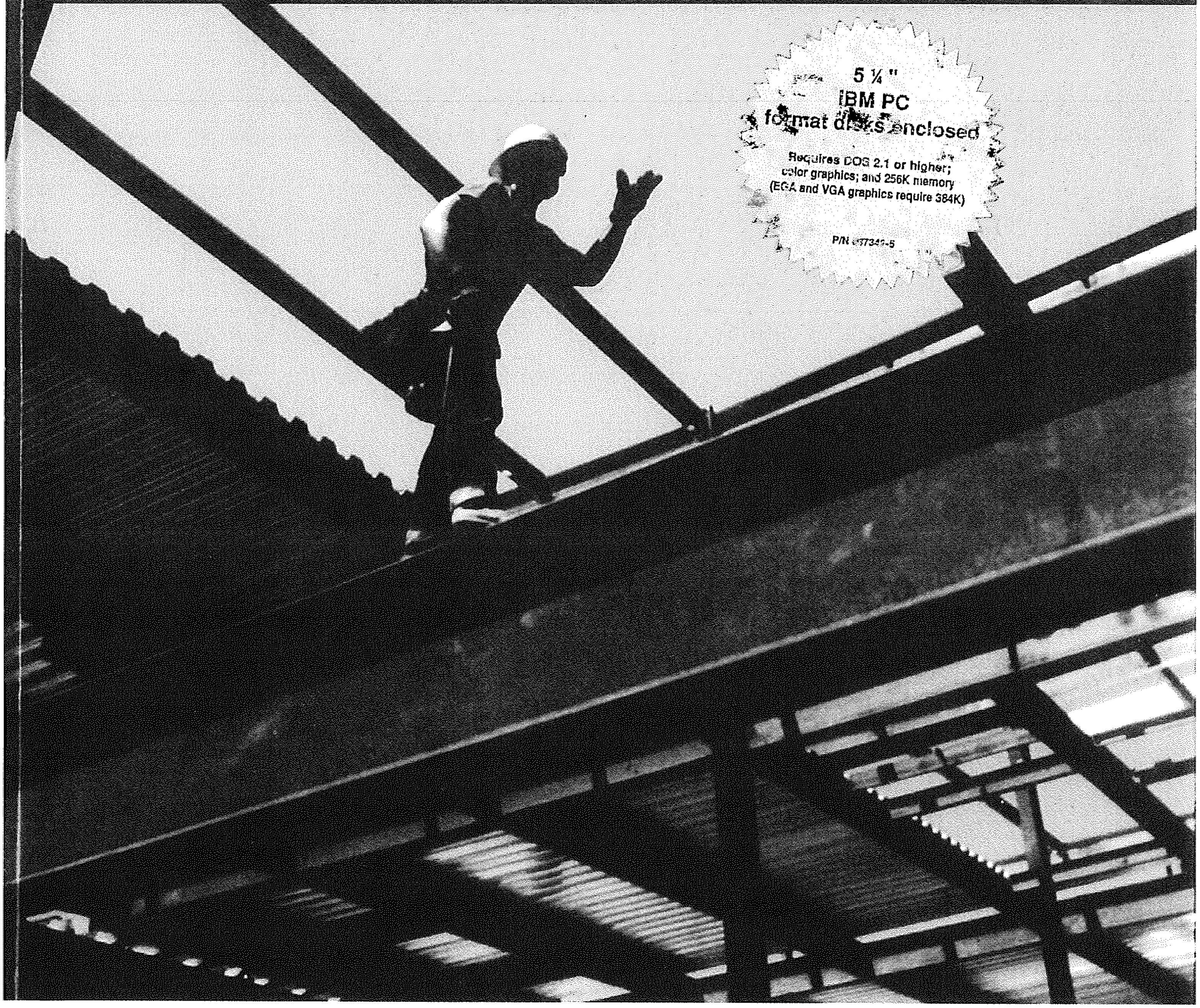
MECHANICS OF MATERIALS

SECOND EDITION

FERDINAND P. BEER ▪ E. RUSSELL JOHNSTON, JR.



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C H A P T E R E I G H T

DEFLECTION OF BEAMS BY INTEGRATION

8.1. INTRODUCTION

In the preceding chapter we learned to design beams for strength. In this chapter and in the next, we shall be concerned with another aspect of the design of beams, namely, the determination of the *deflection* of prismatic beams under given loadings. Of particular interest is the determination of the *maximum deflection* of a beam under a given loading, since the design specifications of a beam will generally include a maximum allowable value for its deflection.

We saw in Sec. 4.5 that a prismatic beam subjected to pure bending is bent into an arc of circle and that, within the elastic range, the curvature of the neutral surface may be expressed as

$$\frac{1}{\rho} = \frac{M}{EI} \qquad (4.21)$$

where M is the bending moment, E the modulus of elasticity, and I the moment of inertia of the cross section about its neutral axis.

When a beam is subjected to a transverse loading, Eq. (4.21) remains valid for any given transverse section, provided that Saint-Venant's principle applies. However, both the bending moment and the curvature of the neutral surface will vary from section to section. Denoting by x the distance of the section from the left end of the beam, we shall write

$$\frac{1}{\rho} = \frac{M(x)}{EI} \qquad (8.1)$$

The knowledge of the curvature at various points of the beam will enable us to draw some general conclusions regarding the deformation of the beam under loading (Sec. 8.2).

To determine the slope and deflection of the beam at any given point, we shall first derive the following second-order linear differential equation, which governs the *elastic curve* characterizing the shape of the

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deformed beam (Sec. 8.3):

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

If the bending moment may be represented for all values of x by a single function $M(x)$, as in the case of the beams and loadings shown in Fig. 8.1, the slope $\theta = dy/dx$ and the deflection y at any point of the beam may be obtained through two successive integrations. The two constants of integration introduced in the process will be determined from the boundary conditions indicated in the figure.

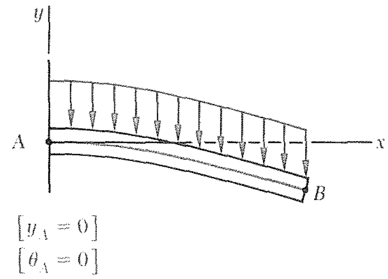
However, if different analytical functions are required to represent the bending moment in various portions of the beam, different differential equations will also be required, leading to different functions defining the elastic curve in the various portions of the beam. In the case of the beam and loading of Fig. 8.2, for example, two differential equations are required, one for the portion of beam AD and the other for the portion DB. The first equation yields the functions θ_1 and y_1, and the second the functions θ_2 and y_2. Altogether, four constants of integration must be determined; two will be obtained by writing that the deflection is zero at A and B, and the other two by expressing that the portions of beam AD and DB have the same slope and the same deflection at D.

We shall observe in Sec. 8.4 that in the case of a beam supporting a distributed load $w(x)$, the elastic curve may be obtained directly from $w(x)$ through four successive integrations. The constants introduced in this process will be determined from the boundary values of V, M, θ, and y.

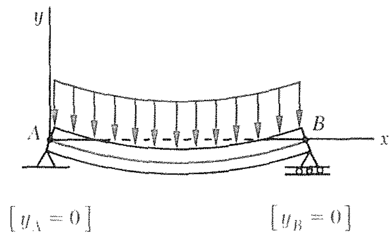
In Sec. 8.5, we shall discuss *statically indeterminate beams*, i.e., beams supported in such a way that the reactions at the supports involve four or more unknowns. Since only three equilibrium equations are available to determine these unknowns, the equilibrium equations must be supplemented by equations obtained from the boundary conditions imposed by the supports.

The method described earlier for the determination of the elastic curve when several functions are required to represent the bending moment M may be quite laborious, since it requires matching slopes and deflections at every transition point. We shall see in Sec. 8.6 that the use of *singularity functions* (previously discussed in Sec. 7.5) considerably simplifies the determination of θ and y at any point of the beam. An alternative method for the determination of θ and y, based on certain geometric properties of the elastic curve and involving the computation of areas and moments of areas under the bending-moment curve, will be discussed in Chap. 9.

The last part of the chapter (Secs. 8.7 and 8.8) is devoted to the *method of superposition*, which consists of determining separately, and then adding, the slope and deflection caused by the various loads applied to a beam. This procedure may be facilitated by the use of the table in Appendix D, which gives the slopes and deflections of beams for various loadings and types of support.



(a) Cantilever beam



(b) Simply supported beam

Fig. 8.1

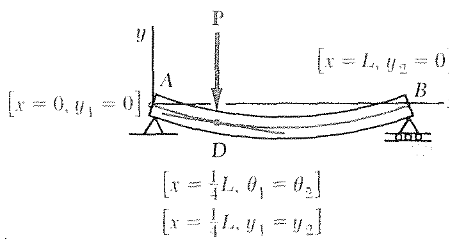


Fig. 8.2

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