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STUDENT EDITION

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QA 273  
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K64  
2000  
COPY 1

Much of this book originally appeared in D. Zwillinger and S. Kokoska, *Standard Probability and Statistics Tables and Formulae*, Chapman & Hall/CRC, 2000. Reprinted courtesy of Chapman & Hall/CRC.

00022661

### Library of Congress Cataloging-in-Publication Data

Catalog record is available from the Library of Congress.

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International Standard Book Number 0-8493-0026-6

# CHAPTER 3

## Probability

### 3.1 ALGEBRA OF SETS

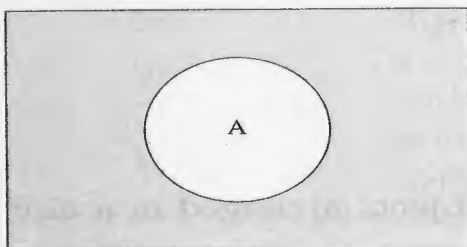


Figure 3.1: Shaded region =  $A'$ .

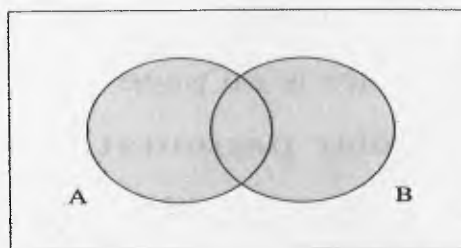


Figure 3.2: Shaded region =  $A \cup B$ .

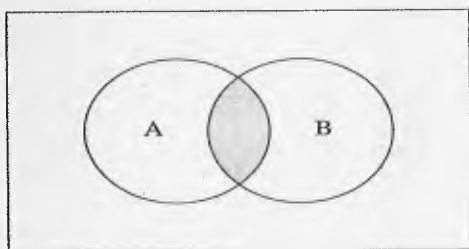


Figure 3.3: Shaded region =  $A \cap B$ .

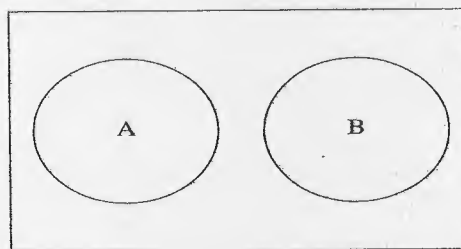


Figure 3.4: Mutually exclusive sets.

### 3.2 COMBINATORIAL METHODS

In an equally likely outcome experiment, computing the probability of an event involves counting. The following techniques are useful for determining the number of outcomes in an event and/or the sample space.

#### 3.2.1 The product rule for ordered pairs

If the first element of an ordered pair can be selected in  $n_1$  ways, and for each of these  $n_1$  ways the second element of the pair can be selected in  $n_2$  ways, then the number of possible pairs is  $n_1 n_2$ .

### 3.2.2 The generalized product rule for $k$ -tuples

Suppose a sample space, or set, consists of ordered collections of  $k$ -tuples. If there are  $n_1$  choices for the first element, and for each choice of the first element there are  $n_2$  choices for the second element,  $\dots$ , and for each of the first  $k - 1$  elements there are  $n_k$  choices for the  $k^{\text{th}}$  element, then there are  $n_1 n_2 \cdots n_k$  possible  $k$ -tuples.

### 3.2.3 Permutations

The number of permutations of  $n$  distinct objects taken  $k$  at a time is

$$P(n, k) = \frac{n!}{(n - k)!}. \quad (3.1)$$

A table of values is on page 210.

### 3.2.4 Circular permutations

The number of permutations of  $n$  distinct objects arranged in a circle is  $(n - 1)!$ .

### 3.2.5 Combinations (binomial coefficients)

The binomial coefficient  $\binom{n}{k}$  is the number of combinations of  $n$  distinct objects taken  $k$  at a time without regard to order:

$$C(n, k) = \binom{n}{k} = \frac{n!}{k!(n - k)!} = \frac{P(n, k)}{k!}. \quad (3.2)$$

See page 210 for a table of values. Other formulas involving binomial coefficients include:

$$(a) \quad \binom{n}{k} = \frac{n(n - 1) \cdots (n - k + 1)}{k!} = \binom{n}{n - k}$$

$$(b) \quad \binom{n}{0} = \binom{n}{n} = 1 \quad \text{and} \quad \binom{n}{1} = n$$

$$(c) \quad \binom{n}{k} = \binom{n - 1}{k} + \binom{n - 1}{k - 1}$$

$$(d) \quad \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$$

$$(e) \quad \binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0$$

**Example 3.8:** For the 5 element set  $\{a, b, c, d, e\}$  find the number of subsets containing exactly 3 elements.

**Solution:**

$$(S1) \quad \text{There are } \binom{5}{3} = \frac{5!}{3!2!} = 10 \text{ subsets containing exactly 3 elements.}$$





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