Topology Preserving Data Simplification with Error Bounds

Chandrajit L. Bajaj

Department of Computer Sciences and TICAM, University of Texas, Austin, TX 78712

Daniel R. Schikore

Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, P.O. Box 808, L-561, Livermore, CA 94550

Many approaches to simplification of triangulated terrains and surfaces have been proposed which permit bounds on the error introduced. A few algorithms additionally bound errors in auxiliary functions defined over the triangulation. We present an approach to simplification of scalar fields over unstructured grids which preserves the topology of functions defined over the triangulation, in addition to bounding of the errors. The topology of a 2D scalar field is defined by critical points (local maxima, local minima, saddle points), in addition to integral curves between them, which together segment the field into regions which vary monotonically. By preserving this shape description, we guarantee that isocontours of the scalar function maintain the correct topology in the simplified model. Methods for topology preserving simplification by both point-insertion (refinement) and point-deletion (coarsening) are presented and compared.

Keywords: Simplification, Scientific Visualization, Scalar Fields

1 Introduction

Scientific data is often sampled or computed over a dense mesh in order to capture high frequency components or achieve a desired error bound. Interactive display and navigation of such large meshes is impeded by the sheer number of triangles required to sufficiently model highly complex data. A number of simplification techniques have been developed which reduce the number of triangles to a particular desired triangle count or until a particular error threshold is met. Given an initial triangulation M of a domain $\mathcal{D}$ and a function $\mathcal{F}(\mathbf{x})$ defined over the triangulation, the simplified mesh can be called M' and the resulting function $\mathcal{F}'(\mathbf{x})$. The measure of error in a simplified mesh M^i is usually represented as:

$$\epsilon(M') = \max_{\mathbf{x} \in \mathcal{D}}(|\mathcal{F}(\mathbf{x}) - \mathcal{F}'(\mathbf{x})|) \qquad (1)$$

The ability to bound the error $\epsilon(M')$ is very important, but the error definition (1) is inherently a local measure, neglecting to consider global features of the data. We introduce new criteria for the simplification of sampled functions which preserves scalar field features in addition to bounding local errors. Two-dimensional scalar field topology is described by the critical points and arcs between them. Preserving the scalar field criticalities maintains an invariant of the connectivity and combinatorial structure (topological genus) of successively simplified isocontours.

In Section 2 we discuss related work in mesh simplification and feature detection. Section 3 introduces the definition of 2D scalar topology as it will be used in our simplification strategy. In Section 4 we introduce two algorithms for simplification with topology preserving characteristics. The first is an extension to existing *coarsening* techniques which iteratively delete vertices or contract edges in the mesh. The second algorithm adopts an inverse approach, iteratively introducing detail (*refinement*) to an initially sparse mesh, preserving the scalar topology of the fine mesh.

2 Related Work

2.1 Simplification

A wide variety of algorithms have been developed for the simplification of meshes. We present a brief overview of the classes of algorithms which have been proposed for geometry and data simplification.

Vertex insertion/deletion

A large class of data and geometry simplification algorithms are based on successive application of one or more topological mesh operators, such as *edge contraction*, which contracts an edge of the mesh to a point, or *vertex deletion*, in which a vertex and adjacent triangles are removed and replaced with a covering of the resulting hole. Point insertion and deletion approaches have been explored by many researchers for application in Geographical Information Systems (GIS). A common technique is to extract key points of data from the originally dense set of points, and compute a Delaunay triangulation [7,11,12,33,39,41]. Silva et al. [37] use a greedy approach for inserting points into an initially sparse mesh. Schroeder et al. [36] compute reduced representations for dense triangular surface meshes such as those computed by Marching Cubes [25] or similar isosurfacing algorithms. Vertices in the dense mesh are examined and classified based on geometric features in the triangulation surrounding the vertex. If error criteria are satisfied, the vertex is deleted and the resulting hole is retriangulated. Retriangulation is guided by local edges detected in the classification stage and aspect ratios of new triangles. Several passes over the object successively remove vertices until no vertex satisfies the criteria for removal. There is no cumulative error measure, and therefore no guarantee on the amount of accumulated error in the final representation. Hamann [15] applies a similar technique in which triangles are considered for deletion based on curvature estimates at the vertices. Reduction may be driven by mesh resolution or, in the case of functional surfaces, root-mean-square (RMS) error. Ronfard et al. [34] apply successive edge contraction operations to compute a wide range of levels-of-detail for triangulated polyhedra. Edges are extracted from a priority queue based on a computed *edge cost* such that edges of lesser significance are removed first. Guéziec [14] introduces a *tolerance volume* for bounding the error resulting from successive edge contraction operations. The resulting merged vertex is positioned such that the volume remains constant. Cohen et al. [6] introduce *Simplification Envelopes* to guide mesh simplification with global error bounds. Envelopes are an extension of *offset surfaces* which serve as an extreme boundary for the desired simplified surface.

Region Merging

Hinker et al. [19] perform "geometric optimization" on triangular surface meshes by grouping faces into contiguous sets which are nearly co-planar. Points interior to a region and points along nearly lin ar boundaries of regions are deleted, and the resulting hole is retriangulated. Kalvin et al. [24] cluster mesh faces into *superfaces*, triangulating the resulting polygons for a simplified representation.

Filtering

Filtering techniques are capable of producing a large range of simplified models through application of grouping and merging rules. An attractive feature of filtering techniques is the ability to simplify objects to a minimal representation through successive applications. Subsampling is a simple type of filtering which is easily applied to subdivision meshes for which there exists a natural remeshing when nested sets of vertices are successively deleted. The major drawback to subsampling is that there is no bound on the error which is introduced through its application. Rossignac et al. [35] use clustering and merging of features of an object based on a regular spatial subdivision. Clustering approaches have the advantage that small features which are geometrically close but not topologically connected can be grouped and merged for higher rates of simplification. In this scheme long, thin objects may collapse to an edge and small objects may collapse to a point. He et al. [16] provide more control over subsampling of regular grids by filtering the simplified mesh at each step. The regular grid corresponds to a sampling of the signed-distance function of a 3D surface. A multi-resolution triangle mesh is extracted from the resulting multi-resolution volume buffer using traditional isosurfacing techniques.

Optimization

Optimization methods define measures of energies for point sets or triangulations based on an original mesh, and use interactive optimization to minimize these energies in forming a simplified mesh. Turk [40] computes simplified polygonal surfaces at a desired number of vertices. Contrast this with the point insertion and deletion methods which are usually driven by error computations rather than desired resolution. Given the desired number of vertices, point repulsion on the polygonal surface spreads the points out. A mutual tessellation of the original triangulation and the introduced points followed by deletion of the original vertices guarantees that the topology of the polygonal surface is maintained. Point repulsion is adjusted based on estimated curvature of the surface, providing an adaptive triangulation which maintains geometric features. Hoppe et al. [21] perform time-intensive mesh optimization based on the definition of an energy function which balances the need for accurate geometry with the desire for compactness in representation. The level of mesh simplification is controlled by a parameter in the energy function which penalizes meshes with large numbers of vertices, as well as a spring constant which helps guide the energy minimization to a desirable result. In [20], Hoppe applies the optimization framework to the simplification of scalar fields.

Multi-resolution analysis

Multi-resolution analysis is a structured mathematical decomposition of functions into multiple levels of representation. Through the use of wavelet transforms [10,27], a hierarchical representation of functions can be obtained by repeatedly breaking the function into a coarser representation in addition to a set of perturbation coefficients which allow the full recovery of the original representation from the coarse representation. Generally, the wavelet basis is chosen such that the perturbation coefficients have desirable attributes such as direct correlation with some measure of error which is introduced at a given level of representation. During reconstruction from the wavelet representation, sufficiently small wavelet coefficients can be left out, resulting in a coarser approximation to the original data, with a known bound on the amount of error [26,8,38]. Further extensions have provided similar basis for the decomposition of surfaces [9]. Muraki [31] applies wavelets in 3D to compute multi-resolution models of 3D volume data. Isosurfaces and planar cross sections of the resulting data show little change in image quality with large reductions in the amount of data representing the volume.

2.2 Feature Detection

The problem of detecting ridges and valleys in digital terrain has been considered in several papers [12]. McCormack et al. [29] consider the problem of detecting drainage patterns in geographic terrain. Interrante et al. [22] used ridge and valley detection on 3D surfaces to enhance the shape of transparently rendered surfaces. Extrema graphs were used by Itoh and Koyamada to speed isocontour extraction [23]. A graph containing extreme points and boundary points of a scalar field can be guaranteed to intersect every isocontour at least once, allowing seed points to be generated by searching only the cells contained in the extrema graph. Helman and Hesselink detect vector field topology by classifying the zeros of a vector field and performing particle tracing from saddle points [18]. The resulting partitioning consists of regions which are topologically equivalent to uniform flow. Globus et aldescribe a software system for 3D vector topology and briefly note that the technique may also be applied to the gradient of a scalar field in order to identify maxima and minima [13]. Bader examines the gradient field of the charge density in a molecular system [1]. The topology of this scalar field represents the bonds linking together the atoms of the molecule. Bader goes on to show how features higher level structures in the topology represent chains, rings, and cages in the molecule.

2.3 Our Approach

Simplification techniques have advanced to the point at which it is useful to now consider preserving global mesh and data features. In the following sec-

DOCKET ALARM

Explore Litigation Insights

Docket Alarm provides insights to develop a more informed litigation strategy and the peace of mind of knowing you're on top of things.

Real-Time Litigation Alerts

Keep your litigation team up-to-date with **real-time alerts** and advanced team management tools built for the enterprise, all while greatly reducing PACER spend.

Our comprehensive service means we can handle Federal, State, and Administrative courts across the country.

Advanced Docket Research

With over 230 million records, Docket Alarm's cloud-native docket research platform finds what other services can't. Coverage includes Federal, State, plus PTAB, TTAB, ITC and NLRB decisions, all in one place.

Identify arguments that have been successful in the past with full text, pinpoint searching. Link to case law cited within any court document via Fastcase.

Analytics At Your Fingertips

Learn what happened the last time a particular judge, opposing counsel or company faced cases similar to yours.

Advanced out-of-the-box PTAB and TTAB analytics are always at your fingertips.

API

Docket Alarm offers a powerful API (application programming interface) to developers that want to integrate case filings into their apps.

LAW FIRMS

Build custom dashboards for your attorneys and clients with live data direct from the court.

Automate many repetitive legal tasks like conflict checks, document management, and marketing.

FINANCIAL INSTITUTIONS

Litigation and bankruptcy checks for companies and debtors.

E-DISCOVERY AND LEGAL VENDORS

Sync your system to PACER to automate legal marketing.

WHAT WILL YOU BUILD? | **sales@docketalarm.com** | **1-866-77-FASTCASE**

fastcase®
Smarter legal research.