A distributed restoration algorithm for multiple-link and node failures of transport networks

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Abstract:
Fast restoration of broadband optical fiber networks from multiple-link and node failu as well as single-link failures, is addressed. A **distributed restoration algorithm** b on message flooding is described. The algorithm is an extension of a previously prop algorithm for single-link failure. It restores the network from multiple-link and node failures, using multidestination flooding and path route monitoring. Computer simula of the algorithm verified that it can find alternate paths within 0.5 s, whenever the message processing delay at a node is 5 ms

Index Terms:
broadband networks optical links broadband optical fiber networks **distributed restoration algorithm** message flooding message processing delay multidestination flooding multiple-li failures node failures path route monitoring single-link failures transport networks

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Performance Analysis of Network Connective Probability of Multihop Network under Correlated Breakage

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Abstract—One of important properties of multihop network is the network connective probability which evaluate the connectivity of the network. The network connective probability is defined as the probability that when some nodes are broken, rest nodes connect each other. Multihop networks are classified to the regular network whose link assignment is regular and the random network whose link assignment is random. It has been shown that the network connective probability of regular network is larger than that of random network. However, all of these results is shown under independent node breakage. In this paper, we analyze the network connective probability of multihop networks under the correlated node breakage. It is shown that regular network has better performance of the network connective probability than random network under the independent breakage, on the other hand, random network has better performance than regular network under the correlated breakage.

1 Introduction

In recent years, multi-hop networks have been widely studied [1]-[8]. These networks must pass messages between source and destination nodes via intermediate links and nodes. Examples of them include ring, shuffle network (SN) [1],[2] and chordal network (CN)[3]. One of the very important performance measure of multi-hop network is the connectivity of the network. If some nodes are broken, it is needed for a network to guarantee the connection among non-broken nodes. Thus, the network connective probability defined as the probability that when some nodes are broken, rest links and nodes construct the connective network, should be a very important property to evaluate the connectivity of the network.

Multi-hop networks are classified to regular network and random network according to the way of link assignment. In the regular network, links are assigned regularly and examples of them include shufflenet and manhattan street network. On the other hand, in random network, link assignment is not regular but somewhat random and examples of them include connective semi-random network (CSRN) [6]. The network connective probabilities of some multi-hop networks have been analyzed and it has been shown that the network connective probability of regular network is larger than that of random network. However, all of them is analyzed under the condition that locations of broken nodes are independent each other. In the real network, there are some case that the locations of broken nodes have correlation, for example, links and nodes are broken in the same area under the case of disaster. Thus, it is significant and great of interest to analyze the network connective probability under the condition when the locations of broken nodes have correlations each other.

In this paper, we analyze the network connective probability of multi-hop network under the condition that locations of broken nodes have correlations each other, where we treat SN, CN and CSRN as the model for analysis. We realize the correlation as follows. At first, we note one node and break it and call this node the center broken node. And next, we note nodes whose links connect to the center broken nodes and break them at some probability. We define this probability as the correlated broken probability. Very interesting result is shown that under independent breakage of node, regular network has better performance of the network connective probability than random network, on the other hand, under the correlated breakage of node, random network has better performance than regular network.

In the section 2, we explain network model of SN, CN and CSRN which we analyze in the section 3. In the section 3, we analyze the network connective probability under the condition when the location of broken nodes have correlation each other. And we compare each of network connective probability in the section 4. In the last, we conclude our study.

2 Multihop network model

In this section, we explain the multihop network models used for analysis of the network connective probability. We treat three networks such as SN, CN and CSRN which consists of N nodes and p unidirected outgoing links per node.

Fig. 1 shows SN with 18 nodes and 2 outgoing links per node. To construct the SN, we arrange $N = kp^k$ ($k = 1, 2, \cdots$; $p = 1, 2, \cdots$) nodes in k columns of p^k nodes each. Moving from left to right, successive columns are connected by p^{k+1} outgoing links, arranged in a fixed shuffle pattern, with the last column connected to the first as if the entire graph were wrapped around a cylinder. Each of the p^k nodes in a column has p outgoing links directed to p different nodes in the next column. Numbering the nodes in a column from 0 to $p^k - 1$, nodes i has outgoing links directed to nodes $j, j+1, \cdots$, and $j + p - 1$ in the next column, where $j = (i \ mod \ p^{k-1})p$. In Fig. 1, p is equal to 2 and k is equal to 2. Since the link assignmentof SN is regular, SN is regular network.

Fig. 2 shows CN with 16 nodes and 2 outgoing links per node. To construct CN, at first, we construct unidirected ring network with N nodes and N unidirected links. And $p-1$ unidirected links are added from each node. Numbering nodes along ring network from 0 to $N - 1$, node i has outgoing links directed to nodes $(i + 1) \ mod \ N, (i + r_1) \ mod \ N, \cdots$, and $(i + r_{p-1}) \ mod \ N$, where r_j ($j = 1, 2, \cdots, p - 1$) is defined as the chordal length. In Fig. 2, r_1 is equal to 3. Since r_i for every i are independent each other, CN is not regular network. However, CN has much regular elements such a symmetrical pattern of network.

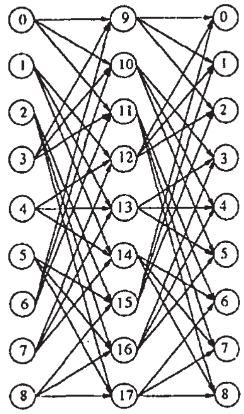


Figure 1. Shuffle network with N = 18 and p = 2.

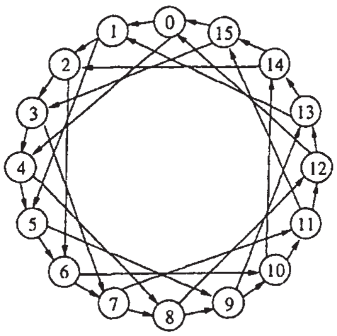


Figure 2. Chordal network with N = 16, p = 2 and r_1 = 3.

Fig. 3 shows CSRN with 16 nodes and 2 outgoing links from a node. Similarly with CN, CSRN includes unidirected ring network with N nodes and N unidirected links. And we add $p - 1$ links from each node whose directed nodes are randomly selected. In CSRN, the number of incoming links per node is not constant, for example, in Fig. 3, the number of incoming links into node 1 is 1 and the one into node 3 is 3. The link assignment of CSRN is random except for the part of ring network, thus CSRN is random network. It has been shown that since the number of incoming links per node is not constant, the network connective probability of CSRN is smaller than those of SN and CN when locations of broken nodes are independent each other. And that of SN is the same as that of CN, because the network connective probability depends on the number of incoming links come into every nodes.

3 Performance Analysis

Here, we analyze the network connective probability of SN, CN and CSRN under the condition that locations of broken nodes have correlation each other. Now, we explain the network connective probability in detail using Fig. 3. This figure shows the connective network which is defined as the network in which all nodes connect to every other nodes directly or indirectly. At first, we consider the case that the node 1 is broken. The node 1 has two outgoing links directed to nodes 2 and 3, and if the node 1 is broken, we can not use them. However, node 2 has two incoming links from nodes 1 and 14, and node 3 has three incoming links from nodes 1, 2 and 11. Therefore, even if node 1 is broken, rest nodes can construct

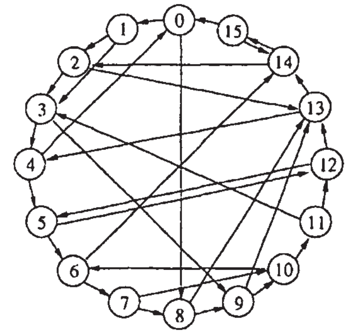


Figure 3. Connective semi-random network with N = 16 and p = 2.

the connective network. Next, we consider the case that node 0 is broken. The node 0 has two outgoing links directed to nodes 1 and 8, and if the node 0 is broken, we can not use them. Since node 1 has only one incoming link from node 0, even if only node 0 is broken, rest nodes can not connect to node 1, that is, they can not construct the connective network. Here, we define the network connective probability as the probability that when some nodes and links are broken, the rest nodes and links can construct the connective network.

Now, we explain the correlated node breakage using Fig. 3. At first, we note one node and break it, where this node is called as the center broken node. And then, we note nodes whose outgoing links come into the center broken node or whose incoming links go out of the center broken node, and break them at a probability defined as the correlated broken probability. In Fig 3, when we assume that the center broken node is the node 3, there are five nodes 1, 2, 4, 9 and 11 which have possibility to become correlated broken node. And they become the broken nodes at the correlated broken probability. It is obvious that none of them is broken when the correlated broken probability is 0 and all of them is broken when the correlated broken probability is 1.

In our study, we analyze the network connective probability that only nodes are broken. And we assume that the number of center broken node is one in the analysis. We denote the correlated broken probability by a and the network connective probability of SN, CN and CSRN by P_{SN}, P_{CN} and P_{CSRN}, respectively.

3.1 Shuffle Network

Because the number of incoming links per node in SN is the constant p, when broken node is only center broken node, the rest nodes can construct the connective network. There are $2p$ nodes have the possibility to become the correlated broken node. All of p nodes which have outgoing link come into the center broken node have the outgoing links directed to the same nodes. For example, in Fig. 1, if we assume that the node 9 is the center broken node, the nodes 0, 3 and 6 has outgoing links to node 9. And each of three nodes have two outgoing links directed to nodes 10 and 11. Therefore, only when all of them are broken, the rest nodes can not construct the connective network. On the other hand, all of outgoing links go out from p nodes which have incoming link from center broken node direct to different nodes. In Fig. 1, nodes 0, 1 and 2 have the incoming link from center broken node 9. And all of the outgoing links from their nodes direct to different nodes, thus even if all of them are broken, the rest nodes can construct the connective network. Thus, the network connective probability of SN is the probability that all of nodes whose outgoing links come into the center broken node are broken, and it is derived as

$$P_{SN} = 1 - a^p . \tag{1}$$

3.2 Chordal Network

The network connective probability of CN with $p = 2$ is different from that with $p \geq 3$. At first, we consider the case with $p = 2$. When p is equal to 2, all of the outgoing links, from the nodes whose incoming links go out from the center broken node, direct to the same node. For example, in Fig. 2, when we assume that the center broken node is node 0, the outgoing links from it direct to nodes 1 and 4. And each of outgoing links from them directs to node 5. Therefore, only when all nodes whose incoming links go out from the center broken node are broken, the rest nodes can not construct the connective network. And we can obtain the network connective probability as

$$P_{CN} = 1 - a^2 \qquad \text{for } p = 2 . \tag{2}$$

And next, we consider the case that $p \geq 3$. In CN, when p is equal to or larger than three and each chordal length is selected properly, all of outgoing links from the nodes whose incoming links go out from the center broken node do not direct to the same nodes. And therefore, even if all of nodes which connect to the center broken nodes with incoming or outgoing links is broken, the rest nodes can construct the connective network, that is,

$$P_{CN} = 1 \qquad \text{for } p \geq 3 . \tag{3}$$

3.3 Connective Semi-Random Network

In CSRN, the number of the incoming links per node is not constant. Since the maximum number of incoming links is $N - 1$ and one link come into a node at least, the probability that the number of the incoming links come into a node is i, denoted as A_i, is

$$A_i = \begin{cases} 0, & \text{for } i = 0 \\ \binom{N-2}{i-1}(\frac{p}{N-2})^{i-1}(1 - \frac{p}{N-2})^{N-1-i} & \text{for } i \geq 1. \end{cases} \tag{4}$$

The nodes which have possibility to become the correlated broken nodes are those which connect to the center broken node by outgoing link or incoming link. When the number of the incoming link come into the center broken node is i, the sum of outgoing links and incoming links it have is $p + i$. However, the number of the nodes which have possibility to become the correlated broken nodes is not always $p + i$, because the p outgoing links have the possibility to overlap with one of i incoming links. For example, in Fig. 3, when the center broken nodes is node 5, the outgoing link to node 12 overlap with the incoming link from node 12. Therefore, in spite of the node 5 has four outgoing and incoming links, the number of the nodes which have possibility to become the correlated broken nodes when the node 5 is the center broken node is three.

And now, we derive the probability that the number of nodes which have possibility to become the correlated broken nodes is j, denoted as B_j. Before derive B_j, we derive the probability that q of p outgoing links which go out of a node overlap with r incoming links come into it, denoted as $C_{p,q,r}$. Here, we define regular link as the link which construct the ring network and random link as other link. We consider the two case. The one is the case that one of the incoming links overlap with the regular outgoing link, and the other case is that none of incoming links overlap with it. Since the regular incoming link never overlap with the regular outgoing link, the probability to become the first case is $(r - 1)/(N - 2)$ and one to become the second case is $1 - (r - 1)/(N - 2)$. In the first case, $C_{p,q,r}$ is the same as the probability that each of $q - 1$ outgoing links among the $p - 1$ outgoing links except for the regular outgoing link overlap one of $r - 1$ incoming links, denoted as $C'_{p-1,q-1,r-1}$. And in the second case, $C_{p,q,r}$ is the same as the probability that each of q outgoing links among the $p - 1$ outgoing links except for the regular outgoing link overlap one of r incoming links, denoted as $C'_{p-1,q,r}$. Using $C'_{p',q',r'}$ given as follows,

$$C'_{p',q',r'} = \begin{cases} 0, & \text{for } q' < 0, r' \leq 0, q' > p', \\ & (p' + r' > N \text{ and} \\ & q' < p' + r' - N) \\ \frac{\binom{p'}{q'} {}_{r'}P_{q'} \; {}_{N-2-r'}P_{p'-q'}}{{}_{N-2}P_{p'}}, & \text{otherwise,} \end{cases} \tag{5}$$

we can derive $C_{p,q,r}$ as

$$C_{p,q,r} = (\frac{r-1}{N-2})C'_{p-1,q-1,r-1} + (1 - \frac{r-1}{N-2})C'_{p-1,q,r} . \tag{6}$$

B_j can be derived as the sum of the probability that when the number of incoming links is $j - p + q$, q of p outgoing links overlap with one of incoming links. Therefore, we can obtain B_j as

$$B_j = \sum_{q=max(0,p+1-j)}^{p} A_{j-p+q} C_{p,q,j-p+q} . \tag{7}$$

Here, we consider two nodes whose regular links connect to the center broken node. We call them regular node (R-node). And we define non-connective node (NC-node) as the node which have no incoming link. Even if a node has many incoming links, when all of source node of them are broken, it becomes NC-node. However, when the number of incoming link is equal to or greater than 2, the probability that all of source nodes of them are broken is very small compared with that when the number of incoming link is 1. Therefore, we assume the NC-node as the node which have only one incoming link and its source node is broken. That is, when the destination node of regular outgoing link of the broken node has only this regular incoming link and this node is not broken, it becomes the NC-node. Fig. 4 shows the center broken node and R-node. (a) shows the case that none of R-node is broken, (b) shows the case that one of them is broken, and (c) shows the case that both of them are broken. It is found that there is only one node which have possibility to become the NC-node in all case. The probability that this node becomes the NC-node is A_1. When the number of broken nodes is k, we can consider the three case with $k = 1$, $k = 2$ and $k > 2$. In $k = 1$, this node is the center broken node and it certainly becomes the case (a) and never becomes the case (b) and (c). In $k = 2$, the one node is the center broken node and the other is the correlated broken node and it becomes the cases (a) or (b). And the probability to become the case (a) is $2/l$ and to become the case (b) is $1 - 2/l$ where l is the number of the nodes have possibility to become the correlated broken nodes. If $k > 2$, it becomes all the case. The number of broken nodes except for R-node in (a), (b) and (c) is k, $k - 1$ and $k - 2$, respectively. Furthermore, when the number of links connect to the center broken node is l, the probability that the number of correlated broken nodes is k, denoted as $t_{l,k}$ is

$$t_{l,k} = B_l \binom{l}{k} a^k (1 - a)^{l-k} . \tag{8}$$

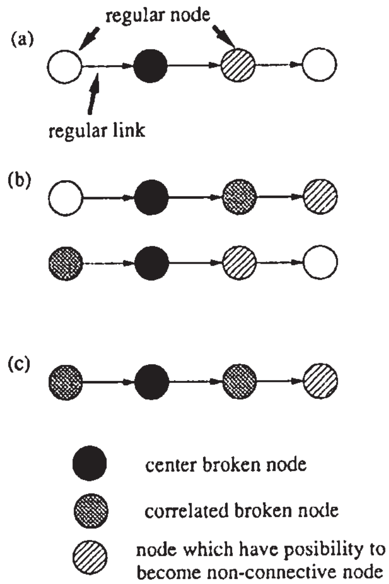


Figure 4. The center broken node and regular nodes.

And in this case, the probability to become the case of (a) is $(\binom{k}{0}\ {}_{l-2}P_k)/{}_lP_k$, to become the case of (b) is $(\binom{k}{1}\ {}_{l-2}P_{k-1})/{}_lP_k$ and to become the case of (c) is $(\binom{k}{2}\ {}_{l-2}P_{k-2})/{}_lP_k$. The network connective probability when the number of broken nodes is l, denoted as E_l, is derived in [8] as follows

$$E_l = \prod_{s=0}^{l-1} \frac{N - NA_1 - s}{N - s} \ . \qquad (9)$$

Therefore, using (8) and (9), we can obtain the network connective probability as

$$\begin{aligned} R_{CSRN} = & \sum_{l=p}^{N-1} t_{l,0}(1 - A_1) \\ & + \sum_{l=p}^{N-1} t_{l,1}\{\frac{2}{l}(1 - A_1) + (1 - \frac{2}{l})(1 - A_1)E_l\} \\ & + \sum_{k=2}^{N-1} \sum_{l=max(p,k)}^{N-1} t_{l,k}\{\frac{\binom{k}{0}\ {}_{l-2}P_k}{{}_lP_k}(1 - A_1)E_k \\ & + \frac{\binom{k}{1}\ {}_{l-2}P_{k-1}}{{}_lP_k}(1 - A_1)E_{k-1} \\ & + \frac{\binom{k}{2}\ {}_{l-2}P_{k-2}}{{}_lP_k}(1 - A_1)E_{k-2}\} \ . \end{aligned} \qquad (10)$$

4 Results

We show computer simulation and theoretical calculation results of the network connective probability under the correlated breakage.

Fig. 5 shows the network connective probability of SN, CN and CSRN with $p = 2$ versus the correlated broken probability. In this figure, the chordal length of CN, r_1 is 50. It is shown that the both the network connective probability of SN and CN is the same in $p = 2$. It is also shown that the network connective probability of CN or SN is larger than that of CSRN in small a, however, in large a, the network connective probability of CN or SN is smaller than that of CSRN.

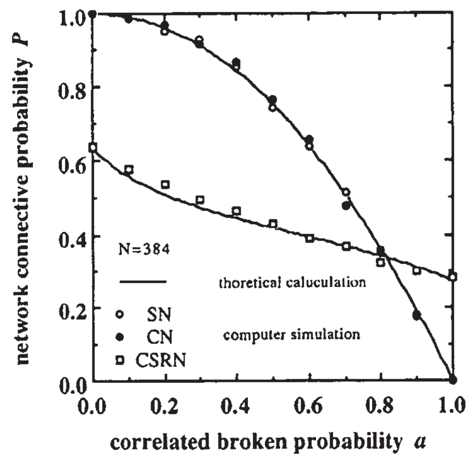


Figure 5. The network connective probability with $p = 2$ versus correlated broken probability.

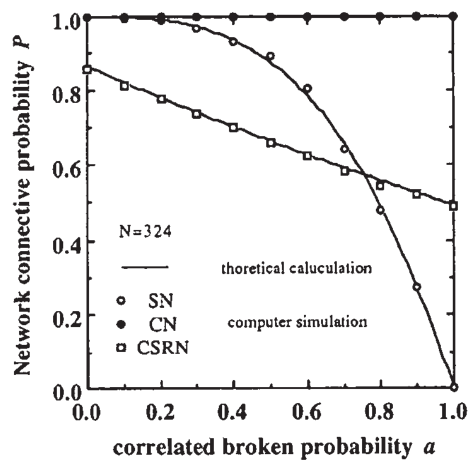
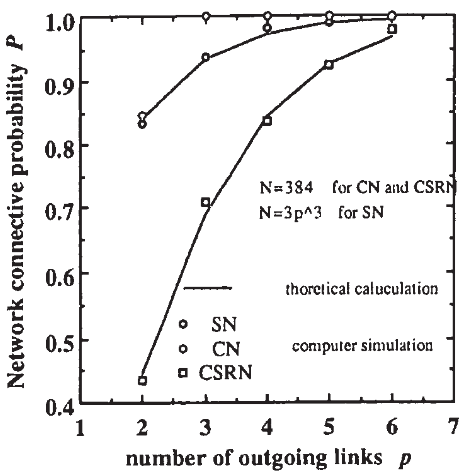


Figure 6. The network connective probability with $p = 3$ versus correlated broken probability.

Fig. 6 shows the network connective probability of SN, CN and CSRN with $p = 3$ versus the correlated broken probability. In this figure, r_1 is 50 and r_2 is 120. The tendency of the network connective probability of SN and CSRN is the same as the case with $p = 2$. However, the tendency of the network connective probability of CN is not different from that with $p = 2$.

In CSRN, because the number of incoming links come into a node is not constant, even if p is large, there are some nodes whose number of incoming links is one. Therefore, the network connective probability itself is small. However, the link assignment of CSRN is random, the condition of correlated breakage is not so different from that of independent breakage. On the other hand, in SN, because the number of incoming links come into a node is constant, the network connective probability under the indepen-



'igure 7. The network connective probability with a = 0.4 versus the number of outgoing links per node.

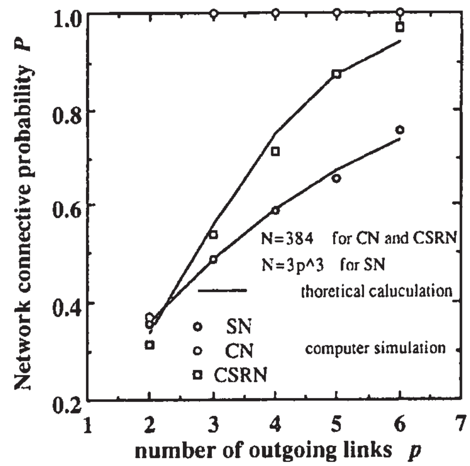


Figure 8. The network connective probability with a = 0.8 versus the number of outgoing links per node.

ent breakage is large. However, because of regularity of the link ssignment, that under the correlated breakage is small. In CN, 'hen p is two, the link assignment is regular, however, when p : larger than two, every chordal length is random and independent each other, and the link assignment is random. Moreover, the umber of incoming links per node of CN is the constant. There-)re, the network connective probability of CN is large under both 1e independent and correlated breakage.

Figs. 7 and 8 show the network connective probability with = 0.4 and 0.8 versus p, respectively. It is shown that the larger is, the smaller difference of network connective probability beveen SN and CSRN is, when a is small. On the other hand, when is large, the larger p is, the larger difference of network conective probability between SN and CSRN is. The reason is as)llows. When a is small, the network connective probability of :SRN is small. However, the larger p is, the smaller the number of odes, whose number of incoming links is 1, is, and the closer to 1 1e network connectivity is. In SN and CN, even if p is small, the etwork connective probability is somewhat large when a is small. Vhen p is large, the network connective probability of CSRN is lmost the same with small p. On the other hand, in SN, the tenency network connectivity versus p is almost the same, however, 1e larger a is, the smaller the value is.

As these results, CN has best performance of network connecvity. However, it has been shown that CN has much poorer per-)rmance of internodal distance than other network. Thus, it is xpecetd for the network to have good performance of both net-'ork connective probability and internodal distance.

; Conclusion

We theoretically analyze the network connective probability f multihop network under the correlated damage of node. We 'eat shuffleNet, chordal network and connective semi-random etwork. It is found that in the independent node breakage, the etwork whose number of incoming links is the constant has good erformance of network connective probability, and found that in 1e correlated node breakage, the network whose link assignment is random has good performance of one.

Acknowledgement

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Abstract:
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Index Terms:
correlation methods network **topology** probability random processes telecommunication network reliability correlated node breakage independent breakage link assignment multih **network** **network connective** probability **node** breakage performance performance analy random network regular network

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On Four-Connecting a Triconnected Graph†
(Extended Abstract)

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Abstract

We consider the problem of finding a smallest set of edges whose addition four-connects a triconnected graph. This is a fundamental graph-theoretic problem that has applications in designing reliable networks.

We present an $O(n\alpha(m,n)+m)$ time sequential algorithm for four-connecting an undirected graph G that is triconnected by adding the smallest number of edges, where n and m are the number of vertices and edges in G, respectively, and $\alpha(m,n)$ is the inverse Ackermann's function.

In deriving our algorithm, we present a new lower bound for the number of edges needed to four-connect a triconnected graph. The form of this lower bound is different from the form of the lower bound known for biconnectivity augmentation and triconnectivity augmentation. Our new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier for the number of edges needed to k-connect a $(k-1)$-connected graph. For $k=4$, we show that this lower bound is tight by giving an efficient algorithm for finding a set of edges with the required size whose addition four-connects a triconnected graph.

1 Introduction

The problem of augmenting a graph to reach a certain connectivity requirement by adding edges has important applications in network reliability [6, 14, 28] and fault-tolerant computing. One version of the augmentation problem is to augment the input graph to reach a given connectivity requirement by adding a smallest set of edges. We refer to this problem as the *smallest augmentation* problem.

†This work was supported in part by NSF Grant CCR-90-23059.

Vertex-Connectivity Augmentations

The following results are known for solving the smallest augmentation problem on an undirected graph to satisfy a vertex-connectivity requirement.

For finding a smallest biconnectivity augmentation, Eswaran & Tarjan [3] gave a lower bound on the smallest number of edges for biconnectivity augmentation and proved that the lower bound can be achieved. Rosenthal & Goldner [26] developed a linear time sequential algorithm for finding a smallest augmentation to biconnect a graph; however, the algorithm in [26] contains an error. Hsu & Ramachandran [11] gave a corrected linear time sequential algorithm. An $O(\log^2 n)$ time parallel algorithm on an EREW PRAM using a linear number of processors for finding a smallest augmentation to biconnect an undirected graph was also given in Hsu & Ramachandran [11], where n is the number of vertices in the input graph. (For more on the PRAM model and PRAM algorithms, see [21].)

For finding a smallest triconnectivity augmentation, Watanabe & Nakamura [33, 35] gave an $O(n(n+m)^2)$ time sequential algorithm for a graph with n vertices and m edges. Hsu & Ramachandran [10, 12] developed a linear time algorithm and an $O(\log^2 n)$ time EREW parallel algorithm using a linear number of processors for this problem. We have been informed that independently, Jordan [15] gave a linear time algorithm for optimally triconnecting a biconnected graph.

For finding a smallest k-connectivity augmentation, for an arbitrary k, there is no polynomial time algorithm known for finding a smallest augmentation to k-connect a graph, for $k>3$. There is also no efficient parallel algorithm known for finding a smallest augmentation to k-connect any nontrivial graph, for $k>3$.

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The above results are for augmenting undirected graphs. For augmenting directed graphs, Masuzawa, Hagihara & Tokura [23] gave an optimal-time sequential algorithm for finding a smallest augmentation to k-connect a rooted directed tree, for an arbitrary k. We are unaware of any results for finding a smallest augmentation to k-connect any nontrivial directed graph other than a rooted directed tree, for $k > 1$.

Other related results on finding smallest vertex-connectivity augmentations are stated in [4, 19].

Edge-Connectivity Augmentations

For the problem of finding a smallest augmentation for a graph to reach a given edge connectivity property, several polynomial time algorithms and efficient parallel algorithms are known. These results can be found in [1, 3, 4, 5, 8, 9, 13, 16, 19, 24, 27, 30, 31, 34, 37].

Augmenting a Weighted Graph

Another version of the problem is to augment a graph, with a weight assigned to each edge, to meet a connectivity requirement using a set of edges with a minimum total cost. Several related problems have been proved to be NP-complete. These results can be found in [3, 5, 7, 20, 22, 32, 33, 36].

Our Result

In this paper, we describe a sequential algorithm for optimally four-connecting a triconnected graph. We first present a lower bound for the number of edges that must be added in order to reach four-connectivity. Note that lower bounds different from the one we give here are known for the number of edges needed to biconnect a connected graph [3] and to triconnect a biconnected graph [10]. It turns out that in both these cases, we can always augment the graph using exactly the number of edges specified in this above lower bound [3, 10]. However, an extension of this type of lower bound for four-connecting a triconnected graph does not always give us the exact number of edges needed [15, 17]. (For details and examples, see Section 3.)

We present a new type of lower bound that equals the exact number of edges needed to four-connect a triconnected graph. By using our new lower bound, we derive an $O(n\alpha(m,n)+m)$ time sequential algorithm for finding a smallest set of edges whose addition four-connects a triconnected graph with n vertices and m edges, where $\alpha(m,n)$ is the inverse Ackermann's function. Our new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier for the number of edges needed to k-connect a $(k-1)$-connected graph. The new lower bound and the algorithm described here may lead to a better understanding of the problem of optimally k-connecting a $(k-1)$-connected graph, for an arbitrary k.

2 Definitions

We give definitions used in this paper.

Vertex-Connectivity

A graph[‡] G with at least $k+1$ vertices is *k-connected*, $k \geq 2$, if and only if G is a complete graph with $k+1$ vertices or the removal of any set of vertices of cardinality less than k does not disconnect G. The *vertex-connectivity* of G is k if G is k-connected, but not $(k+1)$-connected. Let $\mathcal{U}$ be a minimal set of vertices such that the resulting graph obtained from G by removing $\mathcal{U}$ is not connected. The set of vertices $\mathcal{U}$ is a *separating k-set*. If $|\mathcal{U}| = 3$, it is a *separating triplet*. The *degree* of a separating k-set $\mathcal{S}$, $d(\mathcal{S})$, in a k-connected graph G is the number of connected components in the graph obtained from G by removing $\mathcal{S}$. Note that the degree of any separating k-set is ≥ 2.

Wheel and Flower

A set of separating triplets with one common vertex c is called a *wheel* in [18]. A wheel can be represented by the set of vertices $\{c\} \cup \{s_0, s_1, \ldots, s_{q-1}\}$ which satisfies the following conditions: (*i*) $q > 2$; (*ii*) $\forall i \neq j$, $\{c, s_i, s_j\}$ is a separating triplet except in the case that $j = ((i+1) \bmod q)$ and (s_i, s_j) is an edge in G; (*iii*) c is adjacent to a vertex in each of the connected components created by removing any of the separating triplets in the wheel; (*iv*) $\forall j \neq (i+1) \bmod q$, $\{c, s_i, s_j\}$ is a degree-2 separating triplet. The vertex c is the *center* of the wheel [18]. For more details, see [18].

The *degree* of a wheel $W = \{c\} \cup \{s_0, s_1, \ldots, s_{q-1}\}$, $d(W)$, is the number of connected components in $G - \{c, s_0, \ldots, s_{q-1}\}$ plus the number of degree-3 vertices in $\{s_0, s_1, \ldots, s_{q-1}\}$ that are adjacent to c. The degree of a wheel must be at least 3. Note that the number of degree-3 vertices in $\{s_0, s_1, \ldots, s_{q-1}\}$ that are adjacent to c is equal to the number of separating triplets in $\{(c, s_i, s_{(i+2) \bmod q}) \mid 0 \leq i < q$, such that $s_{(i+1) \bmod q}$ is degree 3 in $G\}$. An example is shown in Figure 1.

A separating triplet with degree > 2 or not in a wheel is called a *flower* in [18]. Note that it is possible that two flowers of degree-2 $f_1 = \{a_{1,i} \mid 1 \leq i \leq 3\}$ and $f_2 = \{a_{2,i} \mid 1 \leq i \leq 3\}$ have the property that $\forall i$, $1 \leq i \leq 3$, either $a_{1,i} = a_{2,i}$ or $(a_{1,i}, a_{2,i})$ is an edge in G. We denote $f_1 \mathcal{R} f_2$ if f_1 and f_2 satisfy the above

[‡]Graphs refer to undirected graphs throughout this paper unless specified otherwise.

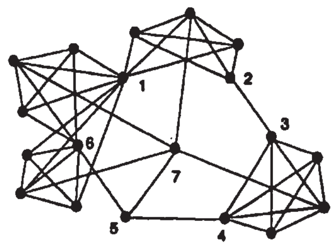

Figure 1: Illustrating a wheel $\{7\} \cup \{1,2,3,4,5,6\}$. The degree of this wheel is 5, i.e. the number of components we got after removing the wheel is 4 and there is one vertex (vertex 5) in the wheel with degree 3.

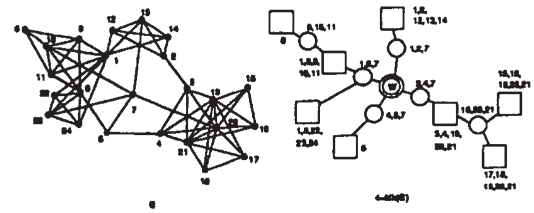

Figure 2: Illustrating a triconnected graph and its 4-$blk(G)$. We use rectangles, circles and two concentric circles to represent R-vertices, F-vertices and W-vertices, respectively. The vertex-numbers beside each vertex in 4-$blk(G)$ represent the set of vertices corresponding to this vertex.

condition. For each flower f, the *flower cluster* $\mathcal{F}_f$ for f is the set of flowers $\{f_1, \ldots, f_x\}$ (including f) such that $f\mathcal{R}f_i$, $\forall i$, $1 \leq i \leq x$.

Each of the separating triplets in a triconnected graph G is either represented by a flower or is in a wheel. We can construct an $O(n)$-space representation for all separating triplets (i.e. flowers and wheels) in a triconnected graph with n vertices and m edges in $O(n\alpha(m,n) + m)$ time [18].

K-Block

Let $G = (V, E)$ be a graph with vertex-connectivity $k-1$. A *k-block* in G is either (i) a minimal set of vertices B in a separating $(k-1)$-set with exactly $k-1$ neighbors in $V \setminus B$ (these are *special k-blocks*) or (ii) a maximal set of vertices B such that there are at least k vertex-disjoint paths in G between any two vertices in B (these are *non-special k-blocks*). Note that a set consisting of a single vertex of degree $k-1$ in G is a k-block. A *k-block leaf* in G is a k-block B_l with exactly $k-1$ neighbors in $V \setminus B_l$. Note also that every special k-block is a k-block leaf. If there is any special 4-block in a separating triplet $\mathcal{S}$, $d(\mathcal{S}) \leq 3$. Given a non-special k-block B leaf, the vertices in B that are not in the flower cluster that separates B are *demanding vertices*. We let every vertex in a special 4-block leaf be a demanding vertex.

Claim 1 *Every non-special k-block leaf contains at least one demanding vertex.* □

Using procedures in [18], we can find all of the 4-block leaves in a triconnected graph with n vertices and m edges in $O(n\alpha(m,n) + m)$ time.

Four-Block Tree

From [18] we know that we can decompose vertices in a triconnected graph into the following 3 types: (i) 4-blocks; (ii) wheels; (iii) separating triplets that are not in a wheel. We modify the decomposition tree in [18] to derive the *four-block tree* 4-$blk(G)$ for a triconnected graph G as follows. We create an R-vertex for each 4-block that is not special (i.e. not in a separating set or in the center of a wheel), an F-vertex for each separating triplet that is not in a wheel, and a W-vertex for each wheel. For each wheel $W = \{c\} \cup \{s_0, s_1, \ldots, s_{q-1}\}$, we also create the following vertices. An F-vertex is created for each separating triplet of the form $\{c, s_i, s_{(i+1) \bmod q}\}$ in W. An R-vertex is created for every degree-3 vertex s in $\{s_0, s_1, \ldots, s_{q-1}\}$ that is adjacent to c and an F-vertex is created for the three vertices that are adjacent to s. There is an edge between an F-vertex f and an R-vertex r if each vertex in the separating triplet corresponding to f is either in the 4-block H_r corresponding to r or adjacent to a vertex in H_r. There is an edge between an F-vertex f and a W-vertex w if the the wheel corresponding to w contains the separating triplet corresponding to f. A *dummy R-vertex* is created and adjacent to each pair of flowers f_1 and f_2 with the properties that f_1 and f_2 are not already connected and either $f_1 \in \mathcal{F}_{f_2}$, $f_2 \in \mathcal{F}_{f_1}$ (i.e. their flower clusters contain each other) or their corresponding separating triplets are overlapped. An example of a 4-block tree is shown in Figure 2.

Note that a degree-1 R-vertex in 4-$blk(G)$ corresponds to a 4-block leaf, but the reverse is not necessarily true, since we do not represent some special 4-block leaves and all degree-3 vertices that are centers of wheels in 4-$blk(G)$. A special 4-block leaf $\{v\}$, where v is a vertex, is represented by an R-vertex in 4-$blk(G)$ if v is not the center of a wheel w and it is in one of separating triplets of w. The degree of a flower F in G is the degree of its corresponding vertex in 4-$blk(G)$. Note also that the degree of a wheel W in

G is equal to the number of components in 4-$blk(G)$ by removing its corresponding W-vertex w and all F-vertices that are adjacent to w. A wheel W in G is a *star wheel* if $d(W)$ equals the number of leaves in 4-$blk(G)$ and every special 4-block leaf in W is either adjacent to or equal to the center. A star wheel W with the center c has the property that every 4-block leaf in G (not including $\{c\}$ if it is a 4-block leaf) can be separated from G by a separating triplet containing the center c. If G contains a star wheel W, then W is the only wheel in G. Note also that the degree of a wheel is less than or equal to the degree of its center in G.

K-connectivity Augmentation Number
The *k-connectivity augmentation number* for a graph G is the smallest number of edges that must be added to G in order to k-connect G.

3 A Lower Bound for the Four-Connectivity Augmentation Number

In this section, we first give a simple lower bound for the four-connectivity augmentation number that is similar to the ones for biconnectivity augmentation [3] and triconnectivity augmentation [10]. We show that this above lower bound is not always equal to the four-connectivity augmentation number [15, 17]. We then give a modified lower bound. This new lower bound turns out to be the exact number of edges that we must add to reach four-connectivity (see proofs in Section 4). Finally, we show relations between the two lower bounds.

3.1 A Simple Lower Bound

Given a graph G with vertex-connectivity $k-1$, it is well known that $\max\{\lceil \frac{l_k}{2} \rceil, d-1\}$ is a lower bound for the k-connectivity augmentation number where l_k is the number of k-block leaves in G and d is the maximum degree among all separating $(k-1)$-sets in G [3]. It is also well known that for $k = 2$ and 3, this lower bound equals the k-connectivity augmentation number [3, 10]. For $k = 4$, however, several researchers [15, 17] have observed that this value is not always equal to the four-connectivity augmentation number. Examples are given in Figure 3. Figure 3.(1) is from [15] and Figure 3.(2) is from [17]. Note that if we apply the above lower bound in each of the three graphs in Figure 3, the values we obtain for Figures 3.(1), 3.(2) and 3.(3) are 3, 3 and 2, respectively, while we need one more edge in each graph to four-connect it.

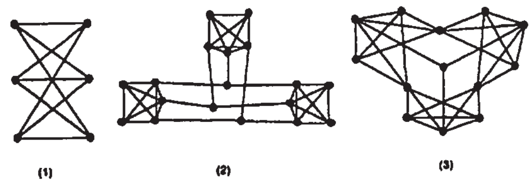

Figure 3: Illustrating three graphs where in each case the value derived by applying a simple lower bound does not equal its four-connectivity augmentation number.

3.2 A Better Lower Bound

Notice that in the previous lower bound, for every separating triplet $\mathcal{S}$ in the triconnected graph $G = \{V, E\}$, we must add at least $d(\mathcal{S}) - 1$ edges between vertices in $V \setminus \mathcal{S}$ to four-connect G, where $d(\mathcal{S})$ is the degree of $\mathcal{S}$ (i.e. the number of connected components in $G - \mathcal{S}$); otherwise, $\mathcal{S}$ remains a separating triplet. Let the set of edges added be $\mathcal{A}_{1,\mathcal{S}}$. We also notice that we must add at least one edge into every 4-block leaf $\mathcal{B}$ to four-connect G; otherwise, $\mathcal{B}$ remains a 4-block leaf. Since it is possible that $\mathcal{S}$ contains some 4-block leaves, we need to know the minimum number of edges needed to eliminate all 4-block leaves inside $\mathcal{S}$. Let the set of edges added be $\mathcal{A}_{2,\mathcal{S}}$. We know that $\mathcal{A}_{1,\mathcal{S}} \cap \mathcal{A}_{2,\mathcal{S}} = \emptyset$. The previous lower bound gives a bound on the cardinality of $\mathcal{A}_{1,\mathcal{S}}$, but not that of $\mathcal{A}_{2,\mathcal{S}}$. In the following paragraph, we define a quantity to measure the cardinality of $\mathcal{A}_{2,\mathcal{S}}$.

Let $Q_{\mathcal{S}}$ be the set of special 4-block leaves that are in the separating triplet $\mathcal{S}$ of a triconnected graph G. Two 4-block leaves $\mathcal{B}_1$ and $\mathcal{B}_2$ are *adjacent* if there is an edge in G between every demanding vertex in $\mathcal{B}_1$ and every demanding vertex in $\mathcal{B}_2$. We create an *augmenting graph for* $\mathcal{S}$, $\mathcal{G}(\mathcal{S})$, as follows. For each special 4-block leaf in $Q_{\mathcal{S}}$, we create a vertex in $\mathcal{G}(\mathcal{S})$. There is an edge between two vertices v_1 and v_2 in $\mathcal{G}(\mathcal{S})$ if their corresponding 4-blocks are adjacent. Let $\overline{\mathcal{G}(\mathcal{S})}$ be the complement graph of $\mathcal{G}(\mathcal{S})$. The seven types of augmenting graphs and their complement graphs are illustrated in Figure 4.

Definition 1 *The* **augmenting number** $a(\mathcal{S})$ *for a separating triplet* $\mathcal{S}$ *in a triconnected graph is the number of edges in a maximum matching* $\mathcal{M}$ *of* $\overline{\mathcal{G}(\mathcal{S})}$ *plus the number of vertices that have no edges in* $\mathcal{M}$ *incident on them.*

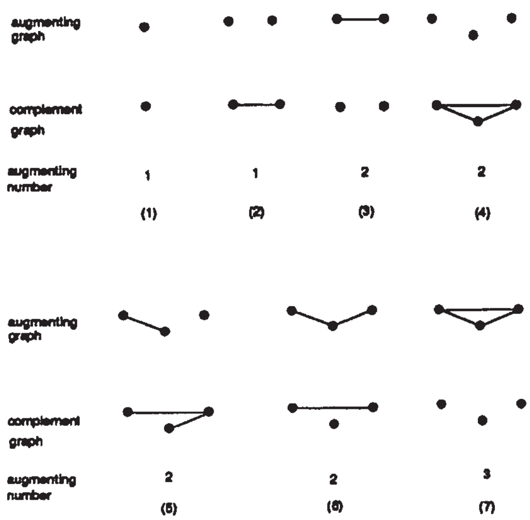

Figure 4: Illustrating the seven types of augmenting graphs, their complement graphs and augmenting numbers that one can get for a separating triplet in a triconnected graph.

The augmenting numbers for the seven types of augmenting graphs are shown in Figure 4. Note that in a triconnected graph, each special 4-block leaf must receive at least one new incoming edge in order to four-connect the input graph. The augmenting number $a(S)$ is exactly the minimum number of edges needed in the separating triplet S in order to four-connect the input graph. The augmenting number of a separating set that does not contain any special 4-block leaf is 0. Note also that we can define the *augmenting number* $a(C)$ for a set C that consists of the center of a wheel using a similar approach. Note that $a(C) \leq 1$.

We need the following definition.

Definition 2 *Let G be a triconnected graph with l 4-block leaves. The* **leaf constraint** *of G, $lc(G)$, is $\lceil \frac{l}{2} \rceil$. The* **degree constraint** *of a separating triplet S in G, $dc(S)$, is $d(S) - 1 + a(S)$, where $d(S)$ is the degree of S and $a(S)$ is the augmenting number of S. The degree constraint of G, $dc(G)$, is the maximum degree constraint among all separating triplets in G. The* **wheel constraint** *of a star wheel W with center c in G, $wc(W)$, is $\lceil \frac{d(W)}{2} \rceil + a(\{c\})$, where $d(W)$ is the degree of W and $a(\{c\})$ is the augmenting number of $\{c\}$. The wheel constraint of G, $wc(G)$, is 0 if there is no star wheel in G; otherwise it is the wheel constraint of the star wheel in G.*

We now give a better lower bound on the 4-connectivity augmentation number for a triconnected graph.

Lemma 1 *We need at least* $\max\{lc(G), dc(G), wc(G)\}$ *edges to four-connect a triconnected graph G.*

__Proof:__ Let $\mathcal{A}$ be a set of edges such that $G' = G \cup \mathcal{A}$ is four-connected. For each 4-block leaf B in G, we need one new incoming edge to a vertex in B; otherwise B is still a 4-block leaf in G'. This gives the first component of the lower bound.

For each separating triplet S in G, $G - S$ contains $d(S)$ connected components. We need to add at least $d(S) - 1$ edges between vertices in $G - S$, otherwise S is still a separating triplet in G'. In addition to that, we need to add at least $a(S)$ edges such that at least one of the two end points of each new edge is in S; otherwise S contains a special 4-block leaf. This gives the second term of the lower bound.

Given the star wheel W with the center c, $4\text{-}blk(G)$ contains exactly $d(W)$ degree-1 R-vertices. Thus we need to add at least $\lceil \frac{d(W)}{2} \rceil$ edges between vertices in $G - \{c\}$; otherwise, G' contains some 4-block leaves. In addition to that, we need to add $a(\{c\})$ non-self-loop edges such that at least one of the two end points of each new edge is in $\{c\}$; otherwise $\{c\}$ is still a special 4-block leaf. This gives the third term of the lower bound. □

3.3 A Comparison of the Two Lower Bounds

We first observe the following relation between the wheel constraint and the leaf constraint. Note that if there exists a star wheel W with degree $d(W)$, there are exactly $d(W)$ 4-block leaves in G if the center is not degree-3. If the center of the star wheel is degree-3, then there are exactly $d(W) + 1$ 4-block leaves in G. Thus the wheel constraint is greater than the leaf constraint if and only if the star wheel has a degree-3 center. We know that the degree of any wheel is less than or equal to the degree of its center. Thus the value of the above lower bound equals 3.

We state the following claims for the relations between the degree constraint of a separating triplet and the leaf constraint.

Claim 2 *Let S be a separating triplet with degree $d(S)$ and h special 4-block leaves. Then there are at least $h + d(S)$ 4-block leaves in G.* □

Claim 3 *Let $\{a_1, a_2, a_3\}$ be a separating triplet in a triconnected graph G. Then a_i, $1 \leq i \leq 3$, is incident on a vertex in every connected component in $G - \{a_1, a_2, a_3\}$.* □

Corollary 1 *The degree of a separating triplet S is no more than the largest degree among all vertices in S.* □

From Corollary 1, we know that it is not possible that a triconnected graph has type (6) or type (7) of the augmenting graphs as shown in Figure 4, since the degree of their underling separating triplet is 1. We also know that the degree of a separating triplet with a special 4-block leaf is at most 3 and at least 2. Thus $dc(S)$ is greater than $d(S) - 1$ if $dc(S)$ equals either 3 or 4. Thus we have the following lemma.

Lemma 2 *Let $low_1(G)$ be the lower bound given in Section 3.1 for a triconnected graph G and let $low_2(G)$ be the lower bound given in Lemma 1 in Section 3.2. (i) $low_1(G) = low_2(G)$ if $low_2(G) \notin \{3,4\}$. (ii) $low_2(G) - low_1(G) \in \{0,1\}$.* □

Thus the simple lower bound extended from biconnectivity and triconnectivity is in fact a good approximation for the four-connectivity augmentation number.

4 Finding a Smallest Four-Connectivity Augmentation for a Triconnected Graph

We first explore properties of the 4-block tree that we will use in this section to develop an algorithm for finding a smallest 4-connectivity augmentation. Then we describe our algorithm. Graphs discussed in this section are triconnected unless specified otherwise.

4.1 Properties of the Four-Block Tree

Massive Vertex, Critical Vertex and Balanced Graph

A separating triplet S in a graph G is *massive* if $dc(S) > lc(G)$. A separating triplet S in a graph G is *critical* if $dc(S) = lc(G)$. A graph G is *balanced* if there is no massive separating triplet in G. If G is balanced, then its 4-$blk(G)$ is also *balanced*. The following lemma and corollary state the number of massive and critical vertices in 4-$blk(G)$.

Lemma 3 *Let S_1, S_2 and S_3 be any three separating triplets in G such that there is no special 4-block in $S_i \cap S_j$, $1 \le i < j \le 3$. $\sum_{i=1}^{3} dc(S_i) \le l+1$, where l is the number of 4-block leaves in G.*

<u>Proof:</u> G is triconnected. We can modify 4-$blk(G)$ in the following way such that the number of leaves in the resulting tree equals l and the degree of an F-node f equals its degree constraint plus 1 if f corresponds to S_i, $1 \le i \le 3$. For each W-vertex w with a degree-3 center c, we create an R-vertex r_c for c, an F-vertex f_c for the three vertices that are adjacent to c in G. We add edges (w,f_c) and (f_c,r_c). Thus r_c is a leaf. For each F-vertex whose corresponding separating triplet S contains h special 4-block leaves, we attach $a(S)$ subtrees with a total number of h leaves with the constraint that any special 4-block that is in more than one separating triplet will be added only once (to the F-node corresponding to S_i, $1 \le i \le 3$, if possible). From Figure 4 we know that the number of special 4-block leaves in any separating triplet is greater than or equal to its augmenting number. Thus the above addition of subtrees can be done. Let 4-$blk(G)'$ be the resulting graph. Thus the number of leaves in 4-$blk(G)'$ is l. Let f be an F-node in 4-$blk(G)'$ whose corresponding separating triplet is S. We know that the degree of f equals $dc(S)+1$ if $S \in \{S_i \mid 1 \le i \le 3\}$. It is easy to verify that the sum of degrees of any three internal vertices in a tree is less than or equal to 4 plus the number of leaves in a tree. □

Corollary 2 *Let G be a graph with more than two non-special 4-block leaves. (i) There is at most one massive F-vertex in 4-$blk(G)$. (ii) If there is a massive F-vertex, there is no critical F-vertex. (iii) There are at most two critical F-vertices in 4-$blk(G)$.* □

Updating the Four-Block Tree

Let v_i be a demanding vertex or a vertex in a special 4-block leaf, $i \in \{1,2\}$. Let B_i be the 4-block leaf that contains v_i, $i \in \{1,2\}$. Let b_i, $i \in \{1,2\}$, be the vertex in 4-$blk(G)$ such that if v_i is a demanding vertex, then b_i is an R-vertex whose corresponding 4-block contains v_i; if v_i is in a special 4-block leaf in a flower, then b_i is the F-vertex whose corresponding separating triplet contains v_i; if v_i is the center of a wheel w, b_i is the F-vertex that is closet to $b_{(i \bmod 2)+1}$ and is adjacent to w. The vertex b_i is the *implied vertex* for B_i, $i \in \{1,2\}$. The *implied path P between B_1 and B_2* is the path in 4-$blk(G)$ between b_1 and b_2. Given 4-$blk(G)$ and an edge (v_1,v_2) not in G, we can obtain 4-$blk(G \cup \{(v_1,v_2)\})$ by performing local updating operations on P. For details, see [18].

In summary, all 4-blocks corresponding to R-vertices in P are collapsed into a single 4-block. Edges in P are deleted. F-vertices in P are connected to the new R-vertex created. We *crack* wheels in a way that is similar to the cracking of a polygon for updating 3-block graphs (see [2, 10] for details). We say that P is *non-adjacent* on a wheel W, if the cracking of W creates two new wheels. Note that it is possible that a separating triplet S in the original graph is no longer a separating triplet in the resulting graph by adding an edge. Thus some special leaves in the original graph are no longer special, in which case they must be added to 4-$blk(G)$.

Reducing the Degree Constraint of a Separating Triplet
We know that the degree constraint of a separating triplet can be reduced by at most 1 by adding a new edge. From results in [18], we know that we can reduce the degree constraint of a separating triplet S by adding an edge between two non-special 4-block leaves B_1 and B_2 such that the path in 4-$blk(G)$ between the two vertices corresponding to B_1 and B_2 passes through the vertex corresponding to S. We also notice the following corollary from the definitions of 4-$blk(G)$ and the degree constraint.

Corollary 3 *Let S be a separating triplet that contains a special 4-block leaf. (i) We can reduce $dc(S)$ by 1 by adding an edge between two special 4-block leaves B_1 and B_2 in S such that B_1 and B_2 are not adjacent. (ii) If we add an edge between a special 4-block leaf in S and a 4-block leaf B not in S, the degree constraint of every separating triplet corresponding to an internal vertex in the path of 4-$blk(G)$ between vertices corresponding to S and B is reduced by 1.* □

Reducing the Number of Four-Block Leaves
We now consider the conditions under which the adding of an edge reduces the leaf constraint $lc(G)$ by 1. Let *real degree* of an F-node in 4-$blk(G)$ be 1 plus the degree constraint of its corresponding separating triplet. The real degree of a W-node with a degree-3 center in G is 1 plus its degree in 4-$blk(G)$. The real degree of any other node is equal to its degree in 4-$blk(G)$.

Definition 3 (The Leaf-Connecting Condition) *Let B_1 and B_2 be two non-adjacent 4-block leaves in G. Let P be the implied path between B_1 and B_2 in 4-$blk(G)$. Two 4-block leaves B_1 and B_2 satisfy the* **leaf-connecting condition** *if at least one of the following conditions is true. (i) There are at least two vertices of real degree at least 3 in P. (ii) There is at least one R-vertex of degree at least 4 in P. (iii) The path P is non-adjacent on a W-vertex in P. (iv) There is an internal vertex of real degree at least 3 in P and at least one of the 4-block leaves in $\{B_1, B_2\}$ is special. (v) B_1 and B_2 are both special and they do not share the same set of neighbors.*

Lemma 4 *Let B_1 and B_2 be two 4-block leaves in G that satisfy the leaf-connecting condition. We can find vertices v_i in B_i, $i \in \{1,2\}$, such that $lc(G \cup \{(v_1, v_2)\}) = lc(G) - 1$, if $lc(G) \geq 2$.* □

4.2 The Algorithm

We now describe an algorithm for finding a smallest augmentation to four-connect a triconnected graph. Let $\delta = dc(G) - lc(G)$. The algorithm first adds 2δ edges to the graph such that the resulting graph is balanced and the lower bound is reduced by 2δ. If $lc(G) \neq 2$ or $wc(G) \neq 3$, there is no star wheel with a degree-3 center. We add an edge such that the degree constraint $dc(G)$ is reduced by 1 and the number of 4-block leaves is reduced by 2. Since there is no star wheel with a degree-3 center, $wc(G)$ is also reduced by 1 if $wc(G) = lc(G)$. The resulting graph stays balanced each time we add an edge and the lower bound given in Lemma 1 is reduced by 1. If $lc(G) = 2$ and $wc(G) = 3$, then there exists a star wheel with a degree-3 center. We reduce $wc(G)$ by 1 by adding an edge between the degree-3 center and a demanding vertex of a 4-block leaf. Since $lc(G) = 2$ and $wc(G) = 3$, $dc(G)$ is at most 2. Thus the lower bound can be reduced by 1 by adding an edge. We keep adding an edge at a time such that the lower bound given in Lemma 1 is reduced by 1. Thus we can find a smallest augmentation to four-connect a triconnected graph. We now describe our algorithm.

The Input Graph is not Balanced
We use an approach that is similar to the one used in biconnectivity and triconnectivity augmentations to balance the input graph [10, 11, 26]. Given a tree T and a vertex v in T, a *v-chain* [26] is a component in $T - \{v\}$ without any vertex of degree more than 2. The leaf of T in each v-chain is a *v-chain leaf* [26]. Let $\delta = dc(G) - lc(G)$ for a unbalanced graph G and let 4-$blk(G)'$ be the modified 4-block tree given in the proof of Lemma 3. Let f be a massive F-vertex. We can show that either there are at least $2\delta + 2$ f-chains in 4-$blk(G)'$ (i.e. f is the only massive F-vertex) or we can eliminate all massive F-vertices by adding an edge. Let λ_i be a demanding vertex in the ith f-chain leaf. We add the set of edges $\{(\lambda_i, \lambda_{i+1}) \mid 1 \leq i \leq 2\delta\}$. It is also easy to show that the lower bound given in Lemma 1 is reduced by 2δ and the graph is balanced.

The Input Graph is Balanced
We first describe the algorithm. Then we give its proof of correctness. In the description, we need the following definition. Let B be a 4-block leaf whose implied vertex in 4-$blk(G)$ is b and let B' be a 4-block leaf whose implied vertex in 4-$blk(G)$ is b'. B' is a *nearest* 4-block leaf of B if there is no other 4-block leaf whose implied vertex has a distance to b that is shorter than the distance between b and b'.

{* G is triconnected with ≥ 5 vertices; the algorithm finds a smallest four-connectivity augmentation. *}
graph function aug3to4(**graph** G);
{* The algorithmic notation used is from Tarjan [29]. *}
  $T := 4\text{-}blk(G)$; root T at an arbitrary vertex;
  let $\bar{l}$ be the number of degree-1 R-vertices in T;
  do $\exists$ a 4-block leaf in $G \rightarrow$
    if $\exists$ a degree-3 center $c \rightarrow$
1.   **if** $lc(G) = 2$ and $wc(G) = 3 \rightarrow$
        {* Vertex c is the center of the star wheel w. *}
        $u_1 :=$ the 4-block leaf $\{c\}$;
        let u_2 be a a non-special 4-block leaf
      | $\exists$ another degree-3 center c' non-adjacent to $c \rightarrow$
        let u_2 be the 4-block leaf $\{c'\}$
      | $\exists$ a special 4-block leaf b non-adjacent to $u_1 \rightarrow$
        let $u_2 := b$
      | $\nexists$ (degree-3 center or special 4-block leaf) non-adjacent to $u_1 \rightarrow$
        let u_2 be a a 4-block leaf such that $\exists$ an internal vertex with real degree ≥ 3 in their implies path
      fi
    | $lc(G) \neq 2$ or $wc(G) \neq 3 \rightarrow$
      if $\bar{l} > 2$ **and** $\exists$ 2 critical F-vertices f_1 and $f_2 \rightarrow$
2.   find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through f_1 and f_2
      | $l > 2$ **and** $\exists$ only one critical F-vertex $f_1 \rightarrow$
        if $\exists$ two non-adjacent special 4-block leaves in the separating triplet S_1 corresponding to $f_1 \rightarrow$
3.   **let** u_1 and u_2 be two non-adjacent 4-block leaves in S_1
        | $\nexists$ two non-adjacent special 4-block leaves in the separating triplet S_1 corresponding to $f_1 \rightarrow$
4.   **let** v be a vertex with the largest real degree among all vertices in T besides f_1;
          if real degree of v in $T \geq 3 \rightarrow$
            find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through f_1 and v
          fi
          {* The case when the degree of v in $T < 3$ will be handled in step 8. *}
        fi
      | $\exists$ two vertices v_1 and v_2 with real degree $\geq 3 \rightarrow$
5.   find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through v_1 and v_2
      | $\exists$ an R-vertex v of degree $\geq 4 \rightarrow$
6.   find two non-special 4-block leaves u_1 and u_2 such that the implied path between them passes through v
      | $\exists$ a W-vertex v of degree $\geq 4 \rightarrow$
7.   **let** u_1 and u_2 be two non-special 4-block leaves such that the implied path between them is non-adjacent on v
      | $\exists$ only one vertex v in T with real degree $\geq 3 \rightarrow$
        {* T is a star with the center v. *}
8.   find a nearest vertex w of v that contains a 4-block leaf v_1;
        let w' be a nearest vertex of w containing a 4-block leaf non-adjacent to v_1;
        find two 4-block leaves u_1 and u_2 whose implied path passes through w, w' and v
        {* The above step can always be done, since T is a star. *}
      {* Note that T is path for all the cases below. *}
      | $\exists$ two non-adjacent special 4-block leaves in one separating triplet $S \rightarrow$
9.   **let** u_1 and u_2 be two non-adjacent special 4-block leaves in S
      | $\exists$ a special 4-block leaf $u_1 \rightarrow$
10.   find a nearest non-adjacent 4-block leaf u_2
      | $\bar{l} = 2 \rightarrow$
        let u_1 and u_2 be the two 4-block leaves corresponding to the two degree-1 R-vertices in T
      fi
    fi;
    let y_i, $i \in \{1,2\}$, be a demanding vertex in u_i such that (y_1, y_2) is not an edge in the current G;
    $G := G \cup \{(y_1, y_2)\}$;
    update T, $\bar{l}$, $lc(G)$, $wc(G)$ and $dc(G)$
  od;
  return G
end aug3to4;

Before we show the correctness of algorithm aug3to4, we need the following claim and corollaries.

Claim 4 *[26] If $4\text{-}blk(G)$ contains two critical vertices f_1 and f_2, then every leaf is either in an f_1-chain or in an f_2-chain and the degree of any other vertex in $4\text{-}blk(G)$ is at most 2.* □

Corollary 4 *If $4\text{-}blk(G)$ contains two critical vertices f_1 and f_2 and the corresponding separating triplet S_i, $i \in \{1,2\}$, of f_i contains a special 4-block leaf, then its augmenting number equals the number of special 4-block leaves in it.* □

Corollary 5 *Let f_1 and f_2 be two critical F-vertices in $4\text{-}blk(G)$. If the number of degree-1 R-vertices in $4\text{-}blk(G) > 2$ and the corresponding separating triplet of f_i, $i \in \{1,2\}$, contains a 4-block leaf B_i, we can add an edge between a vertex in B_1 and a vertex in B_2 to reduce the lower bound given in Lemma 1 by 1.* □

Theorem 1 *Algorithm aug3to4 adds the smallest number of edges to four-connect a triconnected graph.* □

We now describe an efficient way of implementing algorithm aug3to4. The 4-block tree can be computed in $O(n\alpha(m,n)+m)$ time for a graph with n vertices and m edges [18]. We know that the leaf constraint, the degree constraint of any separating triplet and the wheel constraint of any wheel in G can only be decreased by adding an edge. We also know that $lc(G)$, the sum of degree constraints of all separating triplets and the sum of wheel constraints of all wheels are all $O(n)$. Thus we can use the technique in [26] to maintain the current leaf constraint, the degree constraint for any separating triplet and the wheel constraint for any wheel in $O(n)$ time for the entire execution of the algorithm. We also visit each vertex and each edge in the 4-block tree a constant number of times before deciding to collapse them. There are $O(n)$ 4-block leaves and $O(n)$ vertices and edges in $4\text{-}blk(G)$. In each vertex, we need to use a set-union-find algorithm to maintain the identities of vertices after collapsing. Hence the overall time for updating the 4-block tree is $O(n\alpha(n,n))$. We have the following claim.

Claim 5 *Algorithm aug3to4 can be implemented in $O(n\alpha(m,n)+m)$ time where n and m are the number of vertices and edges in the input graph, respectively and $\alpha(m,n)$ is the inverse Ackermann's function.* □

5 Conclusion

We have given a sequential algorithm for finding a smallest set of edges whose addition four-connects a triconnected graph. The algorithm runs in $O(n\alpha(m,n)+m)$ time using $O(n+m)$ space. The following approach was used in developing our algorithm. We first gave a 4-block tree data structure for a triconnected graph that is similar to the one given in [18]. We then described a lower bound on the smallest number of edges that must be added based on the 4-block tree of the input graph. We further showed that it is possible to decrease this lower bound by 1 by adding an appropriate edge.

The lower bound that we gave here is different from the ones that we have for biconnecting a connected graph [3] and for triconnecting a biconnected graph [10]. We also showed relations between these two lower bounds. This new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier for the number of edges needed to k-connect a $(k-1)$-connected graph. It is likely that techniques presented in this paper may be used in finding the k-connectivity augmentation number of a $(k-1)$-connected graph, for an arbitrary k.

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On four-connecting a triconnected graph

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Abstract:
The author considers the problem of finding a smallest set of edges whose addition f connects a triconnected graph. This is a fundamental graph-theoretic problem that h applications in designing reliable **networks**. He presents an O(nα(m,n)+m) time sequential algorithm for four-connecting an undirected graph G that is triconnected b adding the smallest number of edges, where n and m are the number of vertices anc edges in G, respectively, and α(m, n) is the inverse Ackermann function. He present new lower bound for the number of edges needed to four-connect a triconnected gra The form of this lower bound is different from the form of the lower bound known for biconnectivity augmentation and triconnectivity augmentation. The new lower bound applies for arbitrary k, and gives a tighter lower bound than the one known earlier fc number of edges needed to **k-connect** a (k-1)-connect graph. For k=4, he shows th this lower bound is tight by giving an efficient algorithm for finding a set edges with required size whose addition four-connects a triconnected graph

Index Terms:
computational complexity computational geometry four-connecting graph theory graph-theo problem inverse Ackermann function reliable **networks** triconnected graph computational complexity computational geometry four-connecting graph theory graph-theoretic problem inverse Ackermann function reliable **networks** triconnected graph

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A Flexible Architecture for Multi-Hop Optical Networks

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Abstract

It is desirable to have low diameter logical topologies for multihop lightwave networks. Researchers have investigated regular topologies for such networks. Only a few of these (e.g., GEMNET [8]) are scalable to allow the addition of new nodes to an existing network. Adding new nodes to such networks requires a major change in routing scheme. For example, in a multistar implementation, a large number of retuning of transmitters and receivers and/or renumbering nodes are needed for [8]. In this paper, we present a scalable logical topology which is not regular but it has a low diameter. This topology is interesting since it allows the network to be expanded indefinitely and new nodes can be added with a relatively small change to the network. In this paper we have presented the new topology, an algorithm to add nodes to the network and two routing schemes.

Keywords: *optical networks, multihop networks, scalable logical topology, low diameter networks.*

1. Introduction

Optical networks [1] are interconnections of high-speed broadband fibers using *lightpaths*. Each lightpath provides traverses one or more fibers and uses one wavelength division multiplexed (WDM) channel per fiber. In a multihop network, each node has a small number of lightpaths to a few other nodes in the network. The physical topology of the network determines how the lightpaths get defined. For a multistar implementation of the physical topology, a lightpath $u \rightarrow v$ is established when node u broadcasts to a passive optical coupler at a particular wavelength and the node v picks up the optical signal by tuning its receiver to the same wavelength. For a wavelength routed network, a lightpath $u \rightarrow v$ might be established through one or several fibers interconnected by router nodes. The lightpath definition between the nodes in an optical network is usually represented by a directed graph (or digraph) G = (V, E) (where V is the set of nodes and E is the set of the edges) with each node of G representing a node of the network and each edge (denoted by $u \rightarrow v$) representing a lightpath from u to v. G is usually called the logical topology of the network. When the lightpath $u \rightarrow v$ does not exist, the communication from a node u to a node v occurs by using a (graph-theoretic) path (denoted by $u \rightarrow x_1 \rightarrow x_2 \rightarrow \dots \rightarrow x_{k-1} \rightarrow v$) in G using k hops through the intermediate nodes $x_1, x_2, \dots, x_{k-1}$. The information is buffered at intermediate nodes and, to reduce the communication delay, the number of hops should be small. If a shortest graph-theoretic path is used to establish a communication from u to v, the maximum hop distance is the *diameter* of G. Clearly, the lightpaths need to be defined such that G has a small diameter and low average hop distance. The indegree and outdegree of each node should be low to reduce the network cost. However, a reduction of the degree usually implies an increase in the diameter of the digraph, that is, larger communication delays. The design of the logical topology of a network turns out to be a difficult problem in view of these contradictory requirements. Several different logical topologies have been proposed in the literature. An excellent review of multihop networks is presented in [1].

Both regular and irregular structures have been studied for multihop structures [2], [3], [4], [5], [6], [7]. All the proposed regular topologies(e.g., shuffle nets, de Bruijn graphs, torus, hypercubes) enjoy the property of simple routing algorithms, thereby avoiding the need of complex routing tables. Since the diameter of a digraph with n nodes and maximum outdegree d is of $O(\log_d n)$, most of the topologies attempt to reduce the diameter to $O(\log_d n)$. One common property of these network topologies is the number of nodes in the network must be given by some well-defined formula involving network parameters. This makes the topology non-scalable. In short, addition of a node to an existing network is virtually impossible. In [8], the principle of shuffle interconnection between nodes in a shufflenet [4] is generalized (the generalized version can have any number of nodes in each column) to obtain a scalable network topology called GEMNET. A similar idea of generalizing

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the Kautz graph has been studied in [9] showing a better diameter and network throughput than GEMNET. Both these scalable topologies are given by regular digraphs.

One topology that has been studied for optical networks is the bidirectional ring network. In such networks, each node has two incoming lightpaths and two outgoing lightpaths. In terms of the graph model, each node has one outgoing edge to and one incoming edge from the preceding and the following node in the network. Adding a new node to such a ring network involves redefining a fixed number of edges and can be repeated indefinitely.

Our motivation was to develop a topology which has the advantages of a ring network with respect to scalability and the advantages of a regular topology with respect to low diameter. In other words, our topology has to satisfy the following characteristics:

- The diameter should be small
- The routing strategy should be simple
- It should be possible to add new nodes to the network indefinitely with the least possible perturbation of the network.
- Each node in the network should have a predefined upper limit on the number of incoming and outgoing edges.

In this paper we introduce a new scalable topology for multihop networks where the graph is not, in general, regular. Given integers n and d, our proposed topology can be defined for n nodes with a fixed number of incoming and outgoing edges in the network The major advantage of our scheme is that, as a new node is added to the network, most of the existing edges of the logical topology are not changed, implying that the routing schemes between the existing nodes need little modification. The edges to and from the new added node can be implemented by defining new lightpaths which is small in number, namely, $O(d)$. For multistar implementation, for example, this can be accomplished by retuning $O(d)$ transmitters and receivers.

The paper is organized as follows. In section 2, we describe the proposed topology and derive its pertinent properties. Section 3 presents two routing schemes for the proposed topology and establishes that the diameter is $O(\log_d n)$. Our experiments in section 4 show that, for a network with n nodes and having an indegree of at most $d+1$, an outdegree of d and the average hop distance is approximately $\log_d n$. We have concluded with a critical summary in section 4.

2. Scalable topology for multihop networks

2.1 Proposed interconnection topology

Given two integers n and d, $d \le n$. we define the interconnection topology of the network as a digraph G in the following. As mentioned earlier, the digraph is not regular - the indegree and outdegree of a node varies from 1 to $d+1$. We will assume that there is no k, such that $n = d^k$; if $n = d^k$ for some k, our proposed topology is the same as given by [2]. Let k be the integer such that $d^k < n < d^{k+1}$. Let Z_k be the set of all $(k+1)$-digit strings choosing digits from $Z = \{0, 1, 2, \ldots, d-1\}$ and let any string of Z_k be denoted by $x_0x_1\ldots x_k$. We divide Z_k into $k+2$ sets $S_0, S_1, \ldots, S_{k+1}$ such that all strings in Z_k having x_j as the left most occurrence of 0 is included in S_j, $0 \le j \le k$ and all strings with no occurrence of 0 (i.e. $x_j \ne 0$, $0 \le j \le k$) is included in S_{k+1}. We note that $|S_{k+1}| = (d-1)^{k+1}$ and $|S_j| = (d-1)^j d^{k-j}$, $0 \le j \le k$. We define an ordering relation between every pair of strings in Z_k. Each string in S_i is smaller than each string in S_j if $i < j$. For two strings $\sigma_1, \sigma_2 \in S_j$, $0 \le j \le k+1$, if $\sigma_1 = x_0x_1\ldots x_k$ and $\sigma_2 = y_0y_1\ldots y_k$ and t is the largest integer such that $x_t \ne y_t$ then $\sigma_1 < \sigma_2$ if $x_t < y_t$.

Definition: For any string $\sigma_1 = x_0x_1\ldots x_i\ldots x_j\ldots x_k$, the string $\sigma_2 = x_0x_1\ldots x_j\ldots x_i\ldots x_k$ obtained by interchanging the digits in the i[th] and the j[th] position in σ_1, will be called the *i-j-image* of σ_1.

Clearly, if σ_2 is the i-j-image of σ_1 then σ_1 is the i-j-image of σ_2 and if $x_i = x_j$, σ_1 and σ_2 represent the same node.

We will represent each node of the interconnection topology by a distinct string $x_0x_1\ldots x_k$ of Z_k. As $d^k < n < d^{k+1}$, all strings of Z_k will not be used to represent the nodes in G. We will use n smallest strings from Z_k to represent the nodes of G. Suppose the largest string representing a node is in S_M. We will use a node and its string representation interchangeably. We will use the term *used* string to denote a string of Z_k which has been already used to represent some node in G. All other strings of Z_k will be called *unused* strings.

Property 1: all strings of S_0 are used strings.

Property 2: if $\sigma \in S_j$ is an used string, then all strings of $S_0, S_1, \ldots, S_{j-1}$ are also used strings.

Property 3: If $\sigma_1 = 0x_1\ldots x_k$, σ_2 is the 0-1-image of σ_1 and $x_1 \neq 0$, then $\sigma_2 \in S_1$.

Property 4: If $\sigma_1 = 0x_1\ldots x_k$, $x_1 \neq 0$ and σ_2, the 0-1-image of σ_1, is an unused string, then all strings of the form $x_1x_2\ldots x_kj$, $0 \leq j \leq d-1$ are unused strings.

The proofs for Properties 1 - 4 are trivial and are omitted.

We now define the edge set of the digraph G. Let any node u in G be represented by $x_0x_1\ldots x_k$. The outgoing edges from node u are defined as follows:

- There is an edge $x_0x_1x_2\ldots x_k \rightarrow x_1x_2\ldots x_kj$ whenever $x_1x_2\ldots x_kj$ is an used string, for some $j \in Z$,
- There is an edge $0x_1x_2\ldots x_k \rightarrow x_10x_2\ldots x_k$ whenever the following conditions hold:
  a) $x_1x_2\ldots x_kj$ is an unused string for at least one $j \in Z$ and
  b) $x_10\ldots x_k$, the 0-1-image of u, is an used string
- There is an edge $0x_1x_2\ldots x_k \rightarrow 0x_2\ldots x_kj$ for all $j \in Z$ whenever the following conditions hold:
  a) $x_1 \neq 0$ and
  b) $x_10x_2\ldots x_k$, the 0-1-image of u, is an unused string

We note that if $u \in S_j$, j > 0, node $v = x_1x_2\ldots x_kj$ always exists (from property 2, since $v \in S_{j-1}$). As an example, we show a network with 5 nodes for $d = 2$, $k = 2$ in figure 1. We have used a solid line for an edge of the type $x_0x_1x_2\ldots x_k \rightarrow x_1x_2\ldots x_kj$, a line of dots for and a line of dashes and dots for an edge of the type $0x_1x_2\ldots x_k \rightarrow 0x_2\ldots x_kj$. We note that the edge from 010 to 100 satisfies the condition for both an edge of the type $x_0x_1x_2\ldots x_k \rightarrow x_1x_2\ldots x_kj$ and an edge of the type $0x_1x_2\ldots x_k \rightarrow x_10x_2\ldots x_k$.

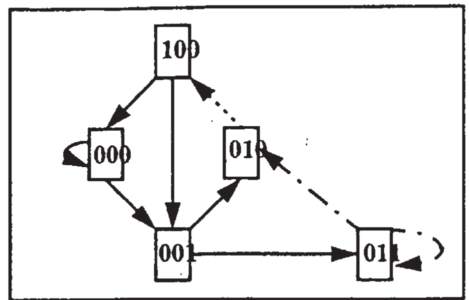

Figure 1: Interconnection topology with $d = 2$, $k = 2$ for $n = 5$ nodes.

2.2 Limits on Nodal Degree

In this section, we derive the upper limits for the indegree and the outdegree of each node in the network. We will show that, by not enforcing the regularity, we can easily achieve scalability. As we add new nodes to the network, minor modifications of the edges in the logical topology suffice, in contrast to large number of changes in the edge-set as required by other proposed methods.

Theorem 1: In the proposed topology, each node has an outdegree of up to d.

Proof: Let u be a node in the network given by $x_0x_1\ldots x_k \in S_j$. We consider the following three cases:

i) $0 < j \leq k$: For every v given by $x_1x_2\ldots x_kt$ for all t, $0 \leq t \leq d-1$ is an used string since $v \in S_{j-1}$. Therefore the edge $u \rightarrow v$ exists in the network. If $u \in S_j$, $j > 0$, these are the only edges from u. Hence, u has outdegree d.

ii) $j = 0$: According to our topology defined above, u will have an edge to $x_1x_2\ldots x_kj$ whenever $x_1x_2\ldots x_kj$ is an used string for some $j \in Z$. We have three subcases to consider:

- If $x_1x_2\ldots x_kj$ is an used string for all j, $0 \leq j < d$ then u has outdegree d.
- Otherwise, if p of the strings $x_1x_2\ldots x_kj$ are used strings, for some j, $0 \leq j < d$ and the 0-1-image of u is also an used string, then u has edges to all the p nodes with used strings of the form $x_1x_2\ldots x_kj$ and to the 0-1-image of u. Hence u has outdegree $p + 1$. Here u has an outdegree of at least 1 and at most d.
- Otherwise, if the 0-1-image of u is an unused string, then all strings of the form $x_1x_2\ldots x_kj$ are unused

strings (**Property** 4) and u has d outgoing edges to nodes of the form $0x_2x_3\ldots x_kj$, $0 \le j < d$. Hence u has outdegree d.

iii) $j = k+1$: If p of the strings $x_1x_2\ldots x_kj$ are used strings, for some j, $0 \le j < d$, then u has outdegree of p. We note that $x_1x_2\ldots x_k0 \in S_k$ is an used string. Therefore $1 \le p \le d$, and u has an outdegree of at least 1 and at most d.

Theorem 2: In the proposed topology, each node has an indegree of up to $d+1$.

Proof: Let us consider the indegree of any node v given by $y_0y_1\ldots y_k \in S_j$. As described in 2.1, there may be three type of edges to node v as follows:

- An edge $ty_0y_1\ldots y_{k-1} \rightarrow y_0y_1\ldots y_k$ whenever $ty_0y_1\ldots y_{k-1}$ is an used string, for some $t \in Z$. There may be at most d edges of this type to v.
- If $y_1 = 0$, $y_0 \ne 0$ there may be an edge $0y_0y_2\ldots y_k \rightarrow y_0y_1\ldots y_k$
- If $y_0 = 0$ and $ty_0y_1\ldots y_{k-1}$ is an unused string for some $t \in Z$, there is an edge $0ty_1\ldots y_{k-1} \rightarrow y_0y_1\ldots y_k$. There may be at most d edges of this type to v.

We have to consider 3 cases, j = 0, j = 1 and j > 1. If j > 1, the only edges are of the type $ty_0y_1\ldots y_{k-1} \rightarrow y_0y_1\ldots y_k$ and there can be up to d such edges. If j = 1, in addition to the edges are of the type $ty_0y_1\ldots y_{k-1} \rightarrow y_0y_1\ldots y_k$, there can be only one edge of the type $0y_0y_2\ldots y_k \rightarrow y_0y_1\ldots y_k$. Thus the total number of edges cannot exceed d + 1, in this case. If j = 0, an edge of the type $0ty_1\ldots y_{k-1} \rightarrow y_0y_1\ldots y_k$ exists if and only if the corresponding edge of type $ty_0y_1\ldots y_{k-1} \rightarrow y_0y_1\ldots y_k$ does *not* exist in the network. Therefore, there are always exactly d incoming edges to v in this case.

2.3 Node Addition to an Existing Network

In this section we consider the changes in the logical topology that should occur when a new node is added to the network. We show that at most $O(d)$ edge changes in **G** would suffice when a new node is added to the network. When a multistar implementation is considered, this means $O(d)$ retuning of transmitters and receivers, whereas for a wavelength routed network, this means redefinition of $O(d)$ lightpaths. In contrast, for other proposed topologies [8], [9] the number of edge modifications needed was $O(nd)$. As discussed in the previous section, the nodes are assigned the smallest strings defined earlier. Addition of a new node u implies that we will assign the smallest unused string to the newly added node. Let the string be $x_0x_1\ldots x_k \in S_j$. We consider the following three cases:

i) $1 < j \le k$: For every v given by $x_1x_2\ldots x_kt$, $0 \le t \le d-1$, $v \in S_{j-1}$. Therefore v is an used string and we have to add a new edge $u \rightarrow v$ to the network. The node given by $w_0 = 0x_0x_1\ldots x_{k-1}$ is guaranteed to be an used string, since $w_0 \in S_0$ and we have to add a new edge $w_0 \rightarrow u$ to the network. If $x_k = d-1$, we have to delete the edge from w_0 to its 0-1-image at this time. For every w given by $tx_0x_1\ldots x_{k-1}$, $1 \le t \le d-1$, $w \in S_{j+1}$ and is an unused string. Therefore w_0 is the only predecessor of u.

ii) $j = k+1$: If $v = x_1x_2\ldots x_kt$, $0 \le t \le p-1$ is an used string, we add a new edge $u \rightarrow v$ to the network. We note that $x_1x_2\ldots x_k0 \in S_k$ is an used string. Therefore, there is at least one v such that $u \rightarrow v$ exists. Similarly, if $w = tx_0x_1\ldots x_{k-1}$, $0 \le t \le p-1$ is an used string, we add a new edge $w \rightarrow u$ to the network. We note that $w_0 = 0x_0x_1\ldots x_{k-1} \in S_0$ is an used string. Therefore, there is at least one w such that $w \rightarrow u$ exists. If $x_k = d-1$, we delete the edge from w_0 to its 0-1-image at this time.

iii) $j = 1$: Let $w_c = 0x_0x_2\ldots x_k$ be the 0-1-image of u. Before inserting u, the node $0x_0x_2\ldots x_k$ was connected to all nodes $v = 0x_2\ldots x_kt$, $0 \le t \le d-1$ (case iii in our topology given in 2.1). We have to

- delete the edge $w_c \rightarrow v$ for each node $v = 0x_2\ldots x_kt$ in the network.
- add an edge $u \rightarrow v$ for each node $v = 0x_2\ldots x_kt$ in the network.
- add a new edge $w_0 = 0x_0x_1\ldots x_{k-1} \rightarrow u$ to the network

- If $w_c \neq w_0$, add an edge $w_c \rightarrow u$ to the network.
- If $x_k = d-1$, and $w_0 \neq 0x_0000...0$ delete the edge from w_0 to its 0-1-image.

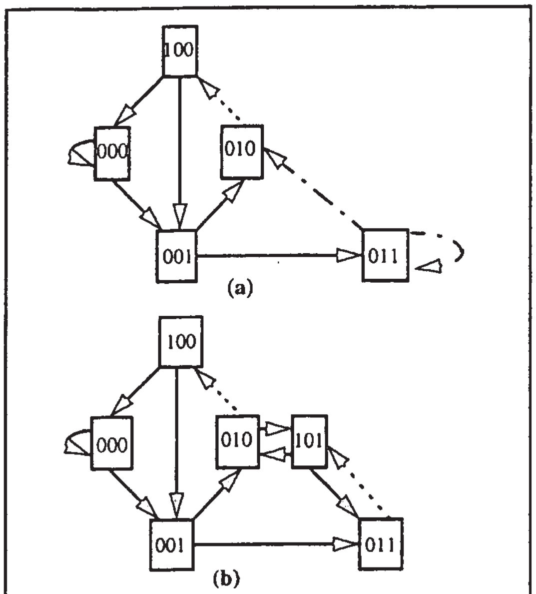

Figure 2: Expanding a topology with $d = 2$, $k = 2$ from (a) $n = 5$ to (b) $n = 6$ nodes.

Figure 2(a) shows again the network with 5 nodes given in Figure 1. We choose the smallest unused string $u = 101$ to represent the new node being inserted. The node u will have outgoing edges (shown by solid lines) to all nodes of the form 01j, to nodes 010 and 011. The 0-1 image of u is node 011. Hence all edges from 011 to nodes 010 and 011 are deleted an a new edge from 101 to 011 is inserted (shown by a dashed line). Also a new edge is inserted from node 010 to 101. The final network is shown in Figure 2(b)

3. Routing strategy

In this section, we present two routing schemes in the proposed topology from any source node S to any destination node D. Let S be given by the string $x_0x_1...x_k \in S_j$ and D be given by the string $y_0y_1...y_k \in S_l$.

3.1 Routing scheme

Let l be the length of the longest suffix of the string $x_0x_1...x_k$ that is also a prefix of $y_0y_1...y_k$ and let $\sigma(S, D)$ denote the string $x_0x_1...x_ky_ly_{l+1}y_{l+2}...y_k$ of length 2(k+1)-l. Since $\sigma(S, D)$ is of length $2(k+1)-l$, it has $(k+1)-l+1$ substrings, each of length $(k+1)$. Two of these substrings represent S and D. Since S and D are nodes in the network, these two substrings are used strings. If all the remaining $k-l$ substrings of $\sigma(S, D)$ having length $k+1$ are also used strings, then a routing path from S to D of length $k+1-l$ exists as given by the sequence of nodes given in (1) below.

$$S = x_0x_1...x_k \rightarrow x_1x_2...x_ky_l \rightarrow x_2...x_{2k-1}x_ky_ly_{l+1} \rightarrow$$

$$... \rightarrow x_ky_l...y_{k-2}y_{k-1} \rightarrow y_0y_1...y_k = D \quad (1)$$

In other words, if all the $k - l + 2$ substrings of $\sigma(S, D)$ are used strings, we can use $\sigma(S, D)$ to represent the path from S to D in (1).

Property 5: If all the $k - l + 2$ substrings of $\sigma(S, D)$ are used strings, $\sigma(S, D)$ represents the shortest path from S to D.

However, if some of the substrings of $\sigma(S, D)$ are not used strings, then some of the corresponding nodes do not currently appear in the network and hence this path does not exist. We note that any two consecutive strings in $\sigma(S, D)$ is given by $\alpha\beta$, where $\alpha = x_ix_{i+1}...x_ky_ly_{l+1}...y_{l+i}$, $0 \leq i \leq k-l-1$,and $\beta = x_{i+1}x_{i+2}...x_ky_ly_{l+1}...y_{l+i}y_{l+i+1}$. Let β be the first unused string in (1). According to our topology, either $\alpha \in S_0$ or $\alpha \in S_{k+1}$.

Property 6: If $\alpha \in S_0$ and $\gamma = x_{i+1}0x_{i+2}...x_ky_ly_{l+1}...y_{l+i}$, the 0-1-image of α is an used string, then

- $\sigma(S, \alpha)$ represents a path from S to α of length i,
- there exists a path $\alpha \rightarrow \gamma \rightarrow \delta = 0x_{i+2}...x_ky_ly_{l+1}...y_{l+i}y_{l+i+1}$
- $\sigma(\delta, D)$ is a string of length $k+2-l$-i

Property 7: If $\alpha \in S_0$ and $\gamma = x_{i+1}0x_{i+2}...x_ky_ly_{l+1}...y_{l+i}$ the 0-1-image of α is an unused string, then

- $\sigma(S, \alpha)$ represents a path from S to α of length i,
- there exists a path

$\alpha \rightarrow \delta = 0x_{i+2} \ldots x_k y_l y_{l+1} \ldots y_{l+i} y_{l+i+1}$

- $\sigma(\delta, D)$ is a string of length $k+2-l-i$

Properties 6 and 7 follow directly from our topology defined in 2.1.

Property 8: If a network contains all nodes in $S_0, S_1, \ldots, S_k$ then

- there exists an edge $S \rightarrow \gamma = x_1 x_2 \ldots x_k 0$ and
- $\sigma(\gamma, D)$ represents a path from α to D of length that cannot exceed k+1.

Proof of Property 8: Since the network contains all nodes in $S_0, S_1, \ldots, S_k$, $\gamma \in S_j$ for some j, $j \le k$ and must exist. Our topology (section 2.1) ensures that the edge $S \rightarrow \gamma$ exists. The path given below consists only strings belonging to groups S_i, $0 \le i \le k$ and hence are used strings:
$\gamma \rightarrow x_2 \ldots x_k 0 y_0 \rightarrow x_3 \ldots x_k 0 y_0 \rightarrow \ldots \rightarrow y_0 y_1 \ldots y_k$. The number of edges in the path is $k+1$, hence the proof.

Theorem 3: The diameter of a network using the proposed topology cannot exceed $2(k+1)$.

Proof: We consider any source-destination pair (S, D). If all the $k - l + 2$ substrings of $\sigma(S, D)$ are used strings, $\sigma(S, D)$ represents the shortest path from S to D and cannot exceed k+1. If β is the first unused string in (1), and α is the preceding string then we have to consider two cases:

Case 1) $\alpha \in S_0$: In this situation we can apply property 6 if 0-1-image of α is an used string. Otherwise we can use property 7. If we can use property 6, it means we need two edges to insert the digit y_{l+i+1}. Alternatively, if we can use property 7, it means we need one edge to insert the digit y_{l+i+1}.

Case 2) $\alpha \in S_{k+1}$: In this situation we discard the partial path from S to α. The first edge in our new path will be $S = x_0 x_1 \ldots x_k \rightarrow x_1 x_2 \ldots x_k 0$. Property 8 guarantees that once we have this situation, we can always start all over again inserting digits $y_0, y_1, \ldots, y_k$ without ever encountering an unused string and requires a maximum of $k+1$ edges. This represents the worst case since there may exist a shorter path by finding the longest suffix of $x_1 x_2 \ldots x_k 0$ that matches the corresponding prefix of D. In this case the path cannot exceed $k + 2$.

Case 1 can appear repeatedly. The worst situation is when we have to apply it to insert every digit of D. In other words, the path in this case can be as long as $2(k+1)$.

3.2 Example of routing

Let us consider the network of Figure 2(b). Suppose, S = 011 and D = 001. Since the only outgoing edge from 011 is to its 0-1-image 101, the first edge in the path is $011 \rightarrow 101$. From 101, we shift in the successive digits of the destination. So, the final path is given by $S = 011 \rightarrow 101 \rightarrow 010 \rightarrow 100 \rightarrow 001 = D$. In this particular example, there are no nodes belonging to group $k+1$. So, case 2 is not used.

4. Experiments to determine the average hop distance

We carried out some experiments to determine the average hop distance $\bar{h}$. In each of these experiments, we have started with a given value of d, the minimum indegree (or outdegree) and a specified value of an integer k. The network with d^k nodes is identical to that given in [8]. We have calculated the average hop distance $\bar{h}$ of this network from the hop distances of every source/destinations pairs using the routing scheme described in the previous section. Then we have added a node to the network and calculated $\bar{h}$ for the new network in the same way. We continued the process of adding nodes until the network contained d^{k+1} nodes. The results of the experiments are shown in Table 1 and reveal the following:

- The average hop distance is approximately k+1.
- The average hop distance starts at approximately k and increases to approximately k+1 as we start adding nodes to the network.

We interpret these results as follows. Even though the diameter is 2(k+1), the number of lightpaths through paths involving 0-1 images, which increase the number of hops, is relatively small. Our network is identical to that in [2] when the number of nodes in the network is d^k or d^{k+1} and, for these values, it is known that the network has a diameter of k and k+1 respectively.

Table 1: Variation of average hop distance with number of nodes

Number of nodes	d	k	average hop $\bar{h}$
10	3	2	2.4333
13	3	2	2.6154
16	3	2	2.6618
19	3	2	2.4954
22	3	2	2.5974
25	3	2	2.5148
10	2	3	2.7000
12	2	3	2.9470
14	2	3	2.8022
16	2	3	2.8333
65	4	3	3.5954
75	4	3	3.8366
85	4	3	4.1077
95	4	3	4.2215
105	4	3	4.5172
115	4	3	4.5506
18	2	4	3.5915
20	2	4	3.67630
22	2	4	3.8636
24	2	4	4.30181
26	2	4	3.7908
28	2	4	3.7169

5. Conclusions

In this paper we have introduced a new graph as a logical network for multihop networks. We have shown that our network has an attractive average hop distance compared to existing networks. The main advantage of our approach is the fact that we can very easily add new nodes to the network. This means that the perturbation of the network in terms of redefining edges in the network is very small in our architecture. The routing scheme in our network is very simple and avoids the use of routing tables.

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A flexible architecture for multihop optical networks

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Number of Pages: xxii+929
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Abstract:
It is desirable to have low diameter logical topologies for multihop lightwave network Researchers have investigated regular topologies for such networks. Only a few of th (e.g., GEMNET) are scalable to allow the addition of new nodes to an existing **netwo** **Adding new nodes** to such **networks** requires a major change in routing scheme. example, in a multistar implementation a large number of retuning of transmitters a receivers anti/or renumbering nodes are needed for GEMNET. We present a scalable logical topology which is not regular but it has a low diameter. This topology is interesting since it allows the network to be expanded indefinitely and new nodes ca added with a relatively small change to the network. We present the new topology, a algorithm to add nodes to the network and two routing schemes

Index Terms:
network topology optical fibre networks optical receivers optical transmitters telecommunica network routing wavelength division multiplexing GEMNET WDM algorithm flexible architecture low diameter logical topologies multihop lightwave networks multihop optical networks multistar implementation network nodes receivers regular topologies retuning routing scheme scalable logical topology transmitters

Documents that cite this document
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PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

IN RE APPLICATION OF: FRED B. HOLT *ET AL.*	EXAMINER: BRADLEY E. EDELMAN
APPLICATION NO.: 09/629,570	ART UNIT: 2153
FILED: JULY 31, 2000	CONF. NO: 5411
FOR: **JOINING A BROADCAST CHANNEL**	

Amendment Under 37 C.F.R. § 1.111

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Technology Center 2100

Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450
Sir:

The present communication responds to the Office Action dated January 12, 2004 in the above-identified application. Please extend the period of time for response to the Office Action by one month to expire on May 12, 2004. Enclosed is a Petition for Extension of Time and the corresponding fee. Please amend the application as follows:

Amendments to the Specification begin on page 2.

Amendments to the Claims are reflected in the listing of claims beginning on page 4.

Remarks/Arguments begin on page 8.

Amendments to the Specification:

In accordance with 37 CFR 1.72(b), an abstract of the disclosure has been included below. In addition, the status of the related cases listed on page 1 of the specification has been updated.

Therefore, please add the Abstract as shown below:

A technique for adding a participant to a network is provided. This technique allows for the simultaneous sharing of information among many participants in a network without the placement of a high overhead on the underlying communication network. To connect to the broadcast channel, a seeking computer first locates a computer that is fully connected to the broadcast channel. The seeking computer then establishes a connection with a number of the computers that are already connected to the broadcast channel. The technique for adding a participant to a network includes identifying a pair of participants that are connected to the network, disconnecting the participants of the identified pair from each other, and connecting each participant of the identified pair of participants to the added participant.

Please amend the "Cross-Reference to Related Applications" to read as follows:

This application is related to U.S. Patent Application No. 09/629,576, entitled "BROADCASTING NETWORK," filed on July 31, 2000 (Attorney Docket No. 030048001 US); U.S. Patent Application No. 09/629,570, entitled "JOINING A BROADCAST CHANNEL," filed on July 31, 2000 (Attorney Docket No. 030048002 US); U.S. Patent Application No. 09/629,577, "LEAVING A BROADCAST CHANNEL," filed on July 31, 2000 (Attorney Docket No. 030048003 US); U.S. Patent Application No. 09/629,575, entitled "BROADCASTING ON A BROADCAST CHANNEL," filed on July 31, 2000 (Attorney Docket No. 030048004 US); U.S. Patent Application No. 09/629,572, entitled "CONTACTING A BROADCAST CHANNEL," filed on July 31, 2000 (Attorney Docket No. 030048005 US);

U.S. Patent Application No. 09/629,023, entitled "DISTRIBUTED AUCTION SYSTEM," filed on July 31, 2000 (Attorney Docket No. 030048006 US); U.S. Patent Application No. 09/629,043, entitled "AN INFORMATION DELIVERY SERVICE," filed on July 31, 2000 (Attorney Docket No. 030048007 US); U.S. Patent Application No. 09/629,024, entitled "DISTRIBUTED CONFERENCING SYSTEM," filed on July 31, 2000 (Attorney Docket No. 030048008 US); and U.S. Patent Application No. 09/629,042, entitled "DISTRIBUTED GAME ENVIRONMENT," filed on July 31, 2000 (Attorney Docket No. 030048009 US), the disclosures of which are incorporated herein by reference.

Amendments to the Claims:

Following is a complete listing of the claims pending in the application, as amended:

1. (Currently amended) A computer-based, non-routing table based, non-switch based method for adding a participant to a network of participants, each participant being connected to three or more other participants, the method comprising:

identifying a pair of participants of the network that are connected wherein a seeking participant contacts a fully connected portal computer, which in turn sends an edge connection request to a number of randomly selected neighboring participants to which the seeking participant is to connect;

disconnecting the participants of the identified pair from each other; and

connecting each participant of the identified pair of participants to ~~the added~~ the seeking participant.

2. (Original) The method of claim 1 wherein each participant is connected to 4 participants.

3. (Original) The method of claim 1 wherein the identifying of a pair includes randomly selecting a pair of participants that are connected.

4. (Original) The method of claim 3 wherein the randomly selecting of a pair includes sending a message through the network on a randomly selected path.

5. (Original) The method of claim 4 wherein when a participant receives the message, the participant sends the message to a randomly selected participant to which it is connected.

6. (Currently amended) The method of claim 4 wherein the randomly selected path is ~~approximately~~ proportional to the diameter of the network.

7. (Original) The method of claim 1 wherein the participant to be added requests a portal computer to initiate the identifying of the pair of participants.

8. (Original) The method of claim 7 wherein the initiating of the identifying of the pair of participants includes the portal computer sending a message to a connected participant requesting an edge connection.

9. (Currently amended) The method of claim 8 wherein the portal computer indicates that the message is to travel a ~~certain~~ distance proportional to the diameter of the network and wherein the participant that receives the message after the message has traveled that ~~certain~~ distance is one of the participants of the identified pair of participants.

10. (Currently amended) The method of claim 9 wherein the certain distance is ~~approximately~~ twice the diameter of the network.

11. (Original) The method of claim 1 wherein the participants are connected via the Internet.

12. (Original) The method of claim 1 wherein the participants are connected via TCP/IP connections.

13. (Original) The method of claim 1 wherein the participants are computer processes.

14. (Currently amended) A computer-based, non-switch based method for adding nodes to a graph that is m-regular and m-connected to maintain the graph as m-regular, where m is four or greater, the method comprising:

identifying p pairs of nodes of the graph that are connected, where p is one half of m, wherein a seeking node contacts a fully connected portal node, which in turn sends an edge connection request to a number of randomly selected neighboring nodes to which the seeking node is to connect;

disconnecting the nodes of each identified pair from each other; and

connecting each node of the identified pairs of nodes to ~~the added~~ the seeking node.

15. (Original) The method of claim 14 wherein identifying of the p pairs of nodes includes randomly selecting a pair of connected nodes.

16. (Original) The method of claim 14 wherein the nodes are computers and the connections are point-to-point communications connections.

17. (Original) The method of claim 14 wherein m is even.

18–31. (Previously cancelled)

32. (Currently amended) A computer-readable medium containing instructions for controlling a computer system to connect a participant to a network of participants, each participant being connected to three or more other participants, the network representing a broadcast channel wherein each participant forwards broadcast messages that it receives to all of its neighbor participants, wherein each participant connected to the broadcast channel receives all messages that are broadcast on the network, the network containing a method wherein messages are numbered sequentially so that messages received out of order are queued and rearranged to be in order, by a method comprising:

identifying a pair of participants of the network that are connected;

disconnecting the participants of the identified pair from each other; and

connecting each participant of the identified pair of participants to ~~the added~~ a seeking participant.

33. (Original) The computer-readable medium of claim 32 wherein each participant is connected to 4 participants.

34. (Original) The computer-readable medium of claim 32 wherein the identifying of a pair includes randomly selecting a pair of participants that are connected.

35. (Original) The computer-readable medium of claim 34 wherein the randomly selecting of a pair includes sending a message through the network on a randomly selected path.

36. (Original) The computer-readable medium of claim 35 wherein when a participant receives the message, the participant sends the message to a randomly selected participant to which it is connected.

37. (Currently amended) The computer-readable medium of claim 35 wherein the randomly selected path is ~~approximately~~ twice a diameter of the network.

38. (Original) The computer-readable medium of claim 32 wherein the participant to be added requests a portal computer to initiate the identifying of the pair of participants.

39. (Original) The computer-readable medium of claim 38 wherein the initiating of the identifying of the pair of participants includes the portal computer sending a message to a connected participant requesting an edge connection.

40. (Currently amended) The computer-readable medium of claim 38 wherein the portal computer indicates that the message is to travel a ~~certain~~ distance <u>that is twice the diameter of the network</u> and wherein the participant that receives the message after the message has traveled that ~~certain~~ distance is one of the identified pair of participants.

41–49. (Previously cancelled)

PTO/SB/21 (05-03)
Approved for use through 04/30/2004. OMB 0651-00321
U.S. Patent and Trademark Office; U.S. DEPARTMENT OF COMMERCE

TRANSMITTAL FORM (*to be used for all correspondence after initial filing*)		
	Application Number	09/629,570
	Filing Date	July 31, 2000
	First Named Inventor	Fred B. Holt
	Art Unit	2153
	Examiner Name	Bradley E. Edelman
Total Number of Pages in This Submission 26	Attorney Docket Number	030048002US

ENCLOSURES (*Check all that apply*)

- [x] Fee Transmittal Form
  - [x] Fee Attached
- [x] Amendment/Reply
  - [ ] After Final
  - [ ] Affidavits/declaration(s)
- [x] Petition for Extension of Time
- [ ] Express Abandonment Request
- [ ] Information Disclosure Statement
- [ ] Certified Copy of Priority Document(s)
- [ ] Response to Missing Parts/ Incomplete Application
  - [ ] Response to Missing Parts under 37 CFR 1.52 or 1.53
- [ ] Drawing(s)
- [ ] Licensing-related Papers
- [ ] Petition
- [ ] Petition to Convert to a Provisional Application
- [ ] Power of Attorney, Revocation Change of Correspondence Address
- [ ] Terminal Disclaimer
- [ ] Request for Refund
- [ ] CD, Number of CD(s) _____
- [ ] After Allowance communication to Group
- [ ] Appeal Communication to Board of Appeals and Interferences
- [ ] Appeal Communication to Group **(Appeal Notice, Brief, Reply Brief)**
- [ ] Proprietary Information
- [ ] Status Letter
- [x] Other Enclosure(s) (please Identify below): Return Postcard

Remarks

SIGNATURE OF APPLICANT, ATTORNEY, OR AGENT

Firm or Individual name	Chun Ng
Signature	[signature]
Date	May 10, 2004

CERTIFICATE OF TRANSMISSION/MAILING

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FEE TRANSMITTAL for FY 2004

Effective 10/01/2003. Patent fees are subject to annual revision.

☐ Applicant claims small entity status. See 37 CFR 1.27

TOTAL AMOUNT OF PAYMENT ($)110

Complete if Known	
Express Mail No.	EV335515821US
Application Number	09/629,570
Filing Date	July 31, 2000
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Examiner Name	Bradley E. Edelman
Art Unit	2153
Attorney Docket No.	030048002US

METHOD OF PAYMENT *(check all that apply)*

☒ Check ☐ Credit card ☐ Money Order ☐ Other ☐ None

☒ Deposit Account:

Deposit Account Number: 50-0665

Deposit Account Name: Perkins Coie LLP

The Commissioner is authorized to: *(check all that apply)*

☐ Charge fee(s) indicated below ☒ Credit any overpayments

☒ Charge any additional fee(s) during the pendency of this application

☐ Charge fee(s) indicated below, **except for the filing fee** to the above-identified deposit account.

FEE CALCULATION

1. BASIC FILING FEE

Large Entity Fee Code	Large Entity Fee ($)	Small Entity Fee Code	Small Entity Fee ($)	Fee Description	Fee Paid
1001	770	2001	385	Utility filing fee	
1002	340	2002	170	Design filing fee	
1003	530	2003	265	Plant filing fee	
1004	770	2004	385	Reissue filing fee	
1205	160	2005	80	Provisional filing fee	

SUBTOTAL (1) ($) 0

2. EXTRA CLAIM FEES FOR UTILITY AND REISSUE

			Extra Claims		Fee from below		Fee Paid
Total Claims	23	-49** =	0	X		=	
Independent Claims	3	- 7** =	0	X		=	
Multiple Dependent						=	

Large Entity Fee Code	Large Entity Fee ($)	Small Entity Fee Code	Small Entity Fee ($)	Fee Description
1202	18	2202	9	Claims in excess of 20
1201	86	2201	43	Independent claims in excess of 3
1203	290	2203	145	Multiple dependent claim, if not paid
1204	86	2204	43	** Reissue independent claims over original patent
1205	18	2205	9	** Reissue claims in excess of 20 and over original patent

SUBTOTAL (2) ($) 0

***or number previously paid, if greater; For Reissues, see above*

FEE CALCULATION (continued)

3. ADDITIONAL FEES

Large Entity Fee Code	Large Entity Fee ($)	Small Entity Fee Code	Small Entity Fee ($)	Fee Description	Fee Paid
1051	130	2051	65	Surcharge – late filing fee or oath	
1052	50	2052	25	Surcharge – late provisional filing fee or cover sheet	
1053	130	1053	130	Non-English Specification	
1812	2,520	1812	2,520	For filing a request for *ex parte* reexamination	
1804	920*	1804	920*	Requesting publication of SIR prior to Examiner action	
1805	1,840*	1805	1,840*	Requesting publication of SIR after Examiner action	
1251	110	2251	55	Extension for reply within first month	110
1252	420	2252	210	Extension for reply within second month	
1253	950	2253	475	Extension for reply within third month	
1254	1,480	2254	740	Extension for reply within fourth month	
1255	2,010	2255	1,005	Extension for reply within fifth month	
1401	330	2401	165	Notice of Appeal	
1402	330	2402	165	Filing a brief in support of an appeal	
1403	290	2403	145	Request for oral hearing	
1451	1,510	1451	1,510	Petition to institute a public use proceeding	
1452	110	2452	55	Petition to revive - unavoidable	
1453	1,330	2453	665	Petition to revive - unintentional	
1501	1,330	2501	665	Utility issue fee (or reissue)	
1502	480	2502	240	Design issue fee	
1503	640	2503	320	Plant issue fee	
1460	130	1460	130	Petitions to the Commissioner	
1807	50	1807	50	Processing fee under 37 CFR 1.17(q)	
1806	180	1806	180	Submission of Information Disclosure Stmt	
8021	40	8021	40	Recording each patent assignment per property (times number of properties)	
1809	770	2809	385	Filing a submission after final rejection (37 CFR 1.129(a))	
1810	770	2810	385	For each additional invention to be examined (37 CFR 1.129(b))	
1801	770	2801	385	Request for Continued Examination (RCE)	
1802	900	1802	900	Request for expedited examination of a design application	

Other fee (specify) ______

*Reduced by Basic Filing Fee Paid **SUBTOTAL (3)** ($) 110

SUBMITTED BY				(Complete *(if applicable)*)	
Name (Print/Type)	Chun Ng	*Registration No. (Attorney/Agent)*	36,878	*Telephone*	206-359-6488
Signature				*Date*	05/10/2004

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[09901-0018/Fee Transmittal.doc]

REMARKS

Reconsideration and withdrawal of the rejections set forth in the Office Action dated January 12, 2004 are respectfully requested.

I. Rejections under 35 U.S.C. § 112, first paragraph

Claims 1, 14, and 32 have been amended to include sufficient antecedent basis. In claim 1, the phrase "the added participant", which appears in the last line of the claim, has been changed to "the seeking participant". In addition, "a seeking participant" precedes "the seeking participant" in an earlier line of claim 1, providing sufficient antecedent basis. In claim 32, the phrase "the added participant", which appears in the last line of the claim, has been changed to "a seeking participant". In claim 14, the phrase "the added node", which appears in the last line of the claim, has been changed to "the seeking node". In addition, "a seeking node" precedes "the seeking node" in an earlier line of claim 14, providing sufficient antecedent basis.

II. Rejections under 35 U.S.C. § 112, second paragraph

Claim 6 has been amended to render the claim definite. The term "approximately proportional" has been changed to "proportional". Claim 10 has also been amended to render the claim definite. The term "approximately twice the diameter" has been changed to "twice the diameter". Claim 37 has been amended to render the claim definite. The term "approximately twice a diameter of the network" has been changed to "twice a diameter of the network".

III. Rejections under 35 U.S.C. § 102

A. The Applied Art

U.S. Patent No. 6,603,742 B1 to Steele, Jr. et al. (*Steele, Jr. et al.*) is directed to a technique for reconfiguring networks while it remains operational. *Steele, Jr. et al.* discloses a method for adding nodes to a network with minimal recabling. Column 3, lines 2-5. An interim routing table is used to route traffic around the part of the network affected by the adding of a node. Column 11, lines 40-45. Each node in the network can connect to five other nodes. Column 4, lines 36-39, Column 4, lines 43-44. To add a node to a network, two links between two pairs of existing nodes are removed and five links are added to connect the new node to the network. Column 11, lines 25-31. For example, when upgrading from 7 to 8 nodes, the network administrator removes two links, 3-1 and 5-2, and adds five links, 7-1, 7-2, 7-3, 7-5, and 7-6. Column 12, lines 45-48.

B. Analysis

Distinctions between claim 1 and *Steele, Jr. et al.* will first be discussed, followed by distinctions between *Steele, Jr. et al.* and the remaining dependent claims.

As noted above, *Steele, Jr. et al.* discloses a technique for reconfiguring networks. Such a technique includes steps for disconnecting the participants of a pair from each other and connecting each participant to a seeking participant but does not include a step for identifying a pair of participants of the network that are fully connected. Column 12, lines 45-49. *Steele, Jr. et al.* fails to disclose a method for identifying a pair of participants of the network that are fully connected.

In contrast, claim 1 as amended includes the limitation of identifying a pair of participants of the network that are connected. For at least this reason, the applicant believes that claim 1 is patentable over *Steele, Jr. et al.*

The invention discloses an identification method in which a seeking participant contacts a fully connected portal computer. The portal computer directs the identification of a number of (for example four), randomly selected neighboring participants to which the seeking participant is to connect. *Steele, Jr. et al.* fails to disclose a portal computer that directs the identification of viable neighboring participants to which the seeking participant is to connect. Claim 1 has been amended to recite, among other limitations, the use of a portal computer for the identifying of "a number of selected neighboring participants to which the seeking participant is to connect." *Steele, Jr. et al.* fails to disclose such a method for identifying neighboring participants for a seeking participant to connect to. For at least this reason, claim 1 is patentable over *Steele, Jr. et al.*

Further, the claimed does not make use of routing tables. *Steele, Jr. et al.* fails to disclose a non-table based routing method. Claim 1 has been amended to recite, among other limitations, "a computer-based, non-routing table based, non-switch based method for adding a participant to a network of participants". For at least this reason, claim 1 is patentable over *Steele, Jr. et al.*

Claim 2 discloses a connection scheme where "each participant is connected to 4 participants". *Steele, Jr. et al.* fails to disclose a connection scheme in which each participant is connected to 4 participants. Instead, *Steele, Jr. et al.* discloses a connection scheme in which each participant is connected to 5 other participants. Column 7, lines 14-33. For at least this reason, claim 2 is patentable over *Steele, Jr. et al.*

Anticipation a claim under 35 U.S.C. § 102 requires that the cited reference must teach every element of the claim.[1] *Steele, Jr. et al.* fails to disclose every limitation recited in claim 1. Since claim 1 is allowable, based on at least the above reasons, the claims that depend on claim 1 are likewise allowable.

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[1] MPEP section 2131, p. 70 (Feb. 2003, Rev. 1). *See also, Ex parte Levy,* 17 U.S.P.Q.2d 1461, 1462 (Bd. Pat. App. & Interf. 1990) (to establish a *prima facie* case of anticipation, the Examiner must identify where "each and every facet of the claimed invention is disclosed in the applied reference."); *Glaverbel Société Anonyme v. Northlake Mktg. & Supply, Inc.*, 45 F.3d 1550, 1554 (Fed. Cir. 1995) (anticipation requires that each claim element must be identical to a corresponding element in the applied reference); *Atlas Powder Co. v. E.I. duPont De Nemours*, 750 F.2d 1569, 1574 (1984) (the failure to mention "a claimed element (in) a prior art reference is enough to negate anticipation by that reference").

IV. Rejections under 35 U.S.C. § 103, first paragraph

A. The Applied Art

A Flood Routing Method for Data Networks by Cho (*Cho*) is directed to a routing algorithm based on a flooding technique. *Cho* discloses a method in which flooding is used to find an optimal route to forward messages through. Flooding refers to a data broadcast technique that sends the duplicate of a packet to all neighboring nodes in a network. In *Cho,* flooding is not used to send the message, but is used to locate the optimal route for the message to be sent through. The method entails flooding a very short packet to explore an optimal route for the transmission of the message and to establish the data path via the selected route. Each node connected to the broadcast channel does not receive all messages that are broadcast on the broadcast channel. When a node receives a message, it does **not** forward that message to all of its neighboring nodes using flooding. In addition, *Cho* fails to disclose a method for rearranging a sequence of messages that are received out of order.

B. Analysis

As noted above, *Steele, Jr. et al.* discloses a method for adding nodes to a network with minimal recabling. *Steele, Jr. et al.* fails to disclose a method in which "each participant forwards broadcast messages that it receives to all of its neighbor participants". Claim 32 has been amended to clarify the language of previously pending claim 32. *Cho* discloses a method in which flooding is used to find an optimal route to forward messages through. *Cho* fails to disclose the use of flooding to forward messages. In *Cho*, flooding is used only to find an optimal route for data transmission and is not used to actually forward messages. *Cho* fails to disclose a system in which "each participant forwards broadcast messages that it receives to all of its neighbor participants". In *Cho*, each participant forwards messages only to a destination node once the optimal route has been selected. *Cho* fails to disclose a system in which "each participant connected to the broadcast channel receives all messages that are broadcast on the network". In addition, Cho fails to disclose a method for addressing a sequence of messages that are received out of order in which "messages are numbered sequentially so that messages received out of order are queued and rearranged to be in order".

As explained below, there is no incentive or teaching to combine *Steele, Jr. et al.* and *Cho*. However, even if they were combined, neither *Steele, Jr. et al.* nor *Cho* teach or suggest the use of flooding to send messages to all nodes connected to a broadcast channel. In addition, neither *Steele, Jr. et al.* nor *Cho* teach or suggest the sequential numbering of messages to rearrange a sequence of messages that are received out of order. The invention of claim 32 includes forwarding messages to all neighboring nodes and numbering each message sequentially so that "messages received out of order are queued and rearranged to be in order", which are not disclosed in either *Steele, Jr. et al.* or *Cho*. For at least this reason, the applicant believes that claim 32 is patentable over the combination of *Steele, Jr. et al.* and *Cho*.

The independent claims are allowable not only because they recite limitations not found in the references (even if combined), but for at least the following additional reasons. For example, there is no motivation to combine the various references as suggested in the Office Action. According to the Manual of Patent Examining Procedure ("MPEP") and controlling case law, the motivation to combine references cannot be based on mere common knowledge and common sense as to benefits that would result from such a combination, but instead must be based on specific teachings in the prior art, such as a specific suggestion in a prior art reference. For example, last year the Federal Circuit rejected an argument by the PTO's Board of Patent Appeals and Interferences that the ability to combine the teachings of two prior art references to produce beneficial results was sufficient motivation to combine them, and thus overturned the

Board's finding of obviousness because of the failure to provide a specific motivation in the prior art to combine the two references.[2] The MPEP provides similar instructions.[3]

Conversely, and in a manner similar to that rejected by the Federal Circuit, the present Office Action lacks any description of a motivation to combine the references. Thus, if the current rejection is maintained, the applicant's representative requests that the Examiner explain with the required specificity where a suggestion or motivation in the references for so combining the references may be found.[4]

Steele et al. deals with a method for adding nodes to a network while *Cho* deals with finding an optimal route to forward messages in a network. The addition of nodes to a network represents a completely separate process from the forwarding of messages in a network. *Steele et al.* contains no specific teachings that would suggest combining *Steele et al.* with *Cho*. In other words, *Steele et al.* contains no specific teachings that would suggest finding an optimal route to forward messages in a network.

One may not use the application as a blueprint to pick and choose teachings from various prior art references to construct the claimed invention ("impermissible hindsight reconstruction").[5] Assuming, for argument's sake, that it would be obvious to combine the teachings of *Steele et al.* with *Cho*, then *Steele et al.* would have done so because it would have

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[2] In re Sang-Su Lee, 277 F.3d 1338, 1341-1343 (Fed. Cir. 2002).

[3] Manual of Patent Examining Procedure, Section 2143 (noting that "the teaching or suggestion to make the claimed combination and the reasonable expectation of success must both be found in the prior art, not in applicant's disclosure," citing in re Vaeck, 947 F.2d 488 (Fed. Cir. 1991).

[4] See, MPEP Section 2144.03.

[5] See, *e.g.,* In re Gorman, 933 F.2d 982,987 (Fed. Cir. 1991), ("One cannot use hindsight construction to pick and choose between isolated disclosures in the prior art to deprecate the claimed invention.").

provided at least some of the advantages of the presently claimed invention. *Steele et al.'s* failure to employ the teachings cited in *Cho* is persuasive proof that the combination recited in claim 32 is unobvious. For at least this reason, the applicant believes that claim 32 is patentable over the combination of *Steele et al.* and *Cho*.

Claim 33 discloses a connection scheme where "each participant is connected to 4 participants". *Steele, Jr. et al.* fails to disclose a connection scheme in which each participant is connected to 4 participants. Instead, *Steele, Jr. et al.* discloses a connection scheme in which each participant is connected to 5 other participants. Column 7, lines 14-33. For at least this reason, claim 33 is patentable over *Steele, Jr. et al.*

Since claim 32 is allowable, based on at least the above reasons, the claims that depend on claim 32 are likewise allowable. Thus, for at least this reason, claim 33 is patentable over the combination of *Steele, Jr. et al.* and *Cho*.

V. Rejections under 35 U.S.C. § 103, second paragraph

A. The Applied Art

U.S. Patent No. 6,490,247 B1 to Gilbert et al. (*Gilbert et al.*) is directed to a ring-ordered, dynamically reconfigurable computer network utilizing an existing communications system. *Gilbert et al.* discloses a method for adding a node to a network using a switching mechanism in which the nodes are ordered in a ring-like configuration as opposed to a hypercube configuration. Column 3, lines 28-35. The first step in adding a seeking node to the network consists of the seeking contacting a portal node that is fully connected to the network. Column 6, lines 31-33. The portal node that is contacted provides information regarding a neighboring node that is adjacent to the seeking node; the selection of the neighboring node is not random. Column 6, lines 40-42. The seeking node then contacts the neighboring node to request a connection. Column 6, lines 57-59. The portal node provides the relevant information regarding the node that is adjacent to the neighboring node that is adjacent to the seeking node but does not request a connection.

U.S. Patent No. 6,553,020 B1 to Hughes et al. (*Hughes et al.*) is directed to a network for interconnecting nodes for communication across the network. *Hughes et al.* fails to disclose a system where a portal computer randomly selects four nodes to serve as neighboring nodes to the seeking node. *Hughes et al.* also fails to disclose a system in which the portal computer sends an edge connection request to the neighboring nodes.

B. Analysis

As noted above, *Gilbert et al.* discloses a method for adding a node to a network using a switching mechanism. *Gilbert et al.* fails to disclose a method in which a portal computer seeks "a number of randomly selected neighboring participants to which the seeking participant is to connect". In *Gilbert et al.*, the selection of the neighboring nodes is not random. Column 6, lines 40-49. Figure 6 of *Gilbert et al.* reveals that node 100 selects nodes 10 and 16; the selection of nodes 10 and 16 is not random since they are purposely adjacent to one another and since node 10 provides node 100 with information regarding the node adjacent to it, node 16. Column 6, lines 42-46. *Gilbert et al.* fails to disclose a method in which a portal computer "sends an edge connection request to a number of randomly selected neighboring participants to which the seeking participant is to connect". In *Gilbert et al.*, the seeking node, not the portal node, contacts the neighboring participants to which the seeking participant is to connect. Column 6, lines 57-61. *Gilbert et al.* fails to disclose a "non-switch based method for adding a participant to a network of participants". Column 3, lines 8-11. *Gilbert et al.* fails to disclose a method in which an additional node contacts "a number of randomly selected neighboring participants". Column 6, lines 30-32. *Hughes et al.* discloses a method in which an additional node contacts four neighboring participants. *Hughes et al.* fails to disclose a method in which a portal computer seeks "four randomly selected neighboring participants to which the seeking participant is to connect". *Hughes et al.* also fails to disclose a method in which a portal computer "sends an edge connection request to four randomly selected neighboring participants to which the seeking participant is to connect".

As explained below, *Gilbert et al* and *Hughes et al.* would not be combined. However, even if they were combined, neither *Gilbert et al* nor *Hughes et al.* teach or suggest the random selection of neighboring participants. Claim 1 has been amended to recite, among other limitations, a method in which a portal computer seeks "four randomly selected neighboring participants to which the seeking participant is to connect". In other words, the invention of claim 1 includes randomly selecting neighboring participants to which the seeking participant is to connect, which is not disclosed in either *Gilbert et al* or *Hughes et al.* Even if they were combined, neither *Gilbert et al* nor *Hughes et al.* teach or suggest the sending of an edge connection request by the portal computer to the randomly selected neighboring participants to which the seeking participant is to connect. Claim 1 has been amended to recite, among other limitations, a method in which a portal computer "sends an edge connection request to four randomly selected neighboring participants to which the seeking participant is to connect". In other words, the invention of claim 1 includes the portal computer sending an edge connection request to the randomly selected neighboring participants to which the seeking participant is to connect, which is not disclosed in either *Gilbert et al* or *Hughes et al.* For at least these reasons, the applicant believes that claim 1 is patentable over the combination of *Gilbert et al* and *Hughes et al.*

In a similar fashion, claim 14 has been amended to recite, among other limitations, a method in which a portal computer seeks "four randomly selected neighboring nodes to which the seeking node is to connect". In other words, the invention of claim 14 includes randomly selecting neighboring nodes to which the seeking node is to connect, which is not disclosed in either *Gilbert et al* or *Hughes et al.* Even if they were combined, neither *Gilbert et al* nor *Hughes et al.* teach or suggest the random selection of neighboring nodes. In addition, even if they were combined, neither *Gilbert et al* nor *Hughes et al.* teach or suggest the sending of an edge connection request by the portal computer to the randomly selected neighboring nodes to which the seeking node is to connect. Claim 14 has been amended to recite, among other limitations, a method in which a portal computer "sends an edge connection request to four randomly selected neighboring nodes to which the seeking node is to connect". In other words, the invention of claim 14 includes the portal computer sending an edge connection request to the randomly selected neighboring nodes to which the seeking node is to connect, which is not disclosed in either *Gilbert et al* or *Hughes et al.* For at least these reasons, the applicant believes that claim 14 is patentable over the combination of *Gilbert et al* and *Hughes et al.*

Since claim 1 is allowable, based on at least the above reasons, the claims that depend on claim 1 are likewise allowable. Thus, for at least this reason, claims 2-5, 7, 8, and 11-13 are patentable over the combination of *Gilbert et al* and *Hughes et al.* Since claim 14 is allowable, based on at least the above reasons, the claims that depend on claim 14 are likewise allowable. Thus, for at least this reason, claims 15-17 are patentable over the combination of *Gilbert et al* and *Hughes et al.*

If the current rejection is maintained, the applicant's representative requests that the Examiner explain with the required specificity where a suggestion or motivation in the references for so combining the references may be found.[6]

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[6] See, MPEP Section 2144.03.

Gilbert et al. deals with a method for adding nodes to a network while *Hughes et al.* deals with a network for interconnecting nodes for communication across the network. The addition of nodes to a network represents a completely separate process from the interconnection of nodes in a network. *Hughes et al.* contains no specific teachings that would suggest combining *Hughes et al.* with *Gilbert et al.* In other words, *Hughes et al.* contains no specific teachings that would suggest adding a node to a network.

As is known, one may not use the application as a blueprint to pick and choose teachings from various prior art references to construct the claimed invention ("impermissible hindsight reconstruction").[7] Assuming, for argument's sake, that it would be obvious to combine the teachings of *Hughes et al.* with *Gilbert et al.*, then *Hughes et al.* would have done so because it would have provided at least some of the advantages of the presently claimed invention. *Hughes et al.'s* failure to employ the teachings cited in *Gilbert et al.* is persuasive proof that the combination is unobvious. For at least this reason, the applicant believes that claims 1 and 14 are patentable over the combination of *Hughes et al.* and *Gilbert et al.*

Since claim 1 is allowable, based on at least the above reasons, the claims that depend on claim 1 are likewise allowable. Thus, for at least this reason, claims 2-5, 7, 8, and 11-13 are patentable over the combination of *Gilbert et al* and *Hughes et al.* Since claim 14 is allowable, based on at least the above reasons, the claims that depend on claim 14 are likewise allowable. Thus, for at least this reason, claims 15-17 are patentable over the combination of *Gilbert et al* and *Hughes et al.*

[7] See, e.g., In re Gorman, 933 F.2d 982,987 (Fed. Cir. 1991), ("One cannot use hindsight construction to pick and choose between isolated disclosures in the prior art to deprecate the claimed invention.").

VI. Rejections under 35 U.S.C. § 103, third paragraph

A. The Applied Art

A Flood Routing Method for Data Networks by Cho (*Cho*), U.S. Patent No. 6,490,247 B1 to Gilbert et al. (*Gilbert et al.*), and U.S. Patent No. 6,553,020 B1 to Hughes et al. (*Hughes et al.*) have already been disclosed in the above descriptions of the applied art.

B. Analysis

As noted previously, *Gilbert et al.* discloses a method for adding nodes to a network while *Hughest et al.* discloses a network for interconnecting nodes for communication across the network. The combination of *Gilbert et al.* and *Hughest et al.* fails to disclose a method in which "each participant forwards broadcast messages that it receives to all of its neighbor participants". *Cho* discloses a method in which flooding is used to find an optimal route to forward messages through. *Cho* fails to disclose the use of flooding to forward messages. In *Cho*, flooding is used only to find an optimal route for data transmission and is not used to actually forward messages. *Cho* fails to disclose a system in which "each participant forwards broadcast messages that it receives to all of its neighbor participants". In *Cho*, each participant forwards messages only to a destination node once the optimal route has been selected. *Cho* fails to disclose a system in which "each participant connected to the broadcast channel receives all messages that are broadcast on the network". In addition, Cho fails to disclose a method for addressing a sequence of messages that are received out of order in which "messages are numbered sequentially so that messages received out of order are queued and rearranged to be in order". Claim 32 has been amended to clarify the inherent language of previously pending claim 32. As explained below, *Gilbert et al, Hughes et al.,* and *Cho* would not be combined. However, even if they were combined, *Gilbert et al, Hughes et al.,* and *Cho* fail to teach or suggest the use of flooding to send messages to all nodes connected to a broadcast channel. In addition, G*ilbert et al, Hughes* *et al.*, and *Cho* fail to teach or suggest the sequential numbering of messages to rearrange a sequence of messages that are received out of order. The invention of claim 32 includes forwarding messages to all neighboring nodes and numbering each message sequentially so that "messages received out of order are queued and rearranged to be in order", which are not disclosed in G*ilbert et al, Hughes et al.*, or *Cho*. For at least these reasons, the applicant believes that claim 32 is patentable over the combination of G*ilbert et al, Hughes et al.*, and *Cho*.

Since claim 32 is allowable, based on at least the above reasons, the claims that depend on claim 32 are likewise allowable. Thus, for at least this reason, claims 33-36, 38, and 39 are patentable over the combination of G*ilbert et al, Hughes et al.*, and *Cho*.

Gilbert et al. deals with a method for adding nodes to a network, *Hughes et al.* deals with a network for interconnecting nodes for communication, and *Cho* deals with finding an optimal route to forward messages in a network. These three prior art references represent separate, distinct processes. The combination of *Gilbert et al.* and *Hughes et al.* contains no specific teachings that would suggest combining *Gilbert et al.* and *Hughes et al.* with *Cho*. In other words, the combination of *Gilbert et al.* and *Hughes et al.* contains no specific teachings that would suggest finding an optimal route to forward messages in a network.

Assuming, for argument's sake, that it would be obvious to combine the teachings of *Gilbert et al.* and *Hughes et al.* with *Cho*, then *Gilbert et al.* and *Hughes et al.* would have done so because it would have provided at least some of the advantages of the presently claimed invention. The failure of *Gilbert et al.* and *Hughes et al.* to employ the teachings cited in *Cho* is persuasive proof that the combination recited in claim 32 is unobvious. For at least this reason, the applicant believes that claim 32 is patentable over the combination of *Gilbert et al.* and *Hughes et al.* in view of *Cho*.

Since claim 32 is allowable, based on at least the above reasons, the claims that depend on claim 32 are likewise allowable. Thus, for at least this reason, claims 33-36, 38, and 39 are patentable over the combination of *Gilbert et al*, *Hughes et al.*, and *Cho*.

VII. Conclusion

In view of the foregoing, the claims pending in the application comply with the requirements of 35 U.S.C. § 112 and patentably define over the applied art. A Notice of Allowance is, therefore, respectfully requested. If the Examiner has any questions or believes a telephone conference would expedite prosecution of this application, the Examiner is encouraged to call the undersigned at (206) 359-6488.

Respectfully submitted,

Perkins Coie LLP

Date: 5/10/04

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PETITION FOR EXTENSION OF TIME UNDER 37 C.F.R. 1.136(a)	Docket Number (Optional) 030048002US

OIPE JC151 MAY 1 0 2004 PATENT & TRADEMARK OFFICE

In re Application of: Fred B. Holt	
Application Number: 09/629,570	Filed: 07/31/2000
For: JOINING A BROADCAST CHANNEL	
Group Art Unit: 2153	Examiner: Bradley E. Edelman

This is a request under the provisions of 37 CFR 1.136(a) to extend the period for filing a reply in the above identified application.

The requested extension and appropriate non-small-entity fee are as follows (check time period desired):

RECEIVED
MAY 1 7 2004
Technology Center 2100

☒	One month (37 CFR 1.17(a)(1))	$ 110
☐	Two months (37 CFR 1.17(a)(2))	$ 420
☐	Three months (37 CFR 1.17(a)(3))	$ 950
☐	Four months (37 CFR 1.17(a)(4))	$ 1,480
☐	Five months (37 CFR 1.17(a)(5))	$ 2,010

☐ Applicant claims small entity status. See 37 CFR 1.27. Therefore, the fee amount shown above is reduced by one-half, and the resulting fee is: $_____.

☒ A check in the amount of the fee is enclosed.

☐ Payment by credit card. Form PTO-2038 is attached.

☐ The Director has already been authorized to charge fees in this application to a Deposit Account.

☒ The Director is hereby authorized to charge any additional fees which may be required, or credit any overpayment, to Deposit Account No. 50-0665.
I have enclosed a duplicate copy of this sheet.

05/13/2004 [illegible] 00000141 09629570
01 FC:1251 110.00 [illegible]

I am the ☐ applicant/inventor

☐ assignee of record of the entire interest. See 37 CFR 3.71.
Statement under 37 CFR 3.73(b) is enclosed. (Form PTO/SB/96).

☐ attorney or agent of record. Registration number _____.

☒ attorney or agent under 37 CFR 1.34(a).
Registration number if acting under 37 CFR 1.34(a): 36,878.

WARNING: Information on this form may become public. Credit card information should not be included on this form. Provide credit card information and authorization on PTO-2038.

05/10/2004	[signature]
Date	Signature
206-359-6488	Chun Ng
Telephone Number	Typed or printed name

NOTE: Signatures of all the inventors or assignees of record of the entire interest or their representative(s) are required. Submit multiple forms if more than one signature is required, see below.

☒ Total of 1 forms is submitted.

[09901-0018/Petition for Extension of Time.doc]

PATENT APPLICATION FEE DETERMINATION RECORD
Effective December 29, 1999

Application or Docket Number: 09/629570

CLAIMS AS FILED - PART I

FOR	NUMBER FILED (Column 1)	NUMBER EXTRA (Column 2)	SMALL ENTITY TYPE RATE	SMALL ENTITY FEE	OR	OTHER THAN SMALL ENTITY RATE	OTHER THAN SMALL ENTITY FEE
BASIC FEE				345.00	OR		690.00
TOTAL CLAIMS	48 minus 20=	* 28	X$ 9=		OR	X$18=	504.00
INDEPENDENT CLAIMS	7 minus 3 =	* 4	X39=		OR	X78=	312.00
MULTIPLE DEPENDENT CLAIM PRESENT			+130=		OR	+260=	
			TOTAL		OR	TOTAL	1506.00

* If the difference in column 1 is less than zero, enter "0" in column 2

CLAIMS AS AMENDED - PART II

AMENDMENT A	CLAIMS REMAINING AFTER AMENDMENT (Column 1)		HIGHEST NUMBER PREVIOUSLY PAID FOR (Column 2)	PRESENT EXTRA (Column 3)	SMALL ENTITY RATE	ADDITIONAL FEE	OR	OTHER THAN SMALL ENTITY RATE	ADDITIONAL FEE
a									
Total	* 26	Minus	** 48	= —	X$ 9=		OR	X$18=	
Independent	* 3	Minus	*** 7	= —	X39=		OR	X78=	
FIRST PRESENTATION OF MULTIPLE DEPENDENT CLAIM					+130=		OR	+260=	
					TOTAL ADDIT. FEE		OR	TOTAL ADDIT. FEE	

B

AMENDMENT B	CLAIMS REMAINING AFTER AMENDMENT (Column 1)		HIGHEST NUMBER PREVIOUSLY PAID FOR (Column 2)	PRESENT EXTRA (Column 3)	SMALL ENTITY RATE	ADDITIONAL FEE	OR	OTHER THAN SMALL ENTITY RATE	ADDITIONAL FEE
Total	* 27	Minus	** 48	=	X$ 9=		OR	X$18=	
Independent	* 3	Minus	*** 7	=	X39=		OR	X78=	
FIRST PRESENTATION OF MULTIPLE DEPENDENT CLAIM					+130=		OR	+260=	
					TOTAL ADDIT. FEE		OR	TOTAL ADDIT. FEE	

AMENDMENT C	CLAIMS REMAINING AFTER AMENDMENT (Column 1)		HIGHEST NUMBER PREVIOUSLY PAID FOR (Column 2)	PRESENT EXTRA (Column 3)	SMALL ENTITY RATE	ADDITIONAL FEE	OR	OTHER THAN SMALL ENTITY RATE	ADDITIONAL FEE
Total	*	Minus	**	=	X$ 9=		OR	X$18=	
Independent	*	Minus	***	=	X39=		OR	X78=	
FIRST PRESENTATION OF MULTIPLE DEPENDENT CLAIM					+130=		OR	+260=	
					TOTAL ADDIT. FEE		OR	TOTAL ADDIT. FEE	

* If the entry in column 1 is less than the entry in column 2, write "0" in column 3.
** If the "Highest Number Previously Paid For" IN THIS SPACE is less than 20, enter "20."
***If the "Highest Number Previously Paid For" IN THIS SPACE is less than 3, enter "3."
The "Highest Number Previously Paid For" (Total or Independent) is the highest number found in the appropriate box in column 1.

FORM PTO-875
(Rev. 12/99)

Patent and Trademark Office, U.S. DEPARTMENT OF COMMERCE

Performance Analysis of Network Connective Probability of Multihop Network under Correlated Breakage

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Abstract—One of important properties of multihop network is the network connective probability which evaluate the connectivity of the network. The network connective probability is defined as the probability that when some nodes are broken, rest nodes connect each other. Multihop networks are classified to the regular network whose link assignment is regular and the random network whose link assignment is random. It has been shown that the network connective probability of regular network is larger than that of random network. However, all of these results is shown under independent node breakage. In this paper, we analyze the network connective probability of multihop networks under the correlated node breakage. It is shown that regular network has better performance of the network connective probability than random network under the independent breakage, on the other hand, random network has better performance than regular network under the correlated breakage.

1 Introduction

In recent years, multi-hop networks have been widely studied [1]-[8]. These networks must pass messages between source and destination nodes via intermediate links and nodes. Examples of them include ring, shuffle network (SN) [1],[2] and chordal network (CN)[3]. One of the very important performance measure of multi-hop network is the connectivity of the network. If some nodes are broken, it is needed for a network to guarantee the connection among non-broken nodes. Thus, the network connective probability defined as the probability that when some nodes are broken, rest links and nodes construct the connective network, should be a very important property to evaluate the connectivity of the network.

Multi-hop networks are classified to regular network and random network according to the way of link assignment. In the regular network, links are assigned regularly and examples of them include shufflenet and manhattan street network. On the other hand, in random network, link assignment is not regular but somewhat random and examples of them include connective semi-random network (CSRN) [6]. The network connective probabilities of some multi-hop networks have been analyzed and it has been shown that the network connective probability of regular network is larger than that of random network. However, all of them is analyzed under the condition that locations of broken nodes are independent each other. In the real network, there are some case that the locations of broken nodes have correlation, for example, links and nodes are broken in the same area under the case of disaster. Thus, it is significant and great of interest to analyze the network connective probability under the condition when the locations of broken nodes have correlations each other.

In this paper, we analyze the network connective probability of multi-hop network under the condition that locations of broken nodes have correlations each other, where we treat SN, CN and CSRN as the model for analysis. We realize the correlation as follows. At first, we note one node and break it and call this node the center broken node. And next, we note nodes whose links connect to the center broken nodes and break them at some probability. We define this probability as the correlated broken probability. Very interesting result is shown that under independent breakage of node, regular network has better performance of the network connective probability than random network, on the other hand, under the correlated breakage of node, random network has better performance than regular network.

In the section 2, we explain network model of SN, CN and CSRN which we analyze in the section 3. In the section 3, we analyze the network connective probability under the condition when the location of broken nodes have correlation each other. And we compare each of network connective probability in the section 4. In the last, we conclude our study.

2 Multihop network model

In this section, we explain the multihop network models used for analysis of the network connective probability. We treat three networks such as SN, CN and CSRN which consists of N nodes and p unidirected outgoing links per node.

Fig. 1 shows SN with 18 nodes and 2 outgoing links per node. To construct the SN, we arrange $N = kp^k$ ($k = 1, 2, \cdots$; $p = 1, 2, \cdots$) nodes in k columns of p^k nodes each. Moving from left to right, successive columns are connected by p^{k+1} outgoing links, arranged in a fixed shuffle pattern, with the last column connected to the first as if the entire graph were wrapped around a cylinder. Each of the p^k nodes in a column has p outgoing links directed to p different nodes in the next column. Numbering the nodes in a column from 0 to $p^k - 1$, nodes i has outgoing links directed to nodes $j, j+1, \cdots$, and $j+p-1$ in the next column, where $j = (i \bmod p^{k-1})p$. In Fig. 1, p is equal to 2 and k is equal to 2. Since the link assignmentof SN is regular, SN is regular network.

Fig. 2 shows CN with 16 nodes and 2 outgoing links per node. To construct CN, at first, we construct unidirected ring network with N nodes and N unidirected links. And $p-1$ unidirected links are added from each node. Numbering nodes along ring network from 0 to $N-1$, node i has outgoing links directed to nodes $(i+1) \bmod N, (i+r_1) \bmod N, \cdots$, and $(i+r_{p-1}) \bmod N$, where r_j ($j = 1, 2, \cdots, p-1$) is defined as the chordal length. In Fig. 2, r_1 is equal to 3. Since r_i for every i are independent each other, CN is not regular network. However, CN has much regular elements such a symmetrical pattern of network.

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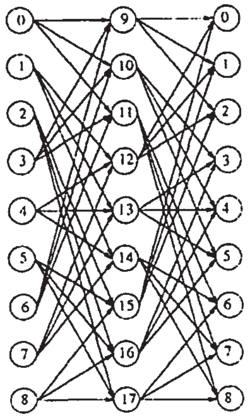


Figure 1. Shuffle network with $N = 18$ and $p = 2$.

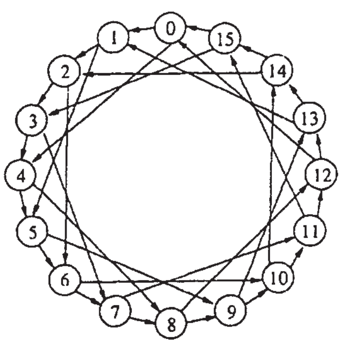


Figure 2. Chordal network with $N = 16$, $p = 2$ and $r_1 = 3$.

Fig. 3 shows CSRN with 16 nodes and 2 outgoing links from a node. Similarly with CN, CSRN includes unidirected ring network with N nodes and N unidirected links. And we add $p - 1$ links from each node whose directed nodes are randomly selected. In CSRN, the number of incoming links per node is not constant, for example, in Fig. 3, the number of incoming links into node 1 is 1 and the one into node 3 is 3. The link assignment of CSRN is random except for the part of ring network, thus CSRN is random network. It has been shown that since the number of incoming links per node is not constant, the network connective probability of CSRN is smaller than those of SN and CN when locations of broken nodes are independent each other. And that of SN is the same as that of CN, because the network connective probability depends on the number of incoming links come into every nodes.

3 Performance Analysis

Here, we analyze the network connective probability of SN, CN and CSRN under the condition that locations of broken nodes have correlation each other. Now, we explain the network connective probability in detail using Fig. 3. This figure shows the connective network which is defined as the network in which all nodes connect to every other nodes directly or indirectly. At first, we consider the case that the node 1 is broken. The node 1 has two outgoing links directed to nodes 2 and 3, and if the node 1 is broken, we can not use them. However, node 2 has two incoming links from nodes 1 and 14, and node 3 has three incoming links from nodes 1, 2 and 11. Therefore, even if node 1 is broken, rest nodes can construct

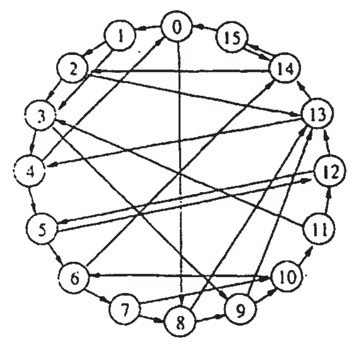


Figure 3. Connective semi-random network with $N = 16$ and $p = 2$.

the connective network. Next, we consider the case that node 0 is broken. The node 0 has two outgoing links directed to nodes 1 and 8, and if the node 0 is broken, we can not use them. Since node 1 has only one incoming link from node 0, even if only node 0 is broken, rest nodes can not connect to node 1, that is, they can not construct the connective network. Here, we define the network connective probability as the probability that when some nodes and links are broken, the rest nodes and links can construct the connective network.

Now, we explain the correlated node breakage using Fig. 3. At first, we note one node and break it, where this node is called as the center broken node. And then, we note nodes whose outgoing links come into the center broken node or whose incoming links go out of the center broken node, and break them at a probability defined as the correlated broken probability. In Fig 3, when we assume that the center broken node is the node 3, there are five nodes 1, 2, 4, 9 and 11 which have possibility to become correlated broken node. And they become the broken nodes at the correlated broken probability. It is obvious that none of them is broken when the correlated broken probability is 0 and all of them is broken when the correlated broken probability is 1.

In our study, we analyze the network connective probability that only nodes are broken. And we assume that the number of center broken node is one in the analysis. We denote the correlated broken probability by a and the network connective probability of SN, CN and CSRN by P_{SN}, P_{CN} and P_{CSRN}, respectively.

3.1 Shuffle Network

Because the number of incoming links per node in SN is the constant p, when broken node is only center broken node, the rest nodes can construct the connective network. There are $2p$ nodes have the possibility to become the correlated broken node. All of p nodes which have outgoing link come into the center broken node have the outgoing links directed to the same nodes. For example, in Fig. 1, if we assume that the node 9 is the center broken node, the nodes 0, 3 and 6 has outgoing links to node 9. And each of three nodes have two outgoing links directed to nodes 10 and 11. Therefore, only when all of them are broken, the rest nodes can not construct the connective network. On the other hand, all of outgoing links go out from p nodes which have incoming link from center broken node direct to different nodes. In Fig. 1, nodes 0, 1 and 2 have the incoming link from center broken node 9. And all of the outgoing links from their nodes direct to different nodes, thus even if all of them are broken, the rest nodes can construct the connective network. Thus, the network connective probability of SN is the probability that all of nodes whose outgoing links come into the center broken node are broken, and it is derived as

$$P_{SN} = 1 - a^p \,. \tag{1}$$

3.2 Chordal Network

The network connective probability of CN with $p = 2$ is different from that with $p \geq 3$. At first, we consider the case with $p = 2$. When p is equal to 2, all of the outgoing links, from the nodes whose incoming links go out from the center broken node, direct to the same node. For example, in Fig. 2, when we assume that the center broken node is node 0, the outgoing links from it direct to nodes 1 and 4. And each of outgoing links from them directs to node 5. Therefore, only when all nodes whose incoming links go out from the center broken node are broken, the rest nodes can not construct the connective network. And we can obtain the network connective probability as

$$P_{CN} = 1 - a^2 \qquad \text{for } p = 2\,. \tag{2}$$

And next, we consider the case that $p \geq 3$. In CN, when p is equal to or larger than three and each chordal length is selected properly, all of outgoing links from the nodes whose incoming links go out from the center broken node do not direct to the same nodes. And therefore, even if all of nodes which connect to the center broken nodes with incoming or outgoing links is broken, the rest nodes can construct the connective network, that is,

$$P_{CN} = 1 \qquad \text{for } p \geq 3\,. \tag{3}$$

3.3 Connective Semi-Random Network

In CSRN, the number of the incoming links per node is not constant. Since the maximum number of incoming links is $N - 1$ and one link come into a node at least, the probability that the number of the incoming links come into a node is i, denoted as A_i, is

$$A_i = \begin{cases} 0, & \text{for } i = 0 \\ \binom{N-2}{i-1}(\frac{p}{N-2})^{i-1}(1 - \frac{p}{N-2})^{N-1-i} & \text{for } i \geq 1. \end{cases} \tag{4}$$

The nodes which have possibility to become the correlated broken nodes are those which connect to the center broken node by outgoing link or incoming link. When the number of the incoming link come into the center broken node is i, the sum of outgoing links and incoming links it have is $p + i$. However, the number of the nodes which have possibility to become the correlated broken nodes is not always $p + i$, because the p outgoing links have the possibility to overlap with one of i incoming links. For example, in Fig. 3, when the center broken nodes is node 5, the outgoing link to node 12 overlap with the incoming link from node 12. Therefore, in spite of the node 5 has four outgoing and incoming links, the number of the nodes which have possibility to become the correlated broken nodes when the node 5 is the center broken node is three.

And now, we derive the probability that the number of nodes which have possibility to become the correlated broken nodes is j, denoted as B_j. Before derive B_j, we derive the probability that q of p outgoing links which go out of a node overlap with r incoming links come into it, denoted as $C_{p,q,r}$. Here, we define regular link as the link which construct the ring network and random link as other link. We consider the two case. The one is the case that one of the incoming links overlap with the regular outgoing link, and the other case is that none of incoming links overlap with it. Since the regular incoming link never overlap with the regular outgoing link, the probability to become the first case is $(r - 1)/(N - 2)$ and one to become the second case is $1 - (r - 1)/(N - 2)$. In the first case, $C_{p,q,r}$ is the same as the probability that each of $q - 1$ outgoing links among the $p - 1$ outgoing links except for the regular outgoing link overlap one of $r - 1$ incoming links, denoted as $C'_{p-1,q-1,r-1}$. And in the second case, $C_{p,q,r}$ is the same as the probability that each of q outgoing links among the $p - 1$ outgoing links except for the regular outgoing link overlap one of r incoming links, denoted as $C'_{p-1,q,r}$. Using $C'_{p',q',r'}$ given as follows,

$$C'_{p',q',r'} = \begin{cases} 0, & \text{for } q' < 0, r' \leq 0, q' > p', \\ & (p' + r' > N \text{ and} \\ & q' < p' + r' - N) \\ \dfrac{\binom{p'}{q'}\, {}_{r'}P_{q'}\; {}_{N-2-r'}P_{p'-q'}}{{}_{N-2}P_{p'}}, & \text{otherwise,} \end{cases} \tag{5}$$

we can derive $C_{p,q,r}$ as

$$C_{p,q,r} = (\frac{r-1}{N-2})C'_{p-1,q-1,r-1} + (1 - \frac{r-1}{N-2})C'_{p-1,q,r}\,. \tag{6}$$

B_j can be derived as the sum of the probability that when the number of incoming links is $j - p + q$, q of p outgoing links overlap with one of incoming links. Therefore, we can obtain B_j as

$$B_j = \sum_{q=max(0,p+1-j)}^{p} A_{j-p+q} C_{p,q,j-p+q}\,. \tag{7}$$

Here, we consider two nodes whose regular links connect to the center broken node. We call them regular node (R-node). And we define non-connective node (NC-node) as the node which have no incoming link. Even if a node has many incoming links, when all of source node of them are broken, it becomes NC-node. However, when the number of incoming link is equal to or greater than 2, the probability that all of source nodes of them are broken is very small compared with that when the number of incoming link is 1. Therefore, we assume the NC-node as the node which have only one incoming link and its source node is broken. That is, when the destination node of regular outgoing link of the broken node has only this regular incoming link and this node is not broken, it becomes the NC-node. Fig. 4 shows the center broken node and R-node. (a) shows the case that none of R-node is broken, (b) shows the case that one of them is broken, and (c) shows the case that both of them are broken. It is found that there is only one node which have possibility to become the NC-node in all case. The probability that this node becomes the NC-node is A_1. When the number of broken nodes is k, we can consider the three case with $k = 1$, $k = 2$ and $k > 2$. In $k = 1$, this node is the center broken node and it certainly becomes the case (a) and never becomes the case (b) and (c). In $k = 2$, the one node is the center broken node and the other is the correlated broken node and it becomes the cases (a) or (b). And the probability to become the case (a) is $2/l$ and to become the case (b) is $1 - 2/l$ where l is the number of the nodes have possibility to become the correlated broken nodes. If $k > 2$, it becomes all the case. The number of broken nodes except for R-node in (a), (b) and (c) is k, $k-1$ and $k-2$, respectively. Furthermore, when the number of links connect to the center broken node is l, the probability that the number of correlated broken nodes is k, denoted as $t_{l,k}$ is

$$t_{l,k} = B_l \binom{l}{k} a^k (1 - a)^{l-k}\,. \tag{8}$$

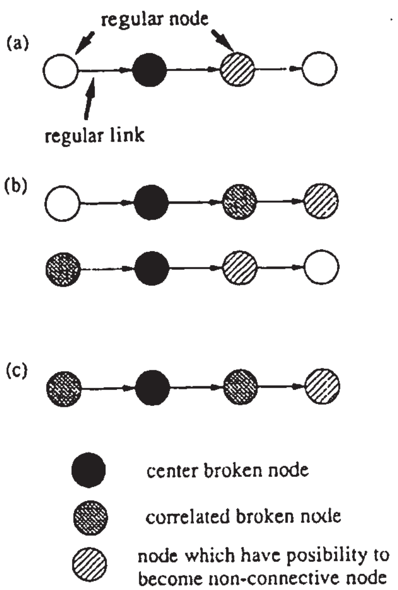


Figure 4. The center broken node and regular nodes.

And in this case, the probability to become the case of (a) is $(\binom{k}{0}\ {}_{l-2}P_k)/{}_lP_k$, to become the case of (b) is $(\binom{k}{1}\ {}_{l-2}P_{k-1})/{}_lP_k$ and to become the case of (c) is $(\binom{k}{2}\ {}_{l-2}P_{k-2})/{}_lP_k$. The network connective probability when the number of broken nodes is l, denoted as E_l, is derived in [8] as follows

$$E_l = \prod_{s=0}^{l-1} \frac{N - NA_1 - s}{N - s} \quad . \qquad (9)$$

Therefore, using (8) and (9), we can obtain the network connective probability as

$$\begin{aligned} R_{CSRN} = & \sum_{l=p}^{N-1} t_{l,0}(1 - A_1) \\ & + \sum_{l=p}^{N-1} t_{l,1}\{\frac{2}{l}(1 - A_1) + (1 - \frac{2}{l})(1 - A_1)E_l\} \\ & + \sum_{k=2}^{N-1} \sum_{l=max(p,k)}^{N-1} t_{l,k}\{\frac{\binom{k}{0}\ {}_{l-2}P_k}{{}_lP_k}(1 - A_1)E_k \\ & \quad + \frac{\binom{k}{1}\ {}_{l-2}P_{k-1}}{{}_lP_k}(1 - A_1)E_{k-1} \\ & \quad + \frac{\binom{k}{2}\ {}_{l-2}P_{k-2}}{{}_lP_k}(1 - A_1)E_{k-2}\} \ . \end{aligned} \qquad (10)$$

4 Results

We show computer simulation and theoretical calculation results of the network connective probability under the correlated breakage.

Fig. 5 shows the network connective probability of SN, CN and CSRN with $p = 2$ versus the correlated broken probability. In this figure, the chordal length of CN, r_1 is 50. It is shown that the both the network connective probability of SN and CN is the same in $p = 2$. It is also shown that the network connective probability of CN or SN is larger than that of CSRN in small a, however, in large a, the network connective probability of CN or SN is smaller than that of CSRN.

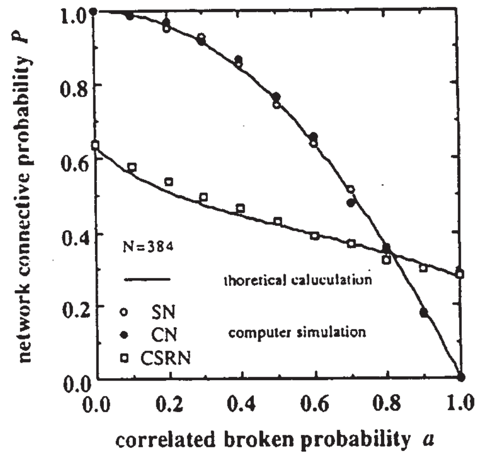


Figure 5. The network connective probability with $p = 2$ versus correlated broken probability.

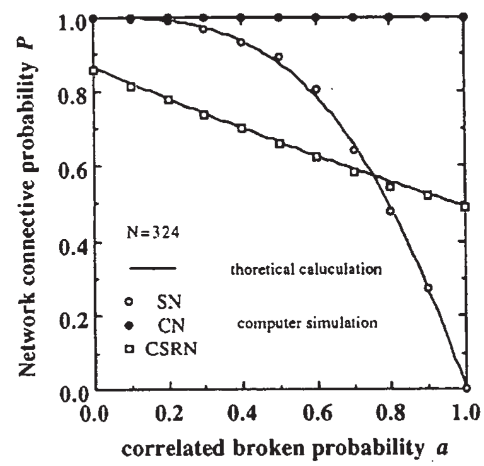
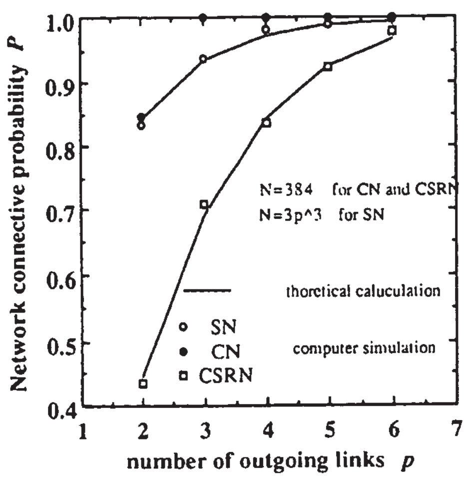


Figure 6. The network connective probability with $p = 3$ versus correlated broken probability.

Fig. 6 shows the network connective probability of SN, CN and CSRN with $p = 3$ versus the correlated broken probability. In this figure, r_1 is 50 and r_2 is 120. The tendency of the network connective probability of SN and CSRN is the same as the case with $p = 2$. However, the tendency of the network connective probability of CN is not different from that with $p = 2$.

In CSRN, because the number of incoming links come into a node is not constant, even if p is large, there are some nodes whose number of incoming links is one. Therefore, the network connective probability itself is small. However, the link assignment of CSRN is random, the condition of correlated breakage is not so different from that of independent breakage. On the other hand, in SN, because the number of incoming links come into a node is constant, the network connective probability under the indepen-



igure 7. The network connective probability with a = 0.4 versus the number of outgoing links per node.

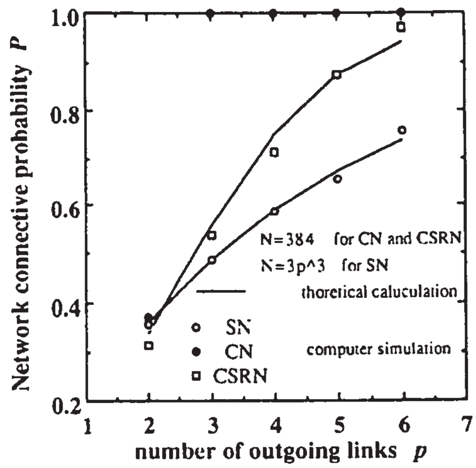


Figure 8. The network connective probability with a = 0.8 versus the number of outgoing links per node.

ent breakage is large. However, because of regularity of the link ssignment, that under the correlated breakage is small. In CN, 'hen p is two, the link assignment is regular, however, when p . larger than two, every chordal length is random and independent each other, and the link assignment is random. Moreover, the umber of incoming links per node of CN is the constant. There- ›re, the network connective probability of CN is large under both ıe independent and correlated breakage.

Figs. 7 and 8 show the network connective probability with = 0.4 and 0.8 versus p, respectively. It is shown that the larger is, the smaller difference of network connective probability beveen SN and CSRN is, when a is small. On the other hand, when is large, the larger p is, the larger difference of network conective probability between SN and CSRN is. The reason is as ›llows. When a is small, the network connective probability of :SRN is small. However, the larger p is, the smaller the number of odes, whose number of incoming links is 1, is, and the closer to 1 ıe network connectivity is. In SN and CN, even if p is small, the etwork connective probability is somewhat large when a is small. Vhen p is large, the network connective probability of CSRN is lmost the same with small p. On the other hand, in SN, the tenency network connectivity versus p is almost the same, however, ıe larger a is, the smaller the value is.

As these results, CN has best performance of network connecvity. However, it has been shown that CN has much poorer per- ›rmance of internodal distance than other network. Thus, it is xpecetd for the network to have good performance of both net- 'ork connective probability and internodal distance.

; Conclusion

We theoretically analyze the network connective probability f multihop network under the correlated damage of node. We 'eat shuffleNet, chordal network and connective semi-random etwork. It is found that in the independent node breakage, the etwork whose number of incoming links is the constant has good erformance of network connective probability, and found that in ıe correlated node breakage, the network whose link assignment is random has good performance of one.

Acknowledgement

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Performance analysis of network connective probability multihop network under correlated breakage

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Abstract:
One of important properties of a multihop network is the network connective probabi which evaluate the connectivity of the network. The network connective probability i: defined as the probability that when some nodes are broken, the rest of the **nodes connect** each other. Multihop **networks** are classified as a regular network whose li assignment is regular and a random network whose link assignment is random. It ha been shown that the network connective probability of a regular network is larger th: that of a random network. However, all of these results is shown under independent breakage. We analyze the network connective probability of multihop networks unde correlated node breakage. It is shown that a regular network has a better performan the network connective probability than a random network under independent break on the other hand, a random network has a better performance than a regular netwc under correlated breakage

Index Terms:
correlation methods network **topology** probability random processes telecommunication network reliability correlated node breakage independent breakage link assignment multih **network** **network connective** probability **node** breakage performance performance analy random network regular network

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A Flood Routing Method for Data Networks

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Abstract

In this paper, a new routing algorithm based on a flooding method is introduced. Flooding techniques have been used previously, e.g. for broadcasting the routing table in the ARPAnet [1] and other special purpose networks [3][4][5]. However, sending data using flooding can often saturate the network [2] and it is usually regarded as an inefficient broadcast mechanism. Our approach is to flood a very short packet to explore an optimal route without relying on a pre-established routing table, and an efficient flood control algorithm to reduce the signalling traffic overhead. This is an inherently robust mechanism in the face of a network configuration change, achieves automatic load sharing across alternative routes, and has potential to solve many contemporary routing problems. An earlier version of this mechanism was originally developed for virtual circuit establishment in the experimental Caroline ATM LAN [6][7] at Monash University.

1. Introduction

Flooding is a data broadcast technique which sends the duplicates of a packet to all neighboring nodes in a network. It is a very reliable method of data transmission because many copies of the original data are generated during the flooding phase, and the destination user can double check the correct reception of the original data. It is also a robust method because no matter how severely the network is damaged, flooding can guarantee at least one copy of the data will be transmitted to the destination, provided a path is available.

While the duplication of packets makes flooding a generally inappropriate method for data transmission, our approach is to take advantage of the simplicity and robustness of flooding for routing purposes. Very short packets are sent over all possible routes to search for the optimal route of the requested QoS and the data path is established via the selected route. Since the Flood Routing algorithm strictly controls the unnecessary packet duplication, the traffic overhead caused from the flooding traffic is minimal.

Use of flooding for routing purposes has been suggested before [3][4][5], and it has been noted that it can be guaranteed to form a shortest path route[10]. And an earlier protocol was proposed and implemented for the experimental local area ATM network (Caroline [6][7]). However the earlier protocol had problems with scaling timer values, and also required complex mechanism to solve potential race and deadlock problem. Our proposal greatly simplifies the previous mechanism and reduces the earlier problems.

Chapter 2 explains the procedure for route establishment and the simulation results are presented in chapter 3. The advantages of the Flood Routing are reviewed specifically in chapter 4. Chapter 5 concludes this paper with suggesting some possible application area and the future study issues.

2. Flood Routing Mechanism

Figure 1, 3, 4 show the stepwise procedure of the route establishment.

In the Figure 1, the host A is requesting a connection set up to the target host B. In the initial

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stage, a short connection request packet (CREQ) is delivered to the first hop router 1 and router 1 starts the flood of the CREQ packets.

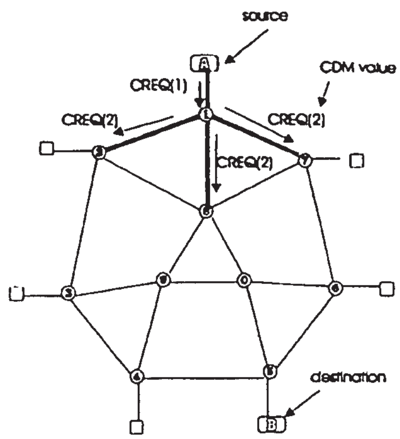

Figure 1

VC number (1byte=0)
Packet Type (1byte="CREQ")
CDM (1byte)
Source Address
Connection No (1byte)
Destination Address
QoS

Figure 2 CREQ Packet Format

Figure 2 shows the format of the CREQ packet. The CREQ packet contains a connection difficulty metric (CDM) field, QoS parameters and the source & destination addresses and connection number. The metric can be any accumulative measure representing the route difficulty, such as hop count, delay, buffer length, etc. The connection number is chosen by the source host to distinguish the different packet floods of the same source and destination.

When a router receives the CREQ packet, the router matches the packet information with the internal Flood Queue to see if the same packet has been received before. If the CREQ packet is new, it records the information in the Flood Queue, increases the CDM value, and forwards the packet to all output links with adequate capacity to meet the QoS except the received one. Thus the flood of CREQ packets propagate through the entire network.

The Flood Queue is a FIFO list which contains the information relating to the best CREQ packet the router has received for each recent flood. As the flood packet of a new connection arrives and the information is pushed into the Flood Queue, the old information gradually moves to the rear and eventually is removed. The queueing delay from the insertion to the deletion depends on the queue size and the call frequency, and provided this delay is enough to cover the time for network wide flood propagation and reply, there is no need for a timer to wait to the completion of the flood.

Since the CDM value is increased as the CREQ packet passes the routers, the metric value represents the route difficulty that the CREQ packet has experienced. Because of the repeated duplication of the packet, a router may receive another copy of the CREQ packet. In this case, the router compares the metric values of the two packets and if the most recently arrived packet has the better metric value, it updates the information in the Flood Queue and repeats the flood action. Otherwise the packet is discarded. As a consequence, all the routers keep the record of the best partial route and the output link to use for setting up the virtual circuit.

Figure 3 shows the intermediate routers 2, 7, 8 have chosen the links toward the router 1 as the best candidate link. If one of them is requested for the path to the source node A, the router will use this link for the virtual circuit set up.

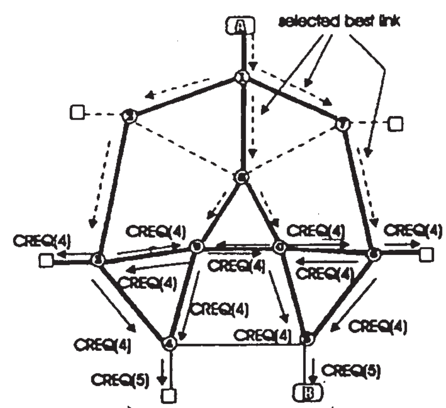

Figure 3

When the destination host receives a CREQ packet, it opens a short time-window to absorb possible further arriving CREQ packets. The expiration of the timer triggers the sending of the connection acceptance (CACC) packet along the best links indicated by the CREQ packet with the lowest CDM. The CACC packet is relayed back to the source host by the routers which at the same time install the virtual circuit via the optimal route. Finally, when the source host receives the CACC packet, the host may initiate data transmission.

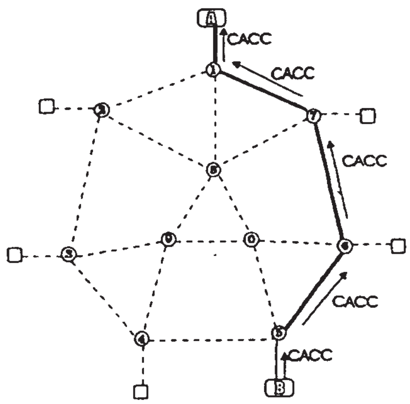

Figure 4

Note that bandwidth reservation occurs during the relay of the CACC packet. It is possible that the available QoS will have dropped below the requested level in one or more links. In this case, the source may either accept the lower QoS, or close the connection and try again.

More implementation details of the flooding protocol can be found in [9].

3. Simulation Result

One concern of Flood Routing is whether it will lead to congestion of the network by the signalling traffic. A simulation was carried out using various network conditions. Figure 5 shows the number of flooding packets produced in a connection trial in a normal traffic condition on a network consisting of 5 switching nodes, 9 hosts and 16 links. The simulation tested the event of 2000 seconds.

The graph shows that the total number of flooding packets per connection converges on the lower bound 18 with some exceptions. This is slightly higher than the number of the network links (16). This shows how the flood control mechanism is efficient in that the routers usually generate only one flooding packet per output link and this duplication process is rarely repeated again. As a result, the total number of flooding packets per connection is nearly same as the number of network links.

Considering the small size of the flooding packet, the bandwidth consumed by the signalling traffic is small. Suppose an ATM network using the Flood Routing generates 1000 calls per seconds, the bandwidth consumption by the signalling traffic will only be about 424 Kbps (= 1 K * 53 byte) per link and this does not include any additional route management traffic such as the routing table update.

From the simulation, it is observed that the average number and the maximum number of the flooding packets depends on the network topology and the traffic condition. If the network is simple topology such as a tree or a star shape, the average number of the flooding packets is nearly identical to the number of the network links. If the network is a complex topology such as a complete mesh topology, and there is a high traffic load, the routers tend to generate more packets because of the racing of the flooding packets.

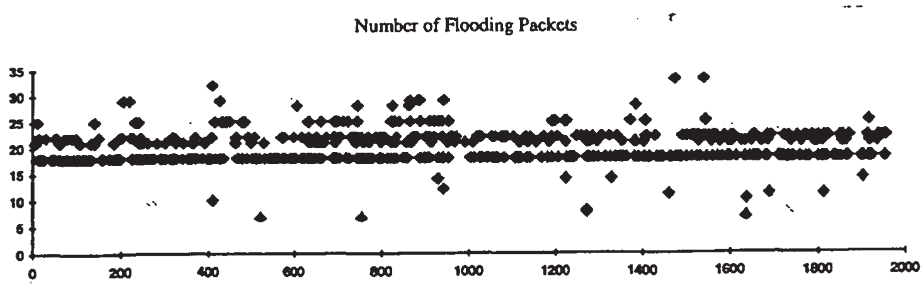

Figure 5

The connections established by Flood Routing successfully avoid busy links and disperse the communication paths to all possible routes. This reduced the chance of congestion and utilizes all network resources efficiently.

4. Advantages of the Flood Routing

The distinctive features of the Flood Routing method are :

(a) It facilitates the load sharing of available network resources. If many possible routes exist between two end points in a network, the Flood Routing can disperse different connections over different routes to share the network load. Figure 6 shows this example.

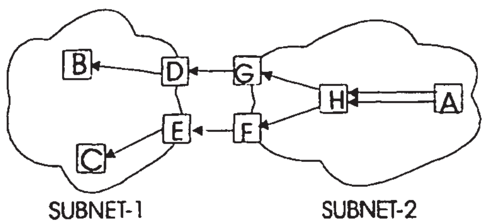

Figure 6 Example of Multipath Connection

In the sample network, there are more than two links exist between node A and H, and the node A used all links for different connections with balancing the load. More than two exterior routers are connecting the subnet 1 and the subnet 2, and the node H distributed the connections to all exterior routers. Therefore, all the network resources are utilized fully in Flood Routing network. This load sharing capability has been considered to be a difficult problem in table based routing algorithms.

(b) It automatically adapts to changes in the network configuration. For example, if the overall traffic between two end points has been increased, the network bandwidth can simply be expanded by adding more links between routers. The Flood Routing algorithm can recognize the additional links and use them for sharing the load in new connections.

(c) The method is robust. The Flood routing can achieve a successful connection even when the network is severely damaged, provided flooding packets can reach the destination. Once a flooding packet reaches the destination, the connection can be established via the un-damaged part of the network which was searched by the packet. This is very useful property in networks which are vulnerable but which require high reliability, such as military networks.

(d) The method is simple to manage, as it makes no use of routing tables. This table-less routing method does not have the problem like "Convergence time" of the Distance Vector routing [8].

(e) It is possible to find the optimal route of the requested bandwidth or the quality of service. While the packet flood is progressing, bandwidth requirement and QoS constraints specified in the flooding packets are examined by the routers and the links that does not meet the requirements are excluded from the routing decision. As a result, the route constructed with the qualified links can meet the bandwidth and the QoS requirements, usually in the first attempt.

(f) It is a loop-free routing algorithm. The only possible case that the route may consist a loop can be caused from the corrupted metric information. However this can be detected by a check sum.

(g) Since the flooding method is basically a broadcast mechanism, it can be used for locating resources in network. Many network applications are best served by a broadcast facility, such as distributed data bases, address resolution, or mobile communications. Implementing broadcast in point-to-point networks is not straight forward. The flooding technique provides a means to solve this problem. In particular, locating a mobile user by Flood Routing, and establishing a dynamic route is an interesting issue. Application to a movable network in which entire network units including both the mobile users as well as the switching nodes and the wireless links is another potential research area.

5. Future Study and Conclusion

In this paper, we introduced a revised Flood Routing technique. Flood Routing is a novel approach to network routing which has the potential to solve many of the routing problems in contemporary networks. The basic Flood Routing presented in this paper has been developed to be used in an ATM style network, however we believe a similar technique can also be applied to IP routing. Another promising area of application of this method would be military or mobile networks which require high mobility and reliability. Research to extend the point-to-point Flood Routing to optimal multi-point routing is now progressing. Further analysis of performance, and application to large scale networks are the future issues.

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