CRC

STANDARD PROBABILITY AND STATISTICS TABLES AND FORMULAE

STUDENT EDITION

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STEPHEN KOKOSKA
Bloomsburg University
Bloomsburg, Pennsylvania

DANIEL ZWILLINGER
Rensselaer Polytechnic Institute

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CHAPTER 3

Probability

3.1 ALGEBRA OF SETS

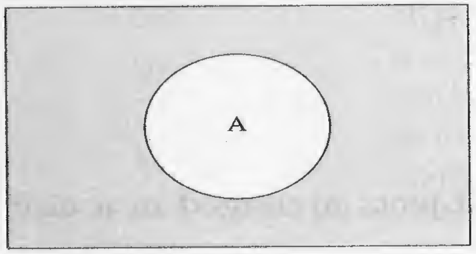

Figure 3.1: Shaded region = A'.

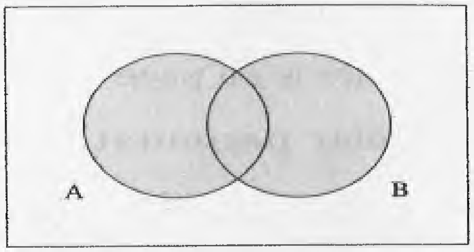

Figure 3.2: Shaded region = $A \cup B$.

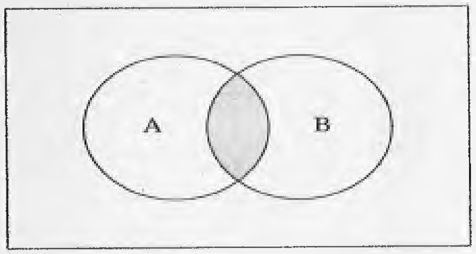

Figure 3.3: Shaded region = $A \cap B$.

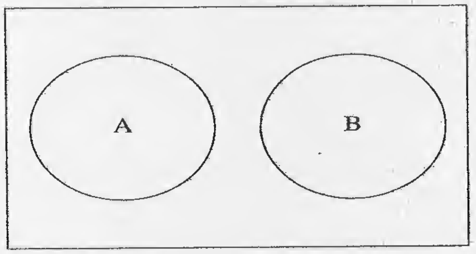

Figure 3.4: Mutually exclusive sets.

3.2 COMBINATORIAL METHODS

In an equally likely outcome experiment, computing the probability of an event involves counting. The following techniques are useful for determining the number of outcomes in an event and/or the sample space.

3.2.1 The product rule for ordered pairs

If the first element of an ordered pair can be selected in n_1 ways, and for each of these n_1 ways the second element of the pair can be selected in n_2 ways, then the number of possible pairs is $n_1 n_2$.

3.2.2 The generalized product rule for k-tuples

Suppose a sample space, or set, consists of ordered collections of k-tuples. If there are n_1 choices for the first element, and for each choice of the first element there are n_2 choices for the second element, ..., and for each of the first $k-1$ elements there are n_k choices for the k^{th} element, then there are $n_1 n_2 \cdots n_k$ possible k-tuples.

3.2.3 Permutations

The number of permutations of n distinct objects taken k at a time is

$$P(n,k) = \frac{n!}{(n-k)!}\,. \tag{3.1}$$

A table of values is on page 210.

3.2.4 Circular permutations

The number of permutations of n distinct objects arranged in a circle is $(n-1)!$.

3.2.5 Combinations (binomial coefficients)

The binomial coefficient $\binom{n}{k}$ is the number of combinations of n distinct objects taken k at a time without regard to order:

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{P(n,k)}{k!}\,. \tag{3.2}$$

See page 210 for a table of values. Other formulas involving binomial coefficients include:

(a) $\binom{n}{k} = \dfrac{n(n-1)\cdots(n-k+1)}{k!} = \binom{n}{n-k}$

(b) $\binom{n}{0} = \binom{n}{n} = 1$ and $\binom{n}{1} = n$

(c) $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

(d) $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$

(e) $\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0$

Example 3.8: For the 5 element set $\{a,b,c,d,e\}$ find the number of subsets containing exactly 3 elements.

Solution:

(S1) There are $\binom{5}{3} = \dfrac{5!}{3!2!} = 10$ subsets containing exactly 3 elements.

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