If v is identical to s, then $\{x, y, z\}$ must be identical to $\{r, t, u\}$. Hence w of Figure 4A must be adjacent to each of the vertices x, y, and z. Thus we have a K_5 in this case.

If v is identical to r, t, or u, then one of the vertices x, y, and z must be identical to s. Without loss of generality let us assume that v is identical to r and that x is identical to s. Then x must be adjacent to w and $\{y, z\}$ must be identical to $\{t, u\}$. Hence each of the vertices y and z must be adjacent to w. Thus we have a K_5 in this case.

Case 2. *v is connected to a star.* In this case all we need show is that no V-irreducible graph G exists such that G has no vertex which is connected to a delta. For if there is a vertex in G which is connected to a delta then this case reduces to Case 1.

Suppose that v is connected to vertices w, x, y, and z. Then these vertices form a star. Hence one of them, say x, is connected to all the rest and Since G is V-irreducible, w is connected to either a delta or a star. But no vertex of G is connected to a delta. Hence w must be connected to a star. Thus w must be connected to y and z as well. But then clearly vertices v, w, and x are connected to deltas. Thus it is impossible to construct a V-irreducible graph such that it has no vertex which is connected to a delta. Q.E.D.

As we can easily see, if a graph contains a B-graph as a subgraph it is H-reducible. (Apply H-reduction to vertices u and v, where u and v are as shown in Figure 3.) Hence we have the following theorem as a corollary to Theorem 1:

THEOREM 2. *A connected graph is either H-reducible or V-reducible if it is not a K_5.*

Thus we can say that H-type and V-type reductions are sufficient to reduce a simple quartic graph to a K_5.

4. CONSTRUCTION OF QUARTIC GRAPHS

In this section a "graph" still means a simple quartic graph.

There are obvious inverse operations of H-type and V-type reductions. We call them *H-type* and *V-type expansions*, respectively. In this section we show that H-type and V-type expansions are necessary and sufficient to construct a graph from a K_5. Since we can obtain a graph from another graph by an H-type or a V-type reduction we need only show that we can always obtain a connected graph by an H-type or a V-type reduction.

REMARK 1. In any graph the number of vertices of odd degrees is even.

REMARK 2. No quartic graph can have a bridge.

For general quartic graphs we have the following theorem.

THEOREM 3. *Every connected graph G_4^m with m vertices can be constructed either from a connected G_4^{m-1} be a V-type expansion or from a connected G_4^{m-2} by an H-type expansion.*

Proof. Since from Theorem 2 we know that G_4^m can be reduced to either G_4^{m-1} or G_4^{m-2}, all we have to do is show that this can be done in such a way that they are actually connected.

Case 1. *A graph G_4^{m-1} obtained from G_4^m by a V-type reduction is not connected.*

Let v be the vertex of G_4^m which is removed to obtain the G_4^{m-1}.

Let w, x, y and z be the adjacent vertices of v.

If G_4^{m-1} is not connected then $G_4^m - \{v\}$ is not connected.

Let us consider possible positions of w, x, y, and z in $G_4^m - \{v\}$.

Suppose that one of them is in one connected component of $G_4^m - \{v\}$ and that others are in different component(s) from the first. Then G_4^m must have a bridge. But by Remark 2 it cannot happen. Hence the only possibility is that two of them, say w and x, are in one connected component of $G_4^m - \{v\}$ and the remaining two are in the other. In this case we add edges wy and xz to $G_4^m - \{v\}$. Then the resultant graph is connected and it is obtained from G_4^m by a V-type reduction. Hence the theorem is true for Case 1.

Case 2. *A graph G_4^{m-2} obtained from G_4^m by an H-type reduction is not connected.*

Let u and v be the vertices of G_4^m which are removed to obtain G_4^{m-2}.

Let q, r, and s be the adjacent vertices of u other than v.

Let x, y, and z be the adjacent vertices of v other than u.

If G_4^{m-2} is not connected then $G_4^m - \{u, v\}$ is not connected.

As we can see, some of $\{q, r, s\}$ may be identical to some of $\{x, y, z\}$. There are three possibilities: The first one is when all of q, r, s, x, y, and z are distinct; the second one is when one of q, r, and s is identical to one of x, y, and z; and the third one is when more than one of q, r, and s are identical to the same number of x, y, and z:

(A) q, r, s, x, y, and z are all distinct. Let us consider possible positions of q, r, s, x, y, and z in $G_4^m - \{u, v\}$. Then by the same reason as for Case 1 each connected component of $G_4^m - \{u, v\}$ must contain at least two of the vertices q, r, s, x, y, and z. Suppose that a connected component of $G_4^m - \{u, v\}$ contains an odd number of them. Then Remark 1 is

violated since the rest of the vertices of the component are of degree
Thus we have the following two possibilities:

(1) Two of them are in one connected component of $G_4^m - \{u, v\}$ and the rest are in the other.

(2) Each connected component of $G_4^m - \{u, v\}$ contains exactly two of them.

For both of the cases (1) and (2) it is easy to find a pairing of the vertices q, r, s, x, y, and z so that the resultant G_4^{m-2} is connected. Hence the theorem is true for Case 2(A).

(B) One of the vertices q, r, and s is identical to one of the vertices x, y and z.

Let us assume that r is identical to x.

By similar considerations to those for Case 2(A) we can see that there are three possibilities concerning the positions of the vertices q, r, s, y, and z:

(1) r is in one connected component of $G_4^m - \{u, v\}$, two of the remaining four are in a second, and the rest are in the third.

(2) r and two of q, s, y, and z are in one connected component of $G_4^m - \{u, v\}$ and the rest are in the other.

(3) r is in one connected component of $G_4^m - \{u, v\}$ and the rest are in the other.

For the cases (1) and (2) it is not difficult to find a pairing of the vertices q, r, s, y, and z so that the resultant graph is a connected G_4^{m-2}. Note that r must be paired with two distinct vertices.

For the case (3), no matter what edges we add to $G_4^m - \{u, v\}$ to produce a G_4^{m-2}, the G_4^{m-2} is always connected.

(C) More than one of the vertices q, r, and s are identical to the same number of vertices of x, y, and z. In this case all G_4^{m-2}'s obtained from G_4^m by an H-type reduction are connected. Hence the theorem is true for this case.

Since all possibilities are exhausted we have the theorem.

5. Conclusion

It has been shown that a simple quartic graph can be reduced to a K_5 by H-type and/or V-type reductions and that one can be constructed by H-type and/or V-type expansions from a K_5.

For regular graphs of an arbitrary even degree it has been shown that one of the two reductions alone is not sufficient to reduce them to a complete graph of the same degree. It remains to be seen if the two are sufficient for regular graphs of even degrees other than 4.

For regular graphs of odd degrees the only known reduction (construction) procedure is that for cubic graphs. While it can be easily seen that H-type reductions reduce some of them, whether they are sufficient is anyone's guess.

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The proofs of Lemma 1 and Case 2 of Theorem 1 are by one of the reviewers and are considerably more elegant and simpler than the original ones by the author. Also pointed out to the author by the same reviewer are Remarks 1 and 2, which greatly simplify the proof of Theorem 3.

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