the message index and attempts restoration from the broadcast phase again.

The reserved tandem node which received a confirmation message changes its status to fixed tandem and connects the reserved spares. In Fig. 5, node F has become fixed tandem, and the failed path between node D and node C is rerouted through the nodes D, F, and C. The other path which failed between node A and node C are also rerouted.

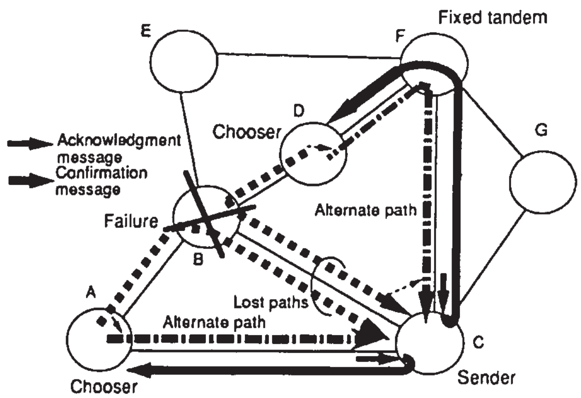

Fig. 5 Confirmation phase

3.2 Path route monitoring

For multi-destination flooding, each node must have route information on the paths passing through the node. One approach is to have the central office distribute such route information to all nodes. However, the routes are changing dynamically under customer control and nodes might receive inconsistent route information because updating route data takes time. We propose a path route monitoring method in which each node collects route information in real time.

The route information required at every node are the ID's of the last two consecutive nodes in every path before the node. This information is collected as follows. Node ID's are sent through assigned space in the path overhead. For every path going through a node, the data in the ID area is shifted and the ID of the node it is going through is written in. In this way, every node receives continuous and real-time route information.

4. Simulation

4.1 Simulation tool and conditions

We evaluated the ability of the algorithm to restore multiple-link and node failures using an event-driven network simulator [4,6] which works on the SUN3 workstation. We used the mesh network model shown in Fig. 6. This network consists of 25 nodes and 40 links. Each link length was generated at random, and the average link length is 184 km. Every link has 35 working paths.

We assumed a transmission speed of 64 kb/s. Messages were 16 bytes long, and the hop limit was 9. In a SONET frame structure, 64 kb/s for transmission speed means that one byte of overhead is used for message communications between nodes. The processing delay time from the arrival of a message to the end of the processing depends on the architecture of the DCS hardware. We assumed a 5 ms delay. This simulation does not include failure detection or crossconnection times.

4.2 Simulation results

Figure 7 shows a cumulative restoration ratio of node failure. The restoration ratio of the network is the ratio of restored to lost paths. For node failure, paths terminating at the failed node are not counted as lost paths because it is impossible to restore them.

We also simulated the algorithm for single-link failure. The result is shown in Fig. 7.

Figure 8 shows the cumulative restoration ratio in a multiple-link failure. There are many link combinations, but only one is shown. Failures between node N8 and N13, and one of the other links, occured simultaneously on two links. The results indicate

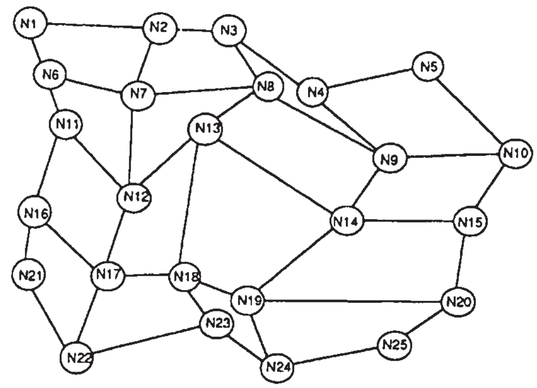

Fig. 6 Network model

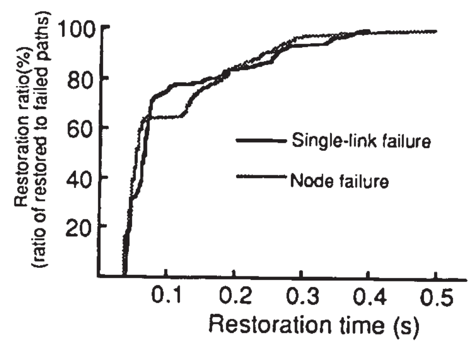

Fig. 7 Simulation results on single-link and node failure

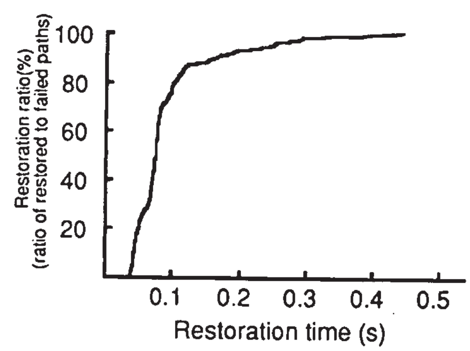

Fig. 8 Simulation result on multiple-link failure

that the proposed algorithm can handle multiple-link and node failure as well as single-link failure. All restorations are completed within 0.5s with message processing delay at the nodes being 5ms.

5. Conclusion

We pointed out problems associated with adapting a restoration algorithm based on flooding to recover from multiple-link and node failures. The main problem is to position the chooser nodes correctly. We proposed multi-destination flooding and path route monitoring. We simulated the algorithm with a mesh network and verified that the algorithm can handle multiple-link and node failures as well as single-link failures.

The message delay within a node depends on the architecture of the DCS and the processing load. The next step will be to analyze these delays and to include restoration time.

Acknowledgment

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A distributed restoration algorithm for multiple-link and node failures of transport networks

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Abstract:
Fast restoration of broadband optical fiber networks from multiple-link and node failu as well as single-link failures, is addressed. A **distributed restoration algorithm** b on message flooding is described. The algorithm is an extension of a previously prop algorithm for single-link failure. It restores the network from multiple-link and node failures, using multidestination flooding and path route monitoring. Computer simula of the algorithm verified that it can find alternate paths within 0.5 s, whenever the message processing delay at a node is 5 ms

Index Terms:
broadband networks optical links broadband optical fiber networks **distributed restoration algorithm** message flooding message processing delay multidestination flooding multiple-li failures node failures path route monitoring single-link failures transport networks

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Performance Analysis of Network Connective Probability of Multihop Network under Correlated Breakage

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Abstract—One of important properties of multihop network is the network connective probability which evaluate the connectivity of the network. The network connective probability is defined as the probability that when some nodes are broken, rest nodes connect each other. Multihop networks are classified to the regular network whose link assignment is regular and the random network whose link assignment is random. It has been shown that the network connective probability of regular network is larger than that of random network. However, all of these results is shown under independent node breakage. In this paper, we analyze the network connective probability of multihop networks under the correlated node breakage. It is shown that regular network has better performance of the network connective probability than random network under the independent breakage, on the other hand, random network has better performance than regular network under the correlated breakage.

1 Introduction

In recent years, multi-hop networks have been widely studied [1]-[8]. These networks must pass messages between source and destination nodes via intermediate links and nodes. Examples of them include ring, shuffle network (SN) [1],[2] and chordal network (CN)[3]. One of the very important performance measure of multi-hop network is the connectivity of the network. If some nodes are broken, it is needed for a network to guarantee the connection among non-broken nodes. Thus, the network connective probability defined as the probability that when some nodes are broken, rest links and nodes construct the connective network, should be a very important property to evaluate the connectivity of the network.

Multi-hop networks are classified to regular network and random network according to the way of link assignment. In the regular network, links are assigned regularly and examples of them include shufflenet and manhattan street network. On the other hand, in random network, link assignment is not regular but somewhat random and examples of them include connective semi-random network (CSRN) [6]. The network connective probabilities of some multi-hop networks have been analyzed and it has been shown that the network connective probability of regular network is larger than that of random network. However, all of them is analyzed under the condition that locations of broken nodes are independent each other. In the real network, there are some case that the locations of broken nodes have correlation, for example, links and nodes are broken in the same area under the case of disaster. Thus, it is significant and great of interest to analyze the network connective probability under the condition when the locations of broken nodes have correlations each other.

In this paper, we analyze the network connective probability of multi-hop network under the condition that locations of broken nodes have correlations each other, where we treat SN, CN and CSRN as the model for analysis. We realize the correlation as follows. At first, we note one node and break it and call this node the center broken node. And next, we note nodes whose links connect to the center broken nodes and break them at some probability. We define this probability as the correlated broken probability. Very interesting result is shown that under independent breakage of node, regular network has better performance of the network connective probability than random network, on the other hand, under the correlated breakage of node, random network has better performance than regular network.

In the section 2, we explain network model of SN, CN and CSRN which we analyze in the section 3. In the section 3, we analyze the network connective probability under the condition when the location of broken nodes have correlation each other. And we compare each of network connective probability in the section 4. In the last, we conclude our study.

2 Multihop network model

In this section, we explain the multihop network models used for analysis of the network connective probability. We treat three networks such as SN, CN and CSRN which consists of N nodes and p unidirected outgoing links per node.

Fig. 1 shows SN with 18 nodes and 2 outgoing links per node. To construct the SN, we arrange $N = kp^k$ ($k = 1, 2, \cdots$; $p = 1, 2, \cdots$) nodes in k columns of p^k nodes each. Moving from left to right, successive columns are connected by p^{k+1} outgoing links, arranged in a fixed shuffle pattern, with the last column connected to the first as if the entire graph were wrapped around a cylinder. Each of the p^k nodes in a column has p outgoing links directed to p different nodes in the next column. Numbering the nodes in a column from 0 to $p^k - 1$, nodes i has outgoing links directed to nodes $j, j+1, \cdots$, and $j + p - 1$ in the next column, where $j = (i \ mod \ p^{k-1})p$. In Fig. 1, p is equal to 2 and k is equal to 2. Since the link assignmentof SN is regular, SN is regular network.

Fig. 2 shows CN with 16 nodes and 2 outgoing links per node. To construct CN, at first, we construct unidirected ring network with N nodes and N unidirected links. And $p-1$ unidirected links are added from each node. Numbering nodes along ring network from 0 to $N - 1$, node i has outgoing links directed to nodes $(i + 1) \ mod \ N, (i + r_1) \ mod \ N, \cdots$, and $(i + r_{p-1}) \ mod \ N$, where r_j ($j = 1, 2, \cdots, p - 1$) is defined as the chordal length. In Fig. 2, r_1 is equal to 3. Since r_i for every i are independent each other, CN is not regular network. However, CN has much regular elements such a symmetrical pattern of network.

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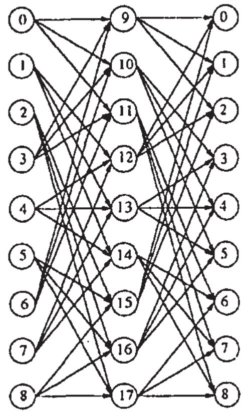


Figure 1. Shuffle network with N = 18 and p = 2.

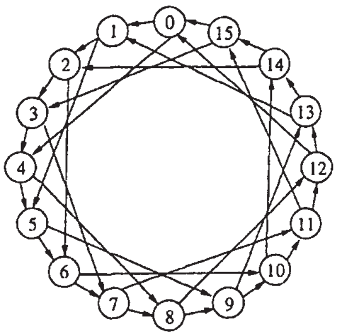


Figure 2. Chordal network with N = 16, p = 2 and r_1 = 3.

Fig. 3 shows CSRN with 16 nodes and 2 outgoing links from a node. Similarly with CN, CSRN includes unidirected ring network with N nodes and N unidirected links. And we add $p - 1$ links from each node whose directed nodes are randomly selected. In CSRN, the number of incoming links per node is not constant, for example, in Fig. 3, the number of incoming links into node 1 is 1 and the one into node 3 is 3. The link assignment of CSRN is random except for the part of ring network, thus CSRN is random network. It has been shown that since the number of incoming links per node is not constant, the network connective probability of CSRN is smaller than those of SN and CN when locations of broken nodes are independent each other. And that of SN is the same as that of CN, because the network connective probability depends on the number of incoming links come into every nodes.

3 Performance Analysis

Here, we analyze the network connective probability of SN, CN and CSRN under the condition that locations of broken nodes have correlation each other. Now, we explain the network connective probability in detail using Fig. 3. This figure shows the connective network which is defined as the network in which all nodes connect to every other nodes directly or indirectly. At first, we consider the case that the node 1 is broken. The node 1 has two outgoing links directed to nodes 2 and 3, and if the node 1 is broken, we can not use them. However, node 2 has two incoming links from nodes 1 and 14, and node 3 has three incoming links from nodes 1, 2 and 11. Therefore, even if node 1 is broken, rest nodes can construct

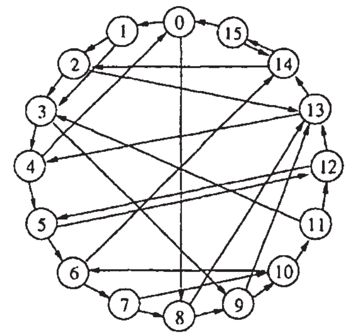


Figure 3. Connective semi-random network with N = 16 and p = 2.

the connective network. Next, we consider the case that node 0 is broken. The node 0 has two outgoing links directed to nodes 1 and 8, and if the node 0 is broken, we can not use them. Since node 1 has only one incoming link from node 0, even if only node 0 is broken, rest nodes can not connect to node 1, that is, they can not construct the connective network. Here, we define the network connective probability as the probability that when some nodes and links are broken, the rest nodes and links can construct the connective network.

Now, we explain the correlated node breakage using Fig. 3. At first, we note one node and break it, where this node is called as the center broken node. And then, we note nodes whose outgoing links come into the center broken node or whose incoming links go out of the center broken node, and break them at a probability defined as the correlated broken probability. In Fig 3, when we assume that the center broken node is the node 3, there are five nodes 1, 2, 4, 9 and 11 which have possibility to become correlated broken node. And they become the broken nodes at the correlated broken probability. It is obvious that none of them is broken when the correlated broken probability is 0 and all of them is broken when the correlated broken probability is 1.

In our study, we analyze the network connective probability that only nodes are broken. And we assume that the number of center broken node is one in the analysis. We denote the correlated broken probability by a and the network connective probability of SN, CN and CSRN by P_{SN}, P_{CN} and P_{CSRN}, respectively.

3.1 Shuffle Network

Because the number of incoming links per node in SN is the constant p, when broken node is only center broken node, the rest nodes can construct the connective network. There are $2p$ nodes have the possibility to become the correlated broken node. All of p nodes which have outgoing link come into the center broken node have the outgoing links directed to the same nodes. For example, in Fig. 1, if we assume that the node 9 is the center broken node, the nodes 0, 3 and 6 has outgoing links to node 9. And each of three nodes have two outgoing links directed to nodes 10 and 11. Therefore, only when all of them are broken, the rest nodes can not construct the connective network. On the other hand, all of outgoing links go out from p nodes which have incoming link from center broken node direct to different nodes. In Fig. 1, nodes 0, 1 and 2 have the incoming link from center broken node 9. And all of the outgoing links from their nodes direct to different nodes, thus even if all of them are broken, the rest nodes can construct the connective network. Thus, the network connective probability of SN is the probability that all of nodes whose outgoing links come into the center broken node are broken, and it is derived as

$$P_{SN} = 1 - a^p \,. \tag{1}$$

3.2 Chordal Network

The network connective probability of CN with $p = 2$ is different from that with $p \geq 3$. At first, we consider the case with $p = 2$. When p is equal to 2, all of the outgoing links, from the nodes whose incoming links go out from the center broken node, direct to the same node. For example, in Fig. 2, when we assume that the center broken node is node 0, the outgoing links from it direct to nodes 1 and 4. And each of outgoing links from them directs to node 5. Therefore, only when all nodes whose incoming links go out from the center broken node are broken, the rest nodes can not construct the connective network. And we can obtain the network connective probability as

$$P_{CN} = 1 - a^2 \qquad \text{for } p = 2\,. \tag{2}$$

And next, we consider the case that $p \geq 3$. In CN, when p is equal to or larger than three and each chordal length is selected properly, all of outgoing links from the nodes whose incoming links go out from the center broken node do not direct to the same nodes. And therefore, even if all of nodes which connect to the center broken nodes with incoming or outgoing links is broken, the rest nodes can construct the connective network, that is,

$$P_{CN} = 1 \qquad \text{for } p \geq 3\,. \tag{3}$$

3.3 Connective Semi-Random Network

In CSRN, the number of the incoming links per node is not constant. Since the maximum number of incoming links is $N - 1$ and one link come into a node at least, the probability that the number of the incoming links come into a node is i, denoted as A_i, is

$$A_i = \begin{cases} 0, & \text{for } i = 0 \\ \binom{N-2}{i-1}(\frac{p}{N-2})^{i-1}(1 - \frac{p}{N-2})^{N-1-i} & \text{for } i \geq 1. \end{cases} \tag{4}$$

The nodes which have possibility to become the correlated broken nodes are those which connect to the center broken node by outgoing link or incoming link. When the number of the incoming link come into the center broken node is i, the sum of outgoing links and incoming links it have is $p + i$. However, the number of the nodes which have possibility to become the correlated broken nodes is not always $p + i$, because the p outgoing links have the possibility to overlap with one of i incoming links. For example, in Fig. 3, when the center broken nodes is node 5, the outgoing link to node 12 overlap with the incoming link from node 12. Therefore, in spite of the node 5 has four outgoing and incoming links, the number of the nodes which have possibility to become the correlated broken nodes when the node 5 is the center broken node is three.

And now, we derive the probability that the number of nodes which have possibility to become the correlated broken nodes is j, denoted as B_j. Before derive B_j, we derive the probability that q of p outgoing links which go out of a node overlap with r incoming links come into it, denoted as $C_{p,q,r}$. Here, we define regular link as the link which construct the ring network and random link as other link. We consider the two case. The one is the case that one of the incoming links overlap with the regular outgoing link, and the other case is that none of incoming links overlap with it. Since the regular incoming link never overlap with the regular outgoing link, the probability to become the first case is $(r - 1)/(N - 2)$ and one to become the second case is $1 - (r - 1)/(N - 2)$. In the first case, $C_{p,q,r}$ is the same as the probability that each of $q - 1$ outgoing links among the $p - 1$ outgoing links except for the regular outgoing link overlap one of $r - 1$ incoming links, denoted as $C'_{p-1,q-1,r-1}$. And in the second case, $C_{p,q,r}$ is the same as the probability that each of q outgoing links among the $p - 1$ outgoing links except for the regular outgoing link overlap one of r incoming links, denoted as $C'_{p-1,q,r}$. Using $C'_{p',q',r'}$ given as follows,

$$C'_{p',q',r'} = \begin{cases} 0, & \text{for } q' < 0, r' \leq 0, q' > p', \\ & (p' + r' > N \text{ and} \\ & q' < p' + r' - N) \\ \frac{\binom{p'}{q'}\, {}_{r'}P_{q'}\; {}_{N-2-r'}P_{p'-q'}}{{}_{N-2}P_{p'}}, & \text{otherwise,} \end{cases} \tag{5}$$

we can derive $C_{p,q,r}$ as

$$C_{p,q,r} = (\frac{r-1}{N-2})C'_{p-1,q-1,r-1} + (1 - \frac{r-1}{N-2})C'_{p-1,q,r}\,. \tag{6}$$

B_j can be derived as the sum of the probability that when the number of incoming links is $j - p + q$, q of p outgoing links overlap with one of incoming links. Therefore, we can obtain B_j as

$$B_j = \sum_{q=max(0,p+1-j)}^{p} A_{j-p+q}C_{p,q,j-p+q}\,. \tag{7}$$

Here, we consider two nodes whose regular links connect to the center broken node. We call them regular node (R-node). And we define non-connective node (NC-node) as the node which have no incoming link. Even if a node has many incoming links, when all of source node of them are broken, it becomes NC-node. However, when the number of incoming link is equal to or greater than 2, the probability that all of source nodes of them are broken is very small compared with that when the number of incoming link is 1. Therefore, we assume the NC-node as the node which have only one incoming link and its source node is broken. That is, when the destination node of regular outgoing link of the broken node has only this regular incoming link and this node is not broken, it becomes the NC-node. Fig. 4 shows the center broken node and R-node. (a) shows the case that none of R-node is broken, (b) shows the case that one of them is broken, and (c) shows the case that both of them are broken. It is found that there is only one node which have possibility to become the NC-node in all case. The probability that this node becomes the NC-node is A_1. When the number of broken nodes is k, we can consider the three case with $k = 1$, $k = 2$ and $k > 2$. In $k = 1$, this node is the center broken node and it certainly becomes the case (a) and never becomes the case (b) and (c). In $k = 2$, the one node is the center broken node and the other is the correlated broken node and it becomes the cases (a) or (b). And the probability to become the case (a) is $2/l$ and to become the case (b) is $1 - 2/l$ where l is the number of the nodes have possibility to become the correlated broken nodes. If $k > 2$, it becomes all the case. The number of broken nodes except for R-node in (a), (b) and (c) is k, $k-1$ and $k-2$, respectively. Furthermore, when the number of links connect to the center broken node is l, the probability that the number of correlated broken nodes is k, denoted as $t_{l,k}$ is

$$t_{l,k} = B_l \binom{l}{k} a^k (1-a)^{l-k}\,. \tag{8}$$

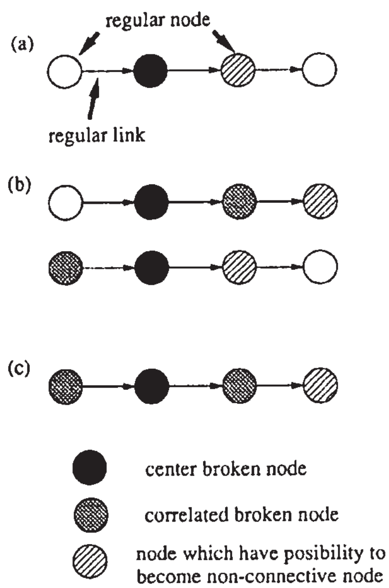


Figure 4. The center broken node and regular nodes.

And in this case, the probability to become the case of (a) is $(\binom{k}{0}\,{}_{l-2}P_k)/{}_lP_k$, to become the case of (b) is $(\binom{k}{1}\,{}_{l-2}P_{k-1})/{}_lP_k$ and to become the case of (c) is $(\binom{k}{2}\,{}_{l-2}P_{k-2})/{}_lP_k$. The network connective probability when the number of broken nodes is l, denoted as E_l, is derived in [8] as follows

$$E_l = \prod_{s=0}^{l-1} \frac{N - NA_1 - s}{N - s} . \tag{9}$$

Therefore, using (8) and (9), we can obtain the network connective probability as

$$\begin{aligned} R_{CSRN} = & \sum_{l=p}^{N-1} t_{l,0}(1 - A_1) \\ & + \sum_{l=p}^{N-1} t_{l,1}\{\frac{2}{l}(1 - A_1) + (1 - \frac{2}{l})(1 - A_1)E_l\} \\ & + \sum_{k=2}^{N-1} \sum_{l=max(p,k)}^{N-1} t_{l,k}\{\frac{\binom{k}{0}\,{}_{l-2}P_k}{{}_lP_k}(1 - A_1)E_k \\ & \qquad + \frac{\binom{k}{1}\,{}_{l-2}P_{k-1}}{{}_lP_k}(1 - A_1)E_{k-1} \\ & \qquad + \frac{\binom{k}{2}\,{}_{l-2}P_{k-2}}{{}_lP_k}(1 - A_1)E_{k-2}\} . \end{aligned} \tag{10}$$

4 Results

We show computer simulation and theoretical calculation results of the network connective probability under the correlated breakage.

Fig. 5 shows the network connective probability of SN, CN and CSRN with $p = 2$ versus the correlated broken probability. In this figure, the chordal length of CN, r_1 is 50. It is shown that the both the network connective probability of SN and CN is the same in $p = 2$. It is also shown that the network connective probability of CN or SN is larger than that of CSRN in small a, however, in large a, the network connective probability of CN or SN is smaller than that of CSRN.

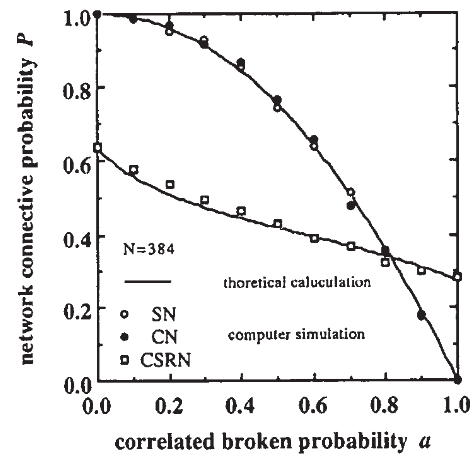


Figure 5. The network connective probability with $p = 2$ versus correlated broken probability.

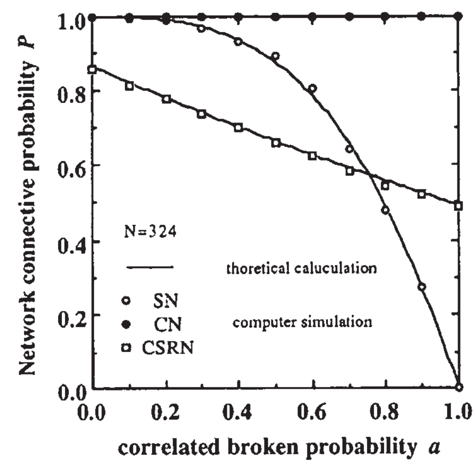
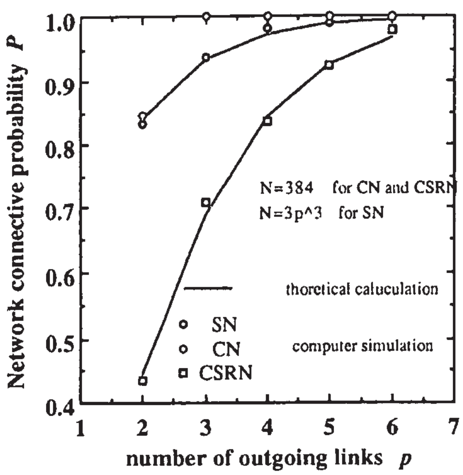


Figure 6. The network connective probability with $p = 3$ versus correlated broken probability.

Fig. 6 shows the network connective probability of SN, CN and CSRN with $p = 3$ versus the correlated broken probability. In this figure, r_1 is 50 and r_2 is 120. The tendency of the network connective probability of SN and CSRN is the same as the case with $p = 2$. However, the tendency of the network connective probability of CN is not different from that with $p = 2$.

In CSRN, because the number of incoming links come into a node is not constant, even if p is large, there are some nodes whose number of incoming links is one. Therefore, the network connective probability itself is small. However, the link assignment of CSRN is random, the condition of correlated breakage is not so different from that of independent breakage. On the other hand, in SN, because the number of incoming links come into a node is constant, the network connective probability under the indepen-



'igure 7. The network connective probability with a = 0.4 versus the number of outgoing links per node.

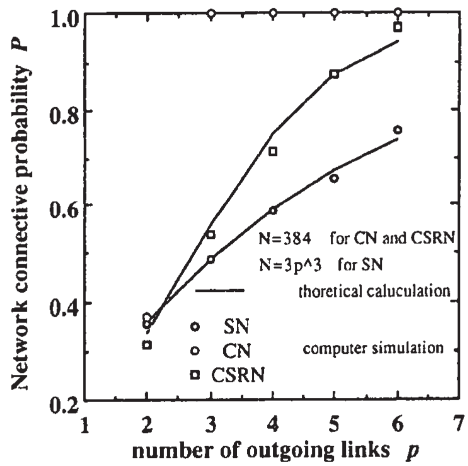


Figure 8. The network connective probability with a = 0.8 versus the number of outgoing links per node.

ent breakage is large. However, because of regularity of the link ssignment, that under the correlated breakage is small. In CN, 'hen p is two, the link assignment is regular, however, when p : larger than two, every chordal length is random and independent each other, and the link assignment is random. Moreover, the umber of incoming links per node of CN is the constant. There›re, the network connective probability of CN is large under both ıe independent and correlated breakage.

Figs. 7 and 8 show the network connective probability with = 0.4 and 0.8 versus p, respectively. It is shown that the larger is, the smaller difference of network connective probability beveen SN and CSRN is, when a is small. On the other hand, when is large, the larger p is, the larger difference of network connective probability between SN and CSRN is. The reason is as ›llows. When a is small, the network connective probability of :SRN is small. However, the larger p is, the smaller the number of odes, whose number of incoming links is 1, is, and the closer to 1 ıe network connectivity is. In SN and CN, even if p is small, the etwork connective probability is somewhat large when a is small. Vhen p is large, the network connective probability of CSRN is lmost the same with small p. On the other hand, in SN, the tenency network connectivity versus p is almost the same, however, ıe larger a is, the smaller the value is.

As these results, CN has best performance of network connecvity. However, it has been shown that CN has much poorer per›rmance of internodal distance than other network. Thus, it is xpecetd for the network to have good performance of both net'ork connective probability and internodal distance.

; Conclusion

We theoretically analyze the network connective probability f multihop network under the correlated damage of node. We 'eat shuffleNet, chordal network and connective semi-random etwork. It is found that in the independent node breakage, the etwork whose number of incoming links is the constant has good erformance of network connective probability, and found that in ıe correlated node breakage, the network whose link assignment is random has good performance of one.

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Performance analysis of network connective probability multihop network under correlated breakage

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Abstract:
One of important properties of a multihop network is the network connective probabi which evaluate the connectivity of the network. The network connective probability i: defined as the probability that when some nodes are broken, the rest of the **nodes connect** each other. Multihop **networks** are classified as a regular network whose li assignment is regular and a random network whose link assignment is random. It ha been shown that the network connective probability of a regular network is larger th: that of a random network. However, all of these results is shown under independent breakage. We analyze the network connective probability of multihop networks unde correlated node breakage. It is shown that a regular network has a better performan the network connective probability than a random network under independent break on the other hand, a random network has a better performance than a regular netwc under correlated breakage

Index Terms:
correlation methods network **topology** probability random processes telecommunication network reliability correlated node breakage independent breakage link assignment multih **network** **network connective** probability **node** breakage performance performance analy random network regular network

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On Four-Connecting a Triconnected Graph[†] (Extended Abstract)

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Abstract

We consider the problem of finding a smallest set of edges whose addition four-connects a triconnected graph. This is a fundamental graph-theoretic problem that has applications in designing reliable networks.

We present an $O(n\alpha(m,n)+m)$ time sequential algorithm for four-connecting an undirected graph G that is triconnected by adding the smallest number of edges, where n and m are the number of vertices and edges in G, respectively, and $\alpha(m,n)$ is the inverse Ackermann's function.

In deriving our algorithm, we present a new lower bound for the number of edges needed to four-connect a triconnected graph. The form of this lower bound is different from the form of the lower bound known for biconnectivity augmentation and triconnectivity augmentation. Our new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier for the number of edges needed to k-connect a $(k-1)$-connected graph. For $k=4$, we show that this lower bound is tight by giving an efficient algorithm for finding a set of edges with the required size whose addition four-connects a triconnected graph.

1 Introduction

The problem of augmenting a graph to reach a certain connectivity requirement by adding edges has important applications in network reliability [6, 14, 28] and fault-tolerant computing. One version of the augmentation problem is to augment the input graph to reach a given connectivity requirement by adding a smallest set of edges. We refer to this problem as the *smallest augmentation* problem.

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Vertex-Connectivity Augmentations
The following results are known for solving the smallest augmentation problem on an undirected graph to satisfy a vertex-connectivity requirement.

For finding a smallest biconnectivity augmentation, Eswaran & Tarjan [3] gave a lower bound on the smallest number of edges for biconnectivity augmentation and proved that the lower bound can be achieved. Rosenthal & Goldner [26] developed a linear time sequential algorithm for finding a smallest augmentation to biconnect a graph; however, the algorithm in [26] contains an error. Hsu & Ramachandran [11] gave a corrected linear time sequential algorithm. An $O(\log^2 n)$ time parallel algorithm on an EREW PRAM using a linear number of processors for finding a smallest augmentation to biconnect an undirected graph was also given in Hsu & Ramachandran [11], where n is the number of vertices in the input graph. (For more on the PRAM model and PRAM algorithms, see [21].)

For finding a smallest triconnectivity augmentation, Watanabe & Nakamura [33, 35] gave an $O(n(n+m)^2)$ time sequential algorithm for a graph with n vertices and m edges. Hsu & Ramachandran [10, 12] developed a linear time algorithm and an $O(\log^2 n)$ time EREW parallel algorithm using a linear number of processors for this problem. We have been informed that independently, Jordan [15] gave a linear time algorithm for optimally triconnecting a biconnected graph.

For finding a smallest k-connectivity augmentation, for an arbitrary k, there is no polynomial time algorithm known for finding a smallest augmentation to k-connect a graph, for $k>3$. There is also no efficient parallel algorithm known for finding a smallest augmentation to k-connect any nontrivial graph, for $k>3$.

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The above results are for augmenting undirected graphs. For augmenting directed graphs, Masuzawa, Hagihara & Tokura [23] gave an optimal-time sequential algorithm for finding a smallest augmentation to k-connect a rooted directed tree, for an arbitrary k. We are unaware of any results for finding a smallest augmentation to k-connect any nontrivial directed graph other than a rooted directed tree, for $k > 1$.

Other related results on finding smallest vertex-connectivity augmentations are stated in [4, 19].

Edge-Connectivity Augmentations

For the problem of finding a smallest augmentation for a graph to reach a given edge connectivity property, several polynomial time algorithms and efficient parallel algorithms are known. These results can be found in [1, 3, 4, 5, 8, 9, 13, 16, 19, 24, 27, 30, 31, 34, 37].

Augmenting a Weighted Graph

Another version of the problem is to augment a graph, with a weight assigned to each edge, to meet a connectivity requirement using a set of edges with a minimum total cost. Several related problems have been proved to be NP-complete. These results can be found in [3, 5, 7, 20, 22, 32, 33, 36].

Our Result

In this paper, we describe a sequential algorithm for optimally four-connecting a triconnected graph. We first present a lower bound for the number of edges that must be added in order to reach four-connectivity. Note that lower bounds different from the one we give here are known for the number of edges needed to biconnect a connected graph [3] and to triconnect a biconnected graph [10]. It turns out that in both these cases, we can always augment the graph using exactly the number of edges specified in this above lower bound [3, 10]. However, an extension of this type of lower bound for four-connecting a triconnected graph does not always give us the exact number of edges needed [15, 17]. (For details and examples, see Section 3.)

We present a new type of lower bound that equals the exact number of edges needed to four-connect a triconnected graph. By using our new lower bound, we derive an $O(n\alpha(m,n)+m)$ time sequential algorithm for finding a smallest set of edges whose addition four-connects a triconnected graph with n vertices and m edges, where $\alpha(m,n)$ is the inverse Ackermann's function. Our new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier for the number of edges needed to k-connect a $(k-1)$-connected graph. The new lower bound and the algorithm described here may lead to a better understanding of the problem of optimally k-connecting a $(k-1)$-connected graph, for an arbitrary k.

2 Definitions

We give definitions used in this paper.

Vertex-Connectivity

A graph[‡] G with at least $k+1$ vertices is *k-connected*, $k \geq 2$, if and only if G is a complete graph with $k+1$ vertices or the removal of any set of vertices of cardinality less than k does not disconnect G. The *vertex-connectivity* of G is k if G is k-connected, but not $(k+1)$-connected. Let $\mathcal{U}$ be a minimal set of vertices such that the resulting graph obtained from G by removing $\mathcal{U}$ is not connected. The set of vertices $\mathcal{U}$ is a *separating k-set*. If $|\mathcal{U}| = 3$, it is a *separating triplet*. The *degree* of a separating k-set $\mathcal{S}$, $d(\mathcal{S})$, in a k-connected graph G is the number of connected components in the graph obtained from G by removing $\mathcal{S}$. Note that the degree of any separating k-set is ≥ 2.

Wheel and Flower

A set of separating triplets with one common vertex c is called a *wheel* in [18]. A wheel can be represented by the set of vertices $\{c\} \cup \{s_0, s_1, \ldots, s_{q-1}\}$ which satisfies the following conditions: (*i*) $q > 2$; (*ii*) $\forall i \neq j$, $\{c, s_i, s_j\}$ is a separating triplet except in the case that $j = ((i+1) \bmod q)$ and (s_i, s_j) is an edge in G; (*iii*) c is adjacent to a vertex in each of the connected components created by removing any of the separating triplets in the wheel; (*iv*) $\forall j \neq (i+1) \bmod q$, $\{c, s_i, s_j\}$ is a degree-2 separating triplet. The vertex c is the *center* of the wheel [18]. For more details, see [18].

The *degree* of a wheel $W = \{c\} \cup \{s_0, s_1, \ldots, s_{q-1}\}$, $d(W)$, is the number of connected components in $G - \{c, s_0, \ldots, s_{q-1}\}$ plus the number of degree-3 vertices in $\{s_0, s_1, \ldots, s_{q-1}\}$ that are adjacent to c. The degree of a wheel must be at least 3. Note that the number of degree-3 vertices in $\{s_0, s_1, \ldots, s_{q-1}\}$ that are adjacent to c is equal to the number of separating triplets in $\{(c, s_i, s_{(i+2) \bmod q}) \mid 0 \leq i < q$, such that $s_{(i+1) \bmod q}$ is degree 3 in $G\}$. An example is shown in Figure 1.

A separating triplet with degree > 2 or not in a wheel is called a *flower* in [18]. Note that it is possible that two flowers of degree-2 $f_1 = \{a_{1,i} \mid 1 \leq i \leq 3\}$ and $f_2 = \{a_{2,i} \mid 1 \leq i \leq 3\}$ have the property that $\forall i$, $1 \leq i \leq 3$, either $a_{1,i} = a_{2,i}$ or $(a_{1,i}, a_{2,i})$ is an edge in G. We denote $f_1 \mathcal{R} f_2$ if f_1 and f_2 satisfy the above

[‡] Graphs refer to undirected graphs throughout this paper unless specified otherwise.

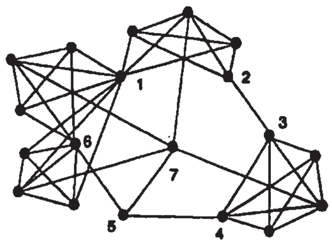

Figure 1: Illustrating a wheel $\{7\} \cup \{1,2,3,4,5,6\}$. The degree of this wheel is 5, i.e. the number of components we got after removing the wheel is 4 and there is one vertex (vertex 5) in the wheel with degree 3.

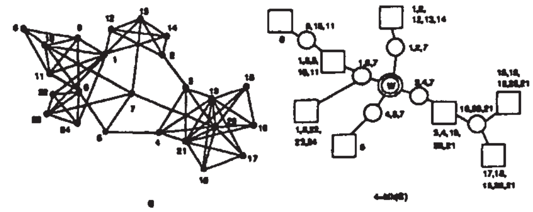

Figure 2: Illustrating a triconnected graph and its 4-$blk(G)$. We use rectangles, circles and two concentric circles to represent R-vertices, F-vertices and W-vertices, respectively. The vertex-numbers beside each vertex in 4-$blk(G)$ represent the set of vertices corresponding to this vertex.

condition. For each flower f, the *flower cluster* $\mathcal{F}_f$ for f is the set of flowers $\{f_1, \ldots, f_x\}$ (including f) such that $f\mathcal{R}f_i$, $\forall i$, $1 \le i \le x$.

Each of the separating triplets in a triconnected graph G is either represented by a flower or is in a wheel. We can construct an $O(n)$-space representation for all separating triplets (i.e. flowers and wheels) in a triconnected graph with n vertices and m edges in $O(n\alpha(m,n) + m)$ time [18].

K-Block

Let $G = (V, E)$ be a graph with vertex-connectivity $k-1$. A *k-block* in G is either (i) a minimal set of vertices B in a separating $(k-1)$-set with exactly $k-1$ neighbors in $V \setminus B$ (these are *special k-blocks*) or (ii) a maximal set of vertices B such that there are at least k vertex-disjoint paths in G between any two vertices in B (these are *non-special k-blocks*). Note that a set consisting of a single vertex of degree $k-1$ in G is a k-block. A *k-block leaf* in G is a k-block B_l with exactly $k-1$ neighbors in $V \setminus B_l$. Note also that every special k-block is a k-block leaf. If there is any special 4-block in a separating triplet $\mathcal{S}$, $d(\mathcal{S}) \le 3$. Given a non-special k-block B leaf, the vertices in B that are not in the flower cluster that separates B are *demanding vertices*. We let every vertex in a special 4-block leaf be a demanding vertex.

Claim 1 *Every non-special k-block leaf contains at least one demanding vertex.* □

Using procedures in [18], we can find all of the 4-block leaves in a triconnected graph with n vertices and m edges in $O(n\alpha(m,n) + m)$ time.

Four-Block Tree

From [18] we know that we can decompose vertices in a triconnected graph into the following 3 types: (i) 4-blocks; (ii) wheels; (iii) separating triplets that are not in a wheel. We modify the decomposition tree in [18] to derive the *four-block tree* 4-$blk(G)$ for a triconnected graph G as follows. We create an R-vertex for each 4-block that is not special (i.e. not in a separating set or in the center of a wheel), an F-vertex for each separating triplet that is not in a wheel, and a W-vertex for each wheel. For each wheel $W = \{c\} \cup \{s_0, s_1, \ldots, s_{q-1}\}$, we also create the following vertices. An F-vertex is created for each separating triplet of the form $\{c, s_i, s_{(i+1) \bmod q}\}$ in W. An R-vertex is created for every degree-3 vertex s in $\{s_0, s_1, \ldots, s_{q-1}\}$ that is adjacent to c and an F-vertex is created for the three vertices that are adjacent to s. There is an edge between an F-vertex f and an R-vertex r if each vertex in the separating triplet corresponding to f is either in the 4-block H_r corresponding to r or adjacent to a vertex in H_r. There is an edge between an F-vertex f and a W-vertex w if the the wheel corresponding to w contains the separating triplet corresponding to f. A *dummy R-vertex* is created and adjacent to each pair of flowers f_1 and f_2 with the properties that f_1 and f_2 are not already connected and either $f_1 \in \mathcal{F}_{f_2}$, $f_2 \in \mathcal{F}_{f_1}$ (i.e. their flower clusters contain each other) or their corresponding separating triplets are overlapped. An example of a 4-block tree is shown in Figure 2.

Note that a degree-1 R-vertex in 4-$blk(G)$ corresponds to a 4-block leaf, but the reverse is not necessarily true, since we do not represent some special 4-block leaves and all degree-3 vertices that are centers of wheels in 4-$blk(G)$. A special 4-block leaf $\{v\}$, where v is a vertex, is represented by an R-vertex in 4-$blk(G)$ if v is not the center of a wheel w and it is in one of separating triplets of w. The degree of a flower F in G is the degree of its corresponding vertex in 4-$blk(G)$. Note also that the degree of a wheel W in

G is equal to the number of components in 4-$blk(G)$ by removing its corresponding W-vertex w and all F-vertices that are adjacent to w. A wheel W in G is a *star wheel* if $d(W)$ equals the number of leaves in 4-$blk(G)$ and every special 4-block leaf in W is either adjacent to or equal to the center. A star wheel W with the center c has the property that every 4-block leaf in G (not including $\{c\}$ if it is a 4-block leaf) can be separated from G by a separating triplet containing the center c. If G contains a star wheel W, then W is the only wheel in G. Note also that the degree of a wheel is less than or equal to the degree of its center in G.

K-connectivity Augmentation Number
The *k-connectivity augmentation number* for a graph G is the smallest number of edges that must be added to G in order to k-connect G.

3 A Lower Bound for the Four-Connectivity Augmentation Number

In this section, we first give a simple lower bound for the four-connectivity augmentation number that is similar to the ones for biconnectivity augmentation [3] and triconnectivity augmentation [10]. We show that this above lower bound is not always equal to the four-connectivity augmentation number [15, 17]. We then give a modified lower bound. This new lower bound turns out to be the exact number of edges that we must add to reach four-connectivity (see proofs in Section 4). Finally, we show relations between the two lower bounds.

3.1 A Simple Lower Bound

Given a graph G with vertex-connectivity $k-1$, it is well known that $\max\{\lceil \frac{l_k}{2} \rceil, d-1\}$ is a lower bound for the k-connectivity augmentation number where l_k is the number of k-block leaves in G and d is the maximum degree among all separating $(k-1)$-sets in G [3]. It is also well known that for $k = 2$ and 3, this lower bound equals the k-connectivity augmentation number [3, 10]. For $k = 4$, however, several researchers [15, 17] have observed that this value is not always equal to the four-connectivity augmentation number. Examples are given in Figure 3. Figure 3.(1) is from [15] and Figure 3.(2) is from [17]. Note that if we apply the above lower bound in each of the three graphs in Figure 3, the values we obtain for Figures 3.(1), 3.(2) and 3.(3) are 3, 3 and 2, respectively, while we need one more edge in each graph to four-connect it.

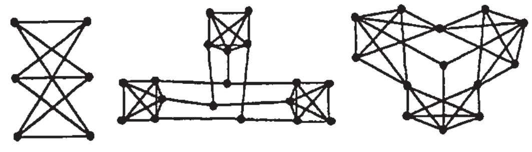

Figure 3: Illustrating three graphs where in each case the value derived by applying a simple lower bound does not equal its four-connectivity augmentation number.

3.2 A Better Lower Bound

Notice that in the previous lower bound, for every separating triplet $\mathcal{S}$ in the triconnected graph $G = \{V, E\}$, we must add at least $d(\mathcal{S}) - 1$ edges between vertices in $V \setminus \mathcal{S}$ to four-connect G, where $d(\mathcal{S})$ is the degree of $\mathcal{S}$ (i.e. the number of connected components in $G - \mathcal{S}$); otherwise, $\mathcal{S}$ remains a separating triplet. Let the set of edges added be $\mathcal{A}_{1,\mathcal{S}}$. We also notice that we must add at least one edge into every 4-block leaf $\mathcal{B}$ to four-connect G; otherwise, $\mathcal{B}$ remains a 4-block leaf. Since it is possible that $\mathcal{S}$ contains some 4-block leaves, we need to know the minimum number of edges needed to eliminate all 4-block leaves inside $\mathcal{S}$. Let the set of edges added be $\mathcal{A}_{2,\mathcal{S}}$. We know that $\mathcal{A}_{1,\mathcal{S}} \cap \mathcal{A}_{2,\mathcal{S}} = \emptyset$. The previous lower bound gives a bound on the cardinality of $\mathcal{A}_{1,\mathcal{S}}$, but not that of $\mathcal{A}_{2,\mathcal{S}}$. In the following paragraph, we define a quantity to measure the cardinality of $\mathcal{A}_{2,\mathcal{S}}$.

Let $Q_{\mathcal{S}}$ be the set of special 4-block leaves that are in the separating triplet $\mathcal{S}$ of a triconnected graph G. Two 4-block leaves $\mathcal{B}_1$ and $\mathcal{B}_2$ are *adjacent* if there is an edge in G between every demanding vertex in $\mathcal{B}_1$ and every demanding vertex in $\mathcal{B}_2$. We create an *augmenting graph for* $\mathcal{S}$, $\mathcal{G}(\mathcal{S})$, as follows. For each special 4-block leaf in $Q_{\mathcal{S}}$, we create a vertex in $\mathcal{G}(\mathcal{S})$. There is an edge between two vertices v_1 and v_2 in $\mathcal{G}(\mathcal{S})$ if their corresponding 4-blocks are adjacent. Let $\overline{\mathcal{G}(\mathcal{S})}$ be the complement graph of $\mathcal{G}(\mathcal{S})$. The seven types of augmenting graphs and their complement graphs are illustrated in Figure 4.

Definition 1 *The* **augmenting number** $a(\mathcal{S})$ *for a separating triplet* $\mathcal{S}$ *in a triconnected graph is the number of edges in a maximum matching* $\mathcal{M}$ *of* $\overline{\mathcal{G}(\mathcal{S})}$ *plus the number of vertices that have no edges in* $\mathcal{M}$ *incident on them.*

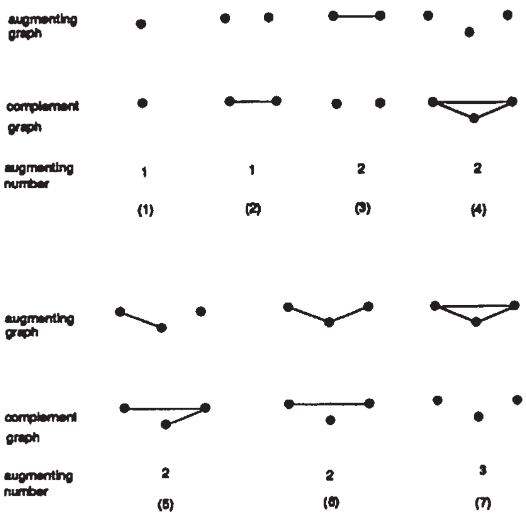

Figure 4: Illustrating the seven types of augmenting graphs, their complement graphs and augmenting numbers that one can get for a separating triplet in a triconnected graph.

The augmenting numbers for the seven types of augmenting graphs are shown in Figure 4. Note that in a triconnected graph, each special 4-block leaf must receive at least one new incoming edge in order to four-connect the input graph. The augmenting number $a(S)$ is exactly the minimum number of edges needed in the separating triplet S in order to four-connect the input graph. The augmenting number of a separating set that does not contain any special 4-block leaf is 0. Note also that we can define the *augmenting number* $a(C)$ for a set C that consists of the center of a wheel using a similar approach. Note that $a(C) \leq 1$.

We need the following definition.

Definition 2 *Let G be a triconnected graph with l 4-block leaves. The* **leaf constraint** *of G, $lc(G)$, is $\lceil \frac{l}{2} \rceil$. The* **degree constraint** *of a separating triplet S in G, $dc(S)$, is $d(S) - 1 + a(S)$, where $d(S)$ is the degree of S and $a(S)$ is the augmenting number of S. The degree constraint of G, $dc(G)$, is the maximum degree constraint among all separating triplets in G. The* **wheel constraint** *of a star wheel W with center c in G, $wc(W)$, is $\lceil \frac{d(W)}{2} \rceil + a(\{c\})$, where $d(W)$ is the degree of W and $a(\{c\})$ is the augmenting number of $\{c\}$. The wheel constraint of G, $wc(G)$, is 0 if there is no star wheel in G; otherwise it is the wheel constraint of the star wheel in G.*

We now give a better lower bound on the 4-connectivity augmentation number for a triconnected graph.

Lemma 1 *We need at least* $\max\{lc(G), dc(G), wc(G)\}$ *edges to four-connect a triconnected graph G.*

<u>**Proof:**</u> Let $\mathcal{A}$ be a set of edges such that $G' = G \cup \mathcal{A}$ is four-connected. For each 4-block leaf B in G, we need one new incoming edge to a vertex in B; otherwise B is still a 4-block leaf in G'. This gives the first component of the lower bound.

For each separating triplet S in G, $G - S$ contains $d(S)$ connected components. We need to add at least $d(S) - 1$ edges between vertices in $G - S$, otherwise S is still a separating triplet in G'. In addition to that, we need to add at least $a(S)$ edges such that at least one of the two end points of each new edge is in S; otherwise S contains a special 4-block leaf. This gives the second term of the lower bound.

Given the star wheel W with the center c, 4-$blk(G)$ contains exactly $d(W)$ degree-1 R-vertices. Thus we need to add at least $\lceil \frac{d(W)}{2} \rceil$ edges between vertices in $G - \{c\}$; otherwise, G' contains some 4-block leaves. In addition to that, we need to add $a(\{c\})$ non-self-loop edges such that at least one of the two end points of each new edge is in $\{c\}$; otherwise $\{c\}$ is still a special 4-block leaf. This gives the third term of the lower bound. □

3.3 A Comparison of the Two Lower Bounds

We first observe the following relation between the wheel constraint and the leaf constraint. Note that if there exists a star wheel W with degree $d(W)$, there are exactly $d(W)$ 4-block leaves in G if the center is not degree-3. If the center of the star wheel is degree-3, then there are exactly $d(W) + 1$ 4-block leaves in G. Thus the wheel constraint is greater than the leaf constraint if and only if the star wheel has a degree-3 center. We know that the degree of any wheel is less than or equal to the degree of its center. Thus the value of the above lower bound equals 3.

We state the following claims for the relations between the degree constraint of a separating triplet and the leaf constraint.

Claim 2 *Let S be a separating triplet with degree $d(S)$ and h special 4-block leaves. Then there are at least $h + d(S)$ 4-block leaves in G.* □

Claim 3 *Let $\{a_1, a_2, a_3\}$ be a separating triplet in a triconnected graph G. Then a_i, $1 \leq i \leq 3$, is incident on a vertex in every connected component in $G - \{a_1, a_2, a_3\}$.* □

Corollary 1 *The degree of a separating triplet S is no more than the largest degree among all vertices in S.* □

From Corollary 1, we know that it is not possible that a triconnected graph has type (6) or type (7) of the augmenting graphs as shown in Figure 4, since the degree of their underling separating triplet is 1. We also know that the degree of a separating triplet with a special 4-block leaf is at most 3 and at least 2. Thus $dc(S)$ is greater than $d(S) - 1$ if $dc(S)$ equals either 3 or 4. Thus we have the following lemma.

Lemma 2 *Let $low_1(G)$ be the lower bound given in Section 3.1 for a triconnected graph G and let $low_2(G)$ be the lower bound given in Lemma 1 in Section 3.2. (i) $low_1(G) = low_2(G)$ if $low_2(G) \notin \{3,4\}$. (ii) $low_2(G) - low_1(G) \in \{0,1\}$.* □

Thus the simple lower bound extended from biconnectivity and triconnectivity is in fact a good approximation for the four-connectivity augmentation number.

4 Finding a Smallest Four-Connectivity Augmentation for a Triconnected Graph

We first explore properties of the 4-block tree that we will use in this section to develop an algorithm for finding a smallest 4-connectivity augmentation. Then we describe our algorithm. Graphs discussed in this section are triconnected unless specified otherwise.

4.1 Properties of the Four-Block Tree

Massive Vertex, Critical Vertex and Balanced Graph

A separating triplet S in a graph G is *massive* if $dc(S) > lc(G)$. A separating triplet S in a graph G is *critical* if $dc(S) = lc(G)$. A graph G is *balanced* if there is no massive separating triplet in G. If G is balanced, then its $4\text{-}blk(G)$ is also *balanced.* The following lemma and corollary state the number of massive and critical vertices in $4\text{-}blk(G)$.

Lemma 3 *Let S_1, S_2 and S_3 be any three separating triplets in G such that there is no special 4-block in $S_i \cap S_j$, $1 \leq i < j \leq 3$. $\sum_{i=1}^{3} dc(S_i) \leq l+1$, where l is the number of 4-block leaves in G.*

Proof: G is triconnected. We can modify $4\text{-}blk(G)$ in the following way such that the number of leaves in the resulting tree equals l and the degree of an F-node f equals its degree constraint plus 1 if f corresponds to S_i, $1 \leq i \leq 3$. For each W-vertex w with a degree-3 center c, we create an R-vertex r_c for c, an F-vertex f_c for the three vertices that are adjacent to c in G. We add edges (w,f_c) and (f_c,r_c). Thus r_c is a leaf. For each F-vertex whose corresponding separating triplet S contains h special 4-block leaves, we attach $a(S)$ subtrees with a total number of h leaves with the constraint that any special 4-block that is in more than one separating triplet will be added only once (to the F-node corresponding to S_i, $1 \leq i \leq 3$, if possible). From Figure 4 we know that the number of special 4-block leaves in any separating triplet is greater than or equal to its augmenting number. Thus the above addition of subtrees can be done. Let $4\text{-}blk(G)'$ be the resulting graph. Thus the number of leaves in $4\text{-}blk(G)'$ is l. Let f be an F-node in $4\text{-}blk(G)'$ whose corresponding separating triplet is S. We know that the degree of f equals $dc(S)+1$ if $S \in \{S_i \mid 1 \leq i \leq 3\}$. It is easy to verify that the sum of degrees of any three internal vertices in a tree is less than or equal to 4 plus the number of leaves in a tree. □

Corollary 2 *Let G be a graph with more than two non-special 4-block leaves. (i) There is at most one massive F-vertex in $4\text{-}blk(G)$. (ii) If there is a massive F-vertex, there is no critical F-vertex. (iii) There are at most two critical F-vertices in $4\text{-}blk(G)$.* □

Updating the Four-Block Tree

Let v_i be a demanding vertex or a vertex in a special 4-block leaf, $i \in \{1,2\}$. Let B_i be the 4-block leaf that contains v_i, $i \in \{1,2\}$. Let b_i, $i \in \{1,2\}$, be the vertex in $4\text{-}blk(G)$ such that if v_i is a demanding vertex, then b_i is an R-vertex whose corresponding 4-block contains v_i; if v_i is in a special 4-block leaf in a flower, then b_i is the F-vertex whose corresponding separating triplet contains v_i; if v_i is the center of a wheel w, b_i is the F-vertex that is closet to $b_{(i \bmod 2)+1}$ and is adjacent to w. The vertex b_i is the *implied vertex* for B_i, $i \in \{1,2\}$. The *implied path P between B_1 and B_2* is the path in $4\text{-}blk(G)$ between b_1 and b_2. Given $4\text{-}blk(G)$ and an edge (v_1,v_2) not in G, we can obtain $4\text{-}blk(G \cup \{(v_1,v_2)\})$ by performing local updating operations on P. For details, see [18].

In summary, all 4-blocks corresponding to R-vertices in P are collapsed into a single 4-block. Edges in P are deleted. F-vertices in P are connected to the new R-vertex created. We *crack* wheels in a way that is similar to the cracking of a polygon for updating 3-block graphs (see [2, 10] for details). We say that P is *non-adjacent* on a wheel W, if the cracking of W creates two new wheels. Note that it is possible that a separating triplet S in the original graph is no longer a separating triplet in the resulting graph by adding an edge. Thus some special leaves in the original graph are no longer special, in which case they must be added to 4-$blk(G)$.

Reducing the Degree Constraint of a Separating Triplet

We know that the degree constraint of a separating triplet can be reduced by at most 1 by adding a new edge. From results in [18], we know that we can reduce the degree constraint of a separating triplet S by adding an edge between two non-special 4-block leaves B_1 and B_2 such that the path in 4-$blk(G)$ between the two vertices corresponding to B_1 and B_2 passes through the vertex corresponding to S. We also notice the following corollary from the definitions of 4-$blk(G)$ and the degree constraint.

Corollary 3 *Let S be a separating triplet that contains a special 4-block leaf. (i) We can reduce $dc(S)$ by 1 by adding an edge between two special 4-block leaves B_1 and B_2 in S such that B_1 and B_2 are not adjacent. (ii) If we add an edge between a special 4-block leaf in S and a 4-block leaf B not in S, the degree constraint of every separating triplet corresponding to an internal vertex in the path of 4-$blk(G)$ between vertices corresponding to S and B is reduced by 1.* □

Reducing the Number of Four-Block Leaves

We now consider the conditions under which the adding of an edge reduces the leaf constraint $lc(G)$ by 1. Let *real degree* of an F-node in 4-$blk(G)$ be 1 plus the degree constraint of its corresponding separating triplet. The real degree of a W-node with a degree-3 center in G is 1 plus its degree in 4-$blk(G)$. The real degree of any other node is equal to its degree in 4-$blk(G)$.

Definition 3 (The Leaf-Connecting Condition) *Let B_1 and B_2 be two non-adjacent 4-block leaves in G. Let P be the implied path between B_1 and B_2 in 4-$blk(G)$. Two 4-block leaves B_1 and B_2 satisfy the* **leaf-connecting condition** *if at least one of the following conditions is true. (i) There are at least two vertices of real degree at least 3 in P. (ii) There is at least one R-vertex of degree at least 4 in P. (iii) The path P is non-adjacent on a W-vertex in P. (iv) There is an internal vertex of real degree at least 3 in P and at least one of the 4-block leaves in $\{B_1, B_2\}$ is special. (v) B_1 and B_2 are both special and they do not share the same set of neighbors.*

Lemma 4 *Let B_1 and B_2 be two 4-block leaves in G that satisfy the leaf-connecting condition. We can find vertices v_i in B_i, $i \in \{1,2\}$, such that $lc(G \cup \{(v_1, v_2)\}) = lc(G) - 1$, if $lc(G) \geq 2$.* □

4.2 The Algorithm

We now describe an algorithm for finding a smallest augmentation to four-connect a triconnected graph. Let $\delta = dc(G) - lc(G)$. The algorithm first adds 2δ edges to the graph such that the resulting graph is balanced and the lower bound is reduced by 2δ. If $lc(G) \neq 2$ or $wc(G) \neq 3$, there is no star wheel with a degree-3 center. We add an edge such that the degree constraint $dc(G)$ is reduced by 1 and the number of 4-block leaves is reduced by 2. Since there is no star wheel with a degree-3 center, $wc(G)$ is also reduced by 1 if $wc(G) = lc(G)$. The resulting graph stays balanced each time we add an edge and the lower bound given in Lemma 1 is reduced by 1. If $lc(G) = 2$ and $wc(G) = 3$, then there exists a star wheel with a degree-3 center. We reduce $wc(G)$ by 1 by adding an edge between the degree-3 center and a demanding vertex of a 4-block leaf. Since $lc(G) = 2$ and $wc(G) = 3$, $dc(G)$ is at most 2. Thus the lower bound can be reduced by 1 by adding an edge. We keep adding an edge at a time such that the lower bound given in Lemma 1 is reduced by 1. Thus we can find a smallest augmentation to four-connect a triconnected graph. We now describe our algorithm.

The Input Graph is not Balanced

We use an approach that is similar to the one used in biconnectivity and triconnectivity augmentations to balance the input graph [10, 11, 26]. Given a tree T and a vertex v in T, a *v-chain* [26] is a component in $T - \{v\}$ without any vertex of degree more than 2. The leaf of T in each v-chain is a *v-chain leaf* [26]. Let $\delta = dc(G) - lc(G)$ for a unbalanced graph G and let 4-$blk(G)'$ be the modified 4-block tree given in the proof of Lemma 3. Let f be a massive F-vertex. We can show that either there are at least $2\delta + 2$ f-chains in 4-$blk(G)'$ (i.e. f is the only massive F-vertex) or we can eliminate all massive F-vertices by adding an edge. Let λ_i be a demanding vertex in the ith f-chain leaf. We add the set of edges $\{(\lambda_i, \lambda_{i+1}) \mid 1 \leq i \leq 2\delta\}$. It is also easy to show that the lower bound given in Lemma 1 is reduced by 2δ and the graph is balanced.

The Input Graph is Balanced

We first describe the algorithm. Then we give its proof of correctness. In the description, we need the following definition. Let B be a 4-block leaf whose implied vertex in 4-$blk(G)$ is b and let B' be a 4-block leaf whose implied vertex in 4-$blk(G)$ is b'. B' is a *nearest* 4-block leaf of B if there is no other 4-block leaf whose implied vertex has a distance to b that is shorter than the distance between b and b'.

{$*$ G is triconnected with ≥ 5 vertices; the algorithm finds a smallest four-connectivity augmentation. $*$}
graph function aug3to4(**graph** G);
{$*$ The algorithmic notation used is from Tarjan [29]. $*$}
$T := 4\text{-}blk(G)$; root T at an arbitrary vertex;
let $\tilde{l}$ be the number of degree-1 R-vertices in T;
do $\exists$ a 4-block leaf in $G \rightarrow$
if $\exists$ a degree-3 center $c \rightarrow$
1. **if** $lc(G) = 2$ and $wc(G) = 3 \rightarrow$
{$*$ Vertex c is the center of the star wheel w. $*$}
$u_1 :=$ the 4-block leaf $\{c\}$;
let u_2 be a a non-special 4-block leaf
| $\exists$ another degree-3 center c' non-adjacent to $c \rightarrow$
let u_2 be the 4-block leaf $\{c'\}$
| $\exists$ a special 4-block leaf b non-adjacent to $u_1 \rightarrow$
let $u_2 := b$
| $\nexists$ (degree-3 center or special 4-block leaf) non-adjacent to $u_1 \rightarrow$
let u_2 be a a 4-block leaf such that $\exists$ an internal vertex with real degree ≥ 3 in their implies path
fi
| $lc(G) \neq 2$ or $wc(G) \neq 3 \rightarrow$
if $\tilde{l} > 2$ **and** $\exists$ 2 critical F-vertices f_1 and $f_2 \rightarrow$
2. find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through f_1 and f_2
| $l > 2$ **and** $\exists$ only one critical F-vertex $f_1 \rightarrow$
if $\exists$ two non-adjacent special 4-block leaves in the separating triplet S_1 corresponding to $f_1 \rightarrow$
3. let u_1 and u_2 be two non-adjacent 4-block leaves in S_1
| $\nexists$ two non-adjacent special 4-block leaves in the separating triplet S_1 corresponding to $f_1 \rightarrow$
4. let v be a vertex with the largest real degree among all vertices in T besides f_1;
if real degree of v in $T \geq 3 \rightarrow$
find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through f_1 and v
fi
{$*$ The case when the degree of v in $T < 3$ will be handled in step 8. $*$}
fi
| $\exists$ two vertices v_1 and v_2 with real degree $\geq 3 \rightarrow$
5. find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through v_1 and v_2
| $\exists$ an R-vertex v of degree $\geq 4 \rightarrow$
6. find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through v
| $\exists$ a W-vertex v of degree $\geq 4 \rightarrow$
7. let u_1 and u_2 be two non-special 4-block leaves such that the implied path between them is non-adjacent on v
| $\exists$ only one vertex v in T with real degree $\geq 3 \rightarrow$
{$*$ T is a star with the center v. $*$}
8. find a nearest vertex w of v that contains a 4-block leaf v_1;
let w' be a nearest vertex of w containing a 4-block leaf non-adjacent to v_1;
find two 4-block leaves u_1 and u_2 whose implied path passes through w, w' and v
{$*$ The above step can always be done, since T is a star. $*$}
{$*$ Note that T is path for all the cases below. $*$}
| $\exists$ two non-adjacent special 4-block leaves in one separating triplet $S \rightarrow$
9. let u_1 and u_2 be two non-adjacent special 4-block leaves in S
| $\exists$ a special 4-block leaf $u_1 \rightarrow$
10. find a nearest non-adjacent 4-block leaf u_2
| $\tilde{l} = 2 \rightarrow$
let u_1 and u_2 be the two 4-block leaves corresponding to the two degree-1 R-vertices in T
fi
fi;
let y_i, $i \in \{1, 2\}$, be a demanding vertex in u_i such that (y_1, y_2) is not an edge in the current G;
$G := G \cup \{(y_1, y_2)\}$;
update T, $\tilde{l}$, $lc(G)$, $wc(G)$ and $dc(G)$
od;
return G
end aug3to4;

Before we show the correctness of algorithm aug3to4, we need the following claim and corollaries.

Claim 4 *[26] If $4\text{-}blk(G)$ contains two critical vertices f_1 and f_2, then every leaf is either in an f_1-chain or in an f_2-chain and the degree of any other vertex in $4\text{-}blk(G)$ is at most 2.* □

Corollary 4 *If $4\text{-}blk(G)$ contains two critical vertices f_1 and f_2 and the corresponding separating triplet S_i, $i \in \{1,2\}$, of f_i contains a special 4-block leaf, then its augmenting number equals the number of special 4-block leaves in it.* □

Corollary 5 *Let f_1 and f_2 be two critical F-vertices in $4\text{-}blk(G)$. If the number of degree-1 R-vertices in $4\text{-}blk(G) > 2$ and the corresponding separating triplet of f_i, $i \in \{1,2\}$, contains a 4-block leaf B_i, we can add an edge between a vertex in B_1 and a vertex in B_2 to reduce the lower bound given in Lemma 1 by 1.* □

Theorem 1 *Algorithm aug3to4 adds the smallest number of edges to four-connect a triconnected graph.* □

We now describe an efficient way of implementing algorithm aug3to4. The 4-block tree can be computed in $O(n\alpha(m,n)+m)$ time for a graph with n vertices and m edges [18]. We know that the leaf constraint, the degree constraint of any separating triplet and the wheel constraint of any wheel in G can only be decreased by adding an edge. We also know that $lc(G)$, the sum of degree constraints of all separating triplets and the sum of wheel constraints of all wheels are all $O(n)$. Thus we can use the technique in [26] to maintain the current leaf constraint, the degree constraint for any separating triplet and the wheel constraint for any wheel in $O(n)$ time for the entire execution of the algorithm. We also visit each vertex and each edge in the 4-block tree a constant number of times before deciding to collapse them. There are $O(n)$ 4-block leaves and $O(n)$ vertices and edges in $4\text{-}blk(G)$. In each vertex, we need to use a set-union-find algorithm to maintain the identities of vertices after collapsing. Hence the overall time for updating the 4-block tree is $O(n\alpha(n,n))$. We have the following claim.

Claim 5 *Algorithm aug3to4 can be implemented in $O(n\alpha(m,n)+m)$ time where n and m are the number of vertices and edges in the input graph, respectively and $\alpha(m,n)$ is the inverse Ackermann's function.* □

5 Conclusion

We have given a sequential algorithm for finding a smallest set of edges whose addition four-connects a triconnected graph. The algorithm runs in $O(n\alpha(m,n)+m)$ time using $O(n+m)$ space. The following approach was used in developing our algorithm. We first gave a 4-block tree data structure for a triconnected graph that is similar to the one given in [18]. We then described a lower bound on the smallest number of edges that must be added based on the 4-block tree of the input graph. We further showed that it is possible to decrease this lower bound by 1 by adding an appropriate edge.

The lower bound that we gave here is different from the ones that we have for biconnecting a connected graph [3] and for triconnecting a biconnected graph [10]. We also showed relations between these two lower bounds. This new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier for the number of edges needed to k-connect a $(k-1)$-connected graph. It is likely that techniques presented in this paper may be used in finding the k-connectivity augmentation number of a $(k-1)$-connected graph, for an arbitrary k.

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