

path length and vulnerability for "ideal" networks against which network designers may measure their networks. For this reason, we too, determine the lower bounds on these performance measures for regular graphs. In addition, the analysis provides a uniform basis by which to compare the various algorithms.

Section 4.2 proposes a set of performance measures, and section 4.3 reviews and extends some properties of regular graphs. Sections 4.4 to 4.8 determine the performance of the various broadcast routing algorithms. Section 4.9 compares the various algorithms, and 4.10 determines average values of the performance measures for the ARPANET topology.

#### 4.2 Performance Measures

This section develops some performance measures for broadcast communication in store-and-forward networks. We develop the performance measures by considering the overhead imposed on the communication subnet by a broadcast, and the delays in performing the broadcast.

An important measure of the amount of bandwidth used in performing a broadcast, and of the processing overhead in the switching (store-and-forward) nodes of the subnet is the Number of Packets Transmitted,  $NPT(i)$ , in broadcasting a message initiated from node  $i$  in the subnet.  $NPT(i)$  is in units of packet-hops per broadcast.

The Broadcast Delay,  $BD(i,j)$ , is the delay before node  $j$  receives a broadcast message initiated from  $i$ .  $BD(i,j)$  is in appropriate units for delay, e.g. seconds or hops. The average Broadcast Delay,  $BDav(i)$ , from a node  $i$  that initiates the broadcast is an estimate of the delay before a receiver hears the message.  $BDav(i)$  is therefore a measure of the ability of the broadcast routing scheme to deliver messages quickly when initiated from  $i$ . In a communication subnet with  $N$  nodes

$$BDav(i) = \frac{1}{N-1} * \sum_{\substack{j=1 \\ j \neq i}}^N BD(i,j). \quad (4.1)$$

Analogously,  $BDmax(i)$  is the maximum Broadcast Delay from a broadcaster  $i$ , and therefore is an indication of the maximum delay before all receivers have heard the message.  $BDmax(i)$  is, therefore, also a measure of the cost of broadcast from node  $i$ .

$$BD_{\max}(i) = \max \{BD(1,j) \mid \forall 1 \leq j \leq N, j \neq 1\}. \quad (4.2)$$

These measures are useful in the design of timeouts for the reinitiation of the broadcast, or recovery techniques in distributed operating systems that use the broadcast routing feature of the communication subnet to locate resources.

The Broadcast Cost, BC, in a communication network is the sum of the cost of broadcast with each node as the initiator. It is assumed that each node has equal probability of initiating a broadcast.  $BD_{\max}(i)$  is the cost of broadcast when initiated from node  $i$ . Hence, if there are  $N$  nodes in the network, then

$$BC = \sum_{i=1}^N BD_{\max}(i). \quad (4.3)$$

These measures of performance can easily be determined for a particular network using a particular broadcast routing technique. The performance measures are a function of the topology of the network. In order to determine lower bounds on these measures, we examine regular graphs.

### 4.3 Regular Graphs

In this section we review and extend some of the definitions and properties of regular graphs, Moore graphs, and generalized Moore graphs.

We assume that the number of nodes in the graph is  $N$ , and that all edge costs are the same and equal to one. The degree of a node in a graph is the number of edges incident to that node. If the degree of all the nodes is same and equal to  $D$ , then the graph is called regular with degree  $D$ . Figure 4.1 shows some regular graphs. A theorem and two corollaries may be stated concerning the relationship between  $N$  and  $D$  in regular graphs. These are simple, and stated without proof.

Theorem 4.1: The sum of the degrees of the nodes of any graph is twice the number of edges in the graph.

Corollary 4.1.1: In any graph the number of nodes of odd degree is even.

Corollary 4.1.2: A regular graph with degree  $D$  and  $N$  nodes can always be constructed if  $N \cdot D$  is even.

Consider a regular graph with degree  $D$ . The maximum number of nodes at distance one from any node is  $D$ , at distance two  $D(D-1)$ , at distance three  $D(D-1)^2$  and so on. Moore graphs are those regular graphs that have exactly  $D(D-1)^{j-1}$  nodes at distance  $j$ , for any node considered. Figure 4.2a illustrates such a tree as seen from one node  $X$ , where the graph has 10 nodes and is of degree 3. Such a picture can

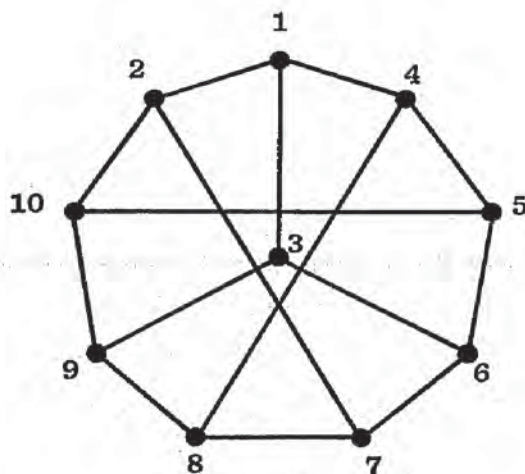
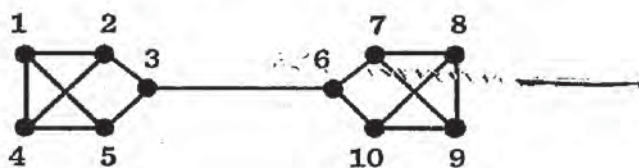
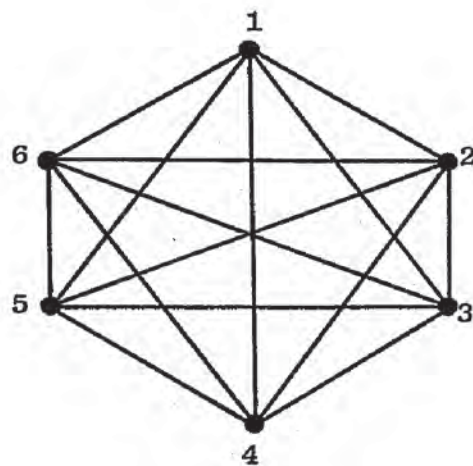


Figure 4.1. SOME REGULAR GRAPHS

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