Thermal Conductivity of Plastic Foams

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A simple equation enabling the prediction of the thermal conductivity of plastic foams, without the aid of adjustable parameters, is proposed. The equation is based on a recurrent method, previously developed, that showed reasonable agreement with experimental results. Ways of decreasing the thermal radiation contribution are shown. In particular, the influence of cell size, radiation transmission through solid membranes, and low-emissivity boundary surfaces are analyzed. Errors involved in steady techniques of measuring the thermal conductivity associated with radiation are discussed.

INTRODUCTION

Cellular materials, and particularly plastic foams, are widely used in insulation in the low-ambient temperature range. The analysis of the modes of heat transfer through the foam should enable one to state conditions for minimizing the apparent thermal conductivity. This would permit one to reduce insulation costs and/or energy requirements.

The aim of this article is to expand our previous results (1), with the following objectives:

- Derive a simple analytical expression without adjustable parameters to predict the apparent thermal conductivity of a plastic foam (In our previous analysis (1), the radiation contribution could not be expressed in a simple form).
- Analyze the contribution of the thermal radiation to the total heat transport, stating conditions enabling it to be a minimum.
- Discuss errors involved in steady experimental techniques of measuring thermal conductivity in relation to the contribution of thermal radiation.

THE THERMAL CONDUCTIVITY OF A PLASTIC FOAM

As previously shown (1), the heat flux through a plastic foam may be expressed as:

$$q = q_{cond}^{sol} f + (q_{cond}^{gas} + q_{rad})(1 - f) \tag{1}$$

where f is the fraction of transversal area occupied by solid (e.g., 2/3 of the solid mass), and $(1 - f)$ is the transverse area corresponding to alternating layers of gas cells and solid layers (e.g., 1/3 of the solid mass). As shown in *Fig. 1*, the specimen thickness L is assumed to be composed of n gas cells of thickness L_g and n solid membranes of thickness L_s:

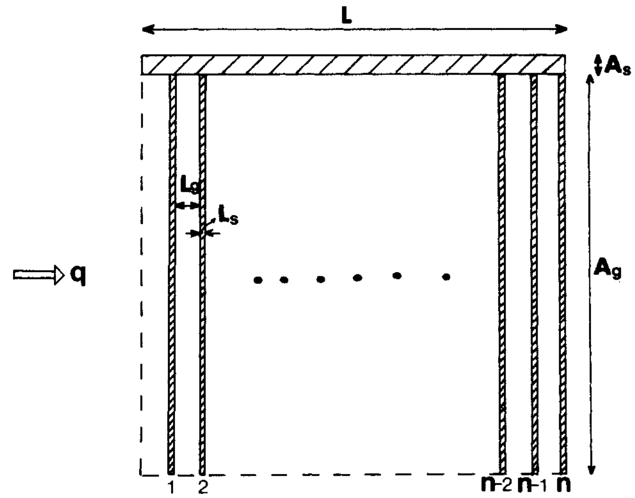

Fig. 1. Geometric model of the plastic foam.

$$L = n(L_g + L_s) \tag{2}$$

Also, since the area A_s occupies 2/3 of the polymer,

$$LA_s = 2nL_sA_g \tag{3}$$

The foam density is given by:

$$\rho_f = \frac{1.5LA_s\rho_s + nL_gA_g\rho_g}{L(A_g + A_s)} \tag{4}$$

From *Eqs 2* to *4*, the following equations arise:

$$f = \frac{A_s}{A_g + A_s} = \frac{(\rho_f - \rho_g)}{1.5(\rho_s - \rho_g)} \simeq \frac{(\rho_f - \rho_g)}{1.5\rho_s} \tag{5}$$

$$n/L = \frac{(1 - 1.5f)}{L_g(1 - f)} \simeq \frac{(\rho_s - \rho_f)}{L_g(\rho_s - \rho_f/1.5)} \tag{6}$$

$$L_s = \frac{Lf}{2n(1 - f)} = \frac{L_g(\rho_f - \rho_g)}{2(\rho_s - \rho_f)} \tag{7}$$

ρ_s and ρ_g are, the densities of the plastic material and the gas filling the cells, respectively. Thus, the area

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specimen length, n/L, and the thickness of a solid membrane, L_s, may be easily calculated as a function of known densities and the average thickness of a gas cell, L_g.

The heat flux conducted through the solid is given by:

$$q_{cond}^{sol} = k_s \Delta T / L \tag{8}$$

while the energy conduction through the gas may be approached by:

$$q_{cond}^{gas} = k_g \Delta T / L \tag{9}$$

where k_s and k_g are the thermal conductivities of the polymer and the gas, respectively.

The net fraction of radiant energy sent forward by a solid membrane of thickness L_s is calculated in Appendix 1. It is given by

$$T_N = \frac{(1-r)}{(1-rt)} \left\{ \frac{(1-r)t}{(1+rt)} + \frac{(1-t)}{2} \right\} \tag{10}$$

The fraction $(1 - T_N)$ is sent back. The coefficient r is the fraction of incident energy reflected by each gas-solid interface. It is related to the refractive index of the plastic, w, by:

$$r = \left\{ \frac{w-1}{w+1} \right\}^2 \tag{11}$$

The coefficient t is the fraction of radiant energy transmitted through the solid membrane, as given by Bouguer's law:

$$t = \exp(-aL_s) \tag{12}$$

The absorption coefficient, a, is taken as the average value over the range of wavelengths of the radiant energy.

Assuming that the foam is placed between two black surfaces at temperatures T_1 and T_2, it is necessary to analyze the net radiant energy transmitted through the set of n solid layers. This calculation, which is shown in Appendix 2, leads to

$$q_{rad} = \frac{\sigma(T_1^4 - T_2^4)}{1 + n(1/T_N - 1)} \simeq \frac{4\sigma \overline{T}^3 \Delta T}{1 + n(1/T_N - 1)} \tag{13}$$

$\overline{T} = (T_1 + T_2)/2$ is the average temperature. Thus, the apparent thermal conductivity of a plastic foam, defined as $k = qL/\Delta T$, is given by:

$$k = k_s f + \left\{ k_g + \frac{4\sigma \overline{T}^3 L}{1 + n(1/T_N - 1)} \right\} (1 - f) \tag{14}$$

For plastic foams used in thermal insulation, $\rho_f \ll \rho_s$, implying $f \to 0$ and $n \to L/L_g$. Then, *Eq 14* reduces to:

$$k = k_g + \frac{4\sigma \overline{T}^3 L}{1 + (L/L_g)(1/T_N - 1)} = k_g + k_r \tag{15}$$

Equation 14 (or *15*) enables one to calculate the thermal conductivity of a plastic foam without the use of adjustable parameters. This equation is the analytical expression of our previous recurrent method (1), which was shown to give a reasonable agreement with experi-

THE CONTRIBUTION OF THERMAL RADIATION

The thermal radiation contribution to the overall heat transfer will be illustrated for a typical polystyrene foam (PS), with air filling the cells, at an average temperature $\overline{T} = 283$ K. For this case, the results (1) are:

$$k_g = 0.02132 \text{ Kcal m}^{-1}{}^\circ\text{C}^{-1}\text{h}^{-1},$$
$$\rho_s = 1052.5 \text{ Kg/m}^3,\ w = 1.6,\ a = 7.53\ 10^{-3}\mu\text{m}^{-1}$$

Commercial materials have average cell sizes in the range of $L_g = 50$ to $300\ \mu$m. From *Eq 7*, the thickness of a solid membrane, L_s, varies in the range 0.2 to 7.3 μm for foam densities in the range $\rho_f = 10$ to 50 Kg/m^3. In fact, there is an inverse correlation between ρ_f and L_g(2), narrowing the actual L_s range. *Equation 12* gives transmission coefficients t varying from 0.95 to 1. Using these values and *Eqs 10* and *11*, the coefficient T_N lies in the range 0.88 to 0.90. This narrow range suggests an average value of $T_N = 0.89$. Thus, solid membranes of typical polystyrene foams allow 89 percent of the incident radiation to pass through them, while 11 percent is sent back.

Figure 2 shows the ratio k_r/k as a function of the specimen thickness L, and different cell sizes L_g. The radiation contribution increases with the specimen thickness, attaining an asymptotic value. For $L > 0.5$ cm, radiation accounts for 7 to 34 percent of the heat transport through a polystyrene foam at $\overline{T} = 283$ K. This fraction varies with temperature and with the nature of the gas filling the cells.

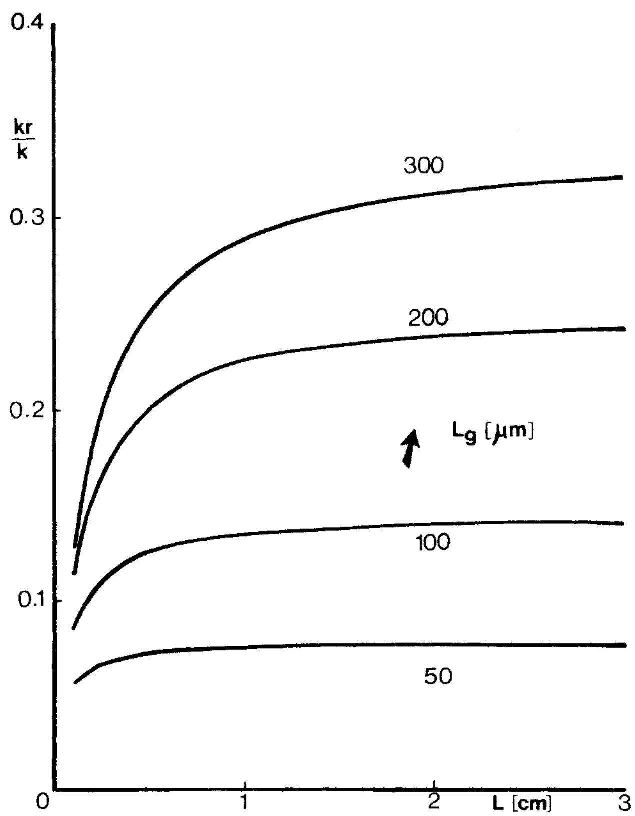


Fig. 2. Relative contribution of radiation to the overall thermal conductivity for a PS foam, as a function of the specimen thick-

It is interesting to analyze the possibilities of decreasing thermal radiation. An obvious way is to reduce the average cell size, requiring modifications in the foaming process. However, in the manufacture of low-density foams, an inverse correlation between foam density and cell size is often observed (2). Then, special techniques must be developed to produce microcellular low-density foams.

Another factor that may be analyzed in the transparency of a solid membrane to radiation, as given by the coefficient T_N. Let us take its limiting values. The refractive indices of plastic materials lie in the range 1.338 to 1.71 (3), giving reflection coefficients $r = 2.09 \cdot 10^{-2}$ to $6.86 \cdot 10^{-2}$. Limiting t values are 0 for a completely opaque material and 1 for a completely transparent material. Then: $T_N = 0.466$ for a plastic foam made from an opaque polymer with the highest refractive index, and $T_N = 0.959$ for a plastic foam made from a transparent polymer with the lowest refractive index.

In order to quantify the influence of T_N, the ratio of asymptotic thermal radiation contributions is taken:

$$\frac{k_r(T_N)}{k_r(T_{N\max})} = \frac{(1/T_{N\max} - 1)}{(1/T_N - 1)} = \frac{4.275 \cdot 10^{-2}}{(1/T_N - 1)} \qquad (16)$$

Figure 3 is a graphical representation of *Eq 16*. At high T_N values, the parametric sensitivity of thermal radiation to T_N is very significant. For example, a 50 percent reduction in the thermal radiation contribution of a polystyrene foam my be achieved if T_N is lowered from 0.89 to 0.80. Additives increasing the average absorption coefficient a (i.e., showing strong I.R. absorption bands in the same wavelength range of the radiation) may be useful for this purpose. Also, black pigments may be tried. For black bodies, $T_N = 0.5$ ($r = t = 0$), in which case, *Fig. 3* shows that radiation may be neglected for practical purposes.

There is still another way to decrease the thermal radiation mode of heat transfer in commercial materials. For example, PS foams used in insulation are frequently covered with surfaces acting as vapor barriers. If these surfaces have low emissivity values, e_1 and e_2, thermal radiation will be attenuated. The analysis of the overall radiation from the boundary at T_1 to the boundary at T_2 is shown in Appendix 3. The result is

$$q_{rad} = \frac{\sigma(T_1^4 - T_2^4)}{1 + n(1/T_N - 1) + (1/e_1 + 1/e_2 - 2)} \qquad (17)$$

Then:

$$k_r = \frac{4\sigma\overline{T}^3 L}{1 + (L/L_g)(1/T_N - 1) + (1/e_1 + 1/e_2 - 2)} \qquad (18)$$

Figure 4 shows the decrease in k_r for a PS foam of average cell size $L_g = 200\ \mu$m, at $\overline{T} = 283$ K, and $e_1 = e_2$. A relevant effect is shown when $e \le 0.2$. However, increasing L or decreasing L_g lowers the relative incidence of using low-emissivity boundaries.

EXPERIMENTAL ERRORS ASSOCIATED WITH THERMAL RADIATION

For plastic foams showing a significant thermal radiation contribution, an experimental error is introduced in usual steady techniques, when hot and cold plaques are not black bodies. In this regard, standard methods usually prescribe blackening the plaques, although no quantification of this effect has been given. *Equation 18*

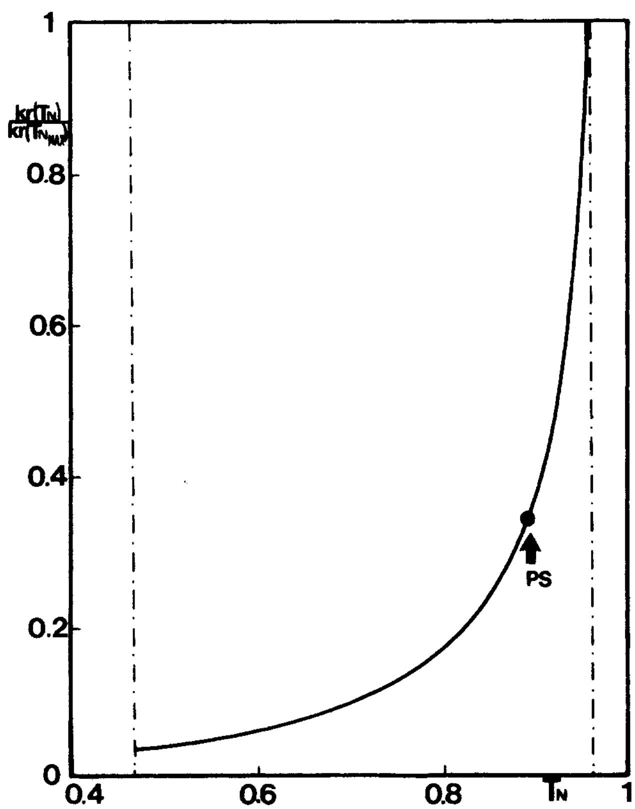


Fig. 3. Influence of the transparency of a solid membrane to radiant energy, T_N, upon the thermal radiation contribution (PS :

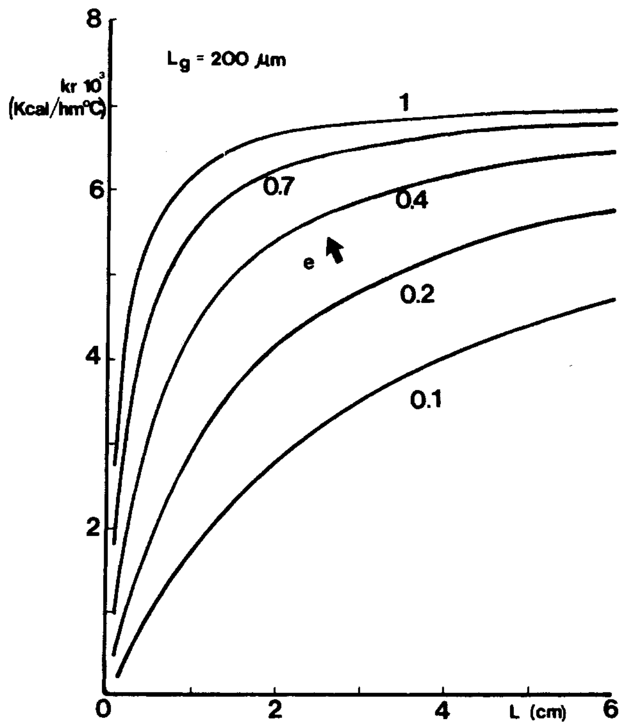


Fig. 4. Radiation contribution to the overall thermal conductivity for a PS foam, as a function of the specimen thickness L, and

gives an estimation of the errors introduced in a particular case. The relative error may be defined as:

$$\epsilon\% = \frac{k_r(e=1) - k_r(e)}{k_g + k_r(e=1)} \cdot 100 \quad (19)$$

Figure 5 shows $\epsilon\%$ for a low-density PS foam, with $L =$ 3 cm, at $\overline{T} = 283$ K, and $e_1 = e_2 = e$. The relative error depends on the average cell size. For $L_g \leq 100$ μm, an emissivity greater than 0.1 will suffice to give results lying within the precision range of the experimental technique. In general, for PS foams at $\overline{T} = 283$ K, an $e \geqslant 0.5$ will decrease the experimental error to less than 5 percent.

It may be concluded that, although blackening the plaques is a reasonable precaution, in most cases, it will not introduce any modification in the experimental results.

CONCLUSIONS

- A simple equation (*Eq 14*) enabling the prediction of the thermal conductivity of plastic foams without using adjustable parameters was proposed. This equation is the analytical expression of a recurrent method developed in a previous article (1). A reasonable comparison with experimental results was already shown (1).
- A significant thermal radiation contribution may be expected for several types of foams. In the case of PS foams at $\overline{T} = 283$ K, radiation accounts for 7 to 34 percent of the overall heat transport. This fraction may be decreased by
  a) diminishing the cell size through a modification in the foaming process;
  b) lowering the coefficient T_N when it has a high original value (i.e., $T_N > 0.8$); this might be achieved by including additives or pigments in the formulation;
  c) using boundary surfaces with low emissivity values (i.e., $e \leqslant 0.2$), for slabs with medium or great cell sizes.
- Blackening the plaques of experimental devices used to measure the thermal conductivity of plastic foams. At steady conditions, this is a reasonable precaution. However, in most cases, it will not introduce any modification in the experimental results (assuming e_1, $e_2 > 0.5$).

APPENDIX I

Net Fraction of Radiant Energy Sent Forward by a Solid Membrane

The situation is depicted in *Fig. 6*. A fraction r of the energy incident upon each air-solid interface is reflected. From the energy radiated through the material, a fraction t reaches the second interface. Then, if S_i is the net radiation flux at the positions indicated in the figure:

$$S_2 = S_1 r + S_4 t(1 - r) \quad (1.1)$$

$$S_3 = S_1(1 - r) + S_4 tr \quad (1.2)$$

$$S_4 = S_3 tr \quad (1.3)$$

$$S_5 = S_3 t(1 - r) \quad (1.4)$$

The energy fraction transmitted through the solid may be obtained from *Eqs 1.1* to *1.4*, as:

$$t_N = S_5/S_1 = \frac{t(1-r)^2}{1 - r^2t^2} \quad (1.5)$$

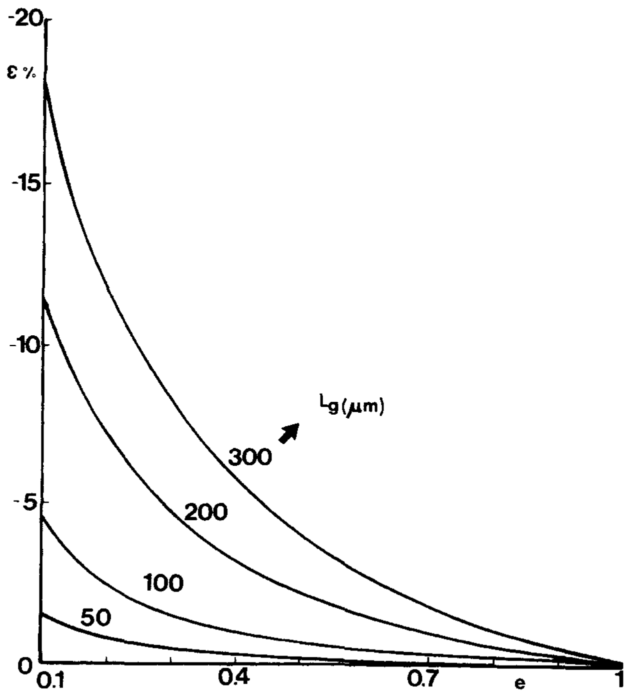
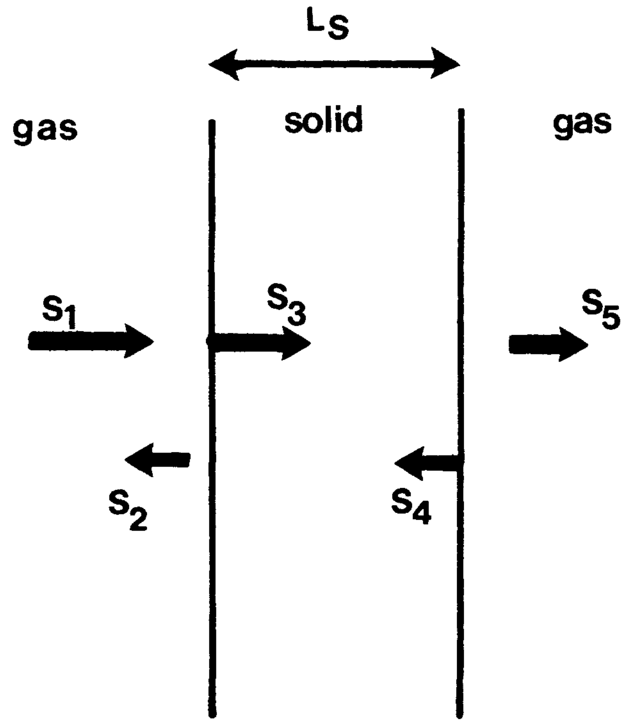


Fig. 5. Relative error in the experimental determination of the thermal conductivity using a steady technique, as a function of the emissivity of the boundary surfaces, and different cell sizes



The energy fraction that is reflected is given by:

$$r_N = S_2/S_1 = \frac{r[1 + t^2(1 - 2r)]}{1 - r^2t^2} \tag{1.6}$$

The fraction that is absorbed is:

$$a_N = (1 - t)(S_3 + S_4)/S_1 = (1 - r)(1 - t)/(1 - rt) \tag{1.7}$$

Obviously, it is verified that:

$$t_N + r_N + a_N = 1 \tag{1.8}$$

At steady state, the absorbed energy is re-emitted. As the temperature of each solid layer is assumed to be uniform, the emission is the same for both directions. Thus, the net fraction of radiant energy sent forward by a solid membrane is given by:

$$T_N = t_N + \frac{a_N}{2} = \frac{(1 - r)}{(1 - rt)}\left\{\frac{(1 - r)t}{(1 + rt)} + \frac{(1 - t)}{2}\right\} \tag{1.9}$$

The fraction $(1 - T_N)$ is sent back.

APPENDIX 2

Net Radiant Energy Through a Set of n Solid Layers, Each One Transmitting a Fraction T_N of the Incident Energy

The situation is shown in *Fig. 7a*, where S_i^+ is the net radiation flux leaving the generic plane i to the right, and S_i^- is the corresponding radiation flux sent back to the left. It can be stated that:

$$S_i^+ = T_N S_{i-1}^+ + (1 - T_N)S_{i+1}^- \tag{2.1}$$

$$S_{i+1}^- = (1 - T_N)S_i^+ + T_N S_{i+2}^- \tag{2.2}$$

$$S_{i+1}^+ = T_N S_i^+ + (1 - T_N)S_{i+2}^- \tag{2.3}$$

From *Eqs 2.1* to *2.3*, the following recurrent law may be obtained:

$$S_i^+/S_{i-1}^+ = 1/(2 - S_{i+1}^+/S_i^+)\ ,\ \text{for } i < n \tag{2.4}$$

For $i = n$, the radiation S_n^+ is entirely absorbed by the black surface at T_2. Then:

$$S_n^+/S_{n-1}^+ = T_N \tag{2.5}$$

The net radiation arising from the black surface at T_1 and, reaching the black surface at T_2, is given by:

$$q_{rad}^+ = S_n^+ = \sigma T_1^4 S_n^+/S_0^+ = \sigma T_1^4 \prod_{i=1}^{n} (S_i^+/S_{i-1}^+) \tag{2.6}$$

where σ is the Stefan-Boltzmann constant.

From *Eqs 2.4* and *2.5*:

$$S_{n-1}^+/S_{n-2}^+ = 1/(2 - S_n^+/S_{n-1}^+) = 1/(2 - T_N) \tag{2.7}$$

Then, the recurrent *Eq 2.4* gives:

$$S_{n-2}^+/S_{n-3}^+ = 1/\{2 - [1/(2 - T_N)]\} = (2 - T_N)/(3 - 2T_N) \tag{2.8}$$

$$S_{n-3}^+/S_{n-4}^+ = (3 - 2T_N)/(4 - 3T_N) \tag{2.9}$$

$$S_{n-4}^+/S_{n-5}^+ = (4 - 3T_N)/(5 - 4T_N) \tag{2.10}$$

Then,

$$\prod_{i=1}^{n} (S_i^+/S_{i-1}^+) = \frac{T_N}{n - (n - 1)T_N} = \frac{1}{1 + n(1/T_N - 1)} \tag{2.12}$$

From *Eq 2.6*,

$$q_{rad}^+ = \frac{\sigma T_1^4}{1 + n(1/T_N - 1)} \tag{2.13}$$

and

$$q_{rad} = q_{rad}^+ - q_{rad}^- = \frac{\sigma(T_1^4 - T_2^4)}{1 + n(1/T_N - 1)} \tag{2.14}$$

APPENDIX 3

Net Radiant Energy between Two Grey Boundaries with Emissivities e_1 and e_2 with a Set of n Interposed Solid Layers, Each One Transmitting a Fraction T_N of the Incident Energy

The situation is depicted in *Fig. 7b*. From *Eq 2.14* of Appendix 2, the net transmission of the set of n solid layers is given by

$$T = \frac{1}{1 + n(1/T_N - 1)} \tag{3.1}$$

Then,

$$S_0^+ = e_1\sigma T_1^4 + (1 - e_1)S_1^- \tag{3.2}$$

$$S_1^- = (1 - T)S_0^+ + TS_{n+1}^- \tag{3.3}$$

$$S_n^+ = TS_0^+ + (1 - T)S_{n+1}^- \tag{3.4}$$

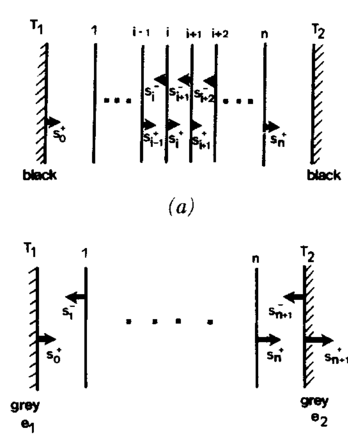
$$S_{n+1}^- = (1 - e_2)S_n^+ \tag{3.5}$$

$$S_{n+1}^+ = e_2 S_n^+ \tag{3.6}$$

The net radiation emitted from the grey surface at T_1, and absorbed by the grey surface at T_2 is:

$$q_{rad}^+ = S_{n+1}^+ \tag{3.7}$$

Solving *Eqs 3.2* to *3.6* to obtain S_{n+1}^+, and substituting into *Eq 3.7*, gives:



(a)

(b)

Fig. 7. (a) Radiant energy through a set of n solid layers with black boundaries. (b) Radiant energy through a set of n solid

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