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Gas Discharge Physics

With 209 Figures

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$\Gamma_e \nu_u - qI$, $\qquad$ (2.50)

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d of the hydrodynamic flux of
fficient.[3] The terms containing
$\ll kT_e$, are neglected; accord-
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the effects caused by *spatial*
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in (2.22, 50–52) in the form
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ount that $\mu_+ \ll \mu_e$, we obtain

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3. Interaction of Electrons in an Ionized Gas with Oscillating Electric Field and Electromagnetic Waves

3.1 The Motion of Electrons in Oscillating Fields

Both the equations of electrodynamics and the equations of motion of electrons are linear with respect to the fields E, H and the velocity v of the electron. For this reason, the *superposition principle* holds. Any periodical field can be resolved into harmonic components, so that it is sufficient to consider only the *sinusoidal* field, all the more so because one normally deals with *monochromatic* fields and waves. In the case of nonrelativistic motion, the magnetic force of the wave, $e(v/c)H$, is much less than the electric force eE. Furthermore, the amplitude of electron oscillations in discharge processes is usually small in comparison with wavelength λ. We assume, therefore, that the electron is in a spatially uniform electric field $E = E_0 \sin \omega t$, $E_0 = \text{const}$.

3.1.1 Free Oscillations

Assume that an electron moves without collisions, an assumption that is meaningful if the electron performs a large number of oscillations in the interval between collisions, $\omega \gg \nu_m$. We integrate the equation of collisionless motion,

$$m\dot{v} = -eE_0 \sin \omega t , \quad \dot{r} = v ,$$

to give

$$v = \frac{eE_0}{m\omega} \cos \omega t + v_0 , \quad r = \frac{eE_0}{m\omega^2} \sin \omega t + v_0 t + r_0 . \qquad (3.1)$$

An electron oscillates at the frequency of the field; these oscillations are superimposed onto an arbitrary translation velocity v_0. The displacement and oscillation velocities are

$$a = \frac{eE_0}{m\omega^2} , \quad u = \frac{eE_0}{m\omega} . \qquad (3.2)$$

The displacement is in phase with the field, while the velocity is out of phase by $\pi/2$. The limiting case of "collisionless" oscillations is approximately realized at optical frequencies, and also at microwave frequencies at low pressures, $p \lesssim 10 \, \text{Torr}$.

3.1.2 Effect of Collisions

Collisions "throw off" the phase, thereby disturbing the purely harmonic course of the electron's oscillations. A sharp change in the direction of motion after

scattering stops the electron from achieving the full range of displacement (3.2) that the applied force can produce; the electron starts oscillating anew after each collision, with a new phase and new angle relative to the instantaneous direction of velocity. In order to take this factor into account, we add the rate of loss of momentum due to collisions to the equation of motion of the "mean" electron. As in the case of constant fields (Sect. 2.1.1), we have the equation for the mean velocity:

$$m\dot{v} = -eE_0 \sin \omega t - mv\nu_m , \quad \dot{r} = v . \tag{3.3}$$

The solution of (3.3), valid after several collisions, is

$$v = \frac{eE_0}{m\sqrt{\omega^2 + \nu_m^2}} \cos(\omega t + \varphi) , \quad \varphi = \arctan\frac{\nu_m}{\omega} ,$$

$$r = \frac{eE_0}{m\omega\sqrt{\omega^2 + \nu_m^2}} \sin(\omega t + \varphi) . \tag{3.4}$$

The amplitudes of displacement and velocity of the electron are less by a factor of $\sqrt{1 + \nu_m^2/\omega^2}$ than those for free oscillations. The higher the effective collision frequency ν_m, the smaller they are (ν_m is determined by the velocity of random motion, which is much greater in discharges than the oscillation velocity; see Sect. 3.2). The displacement is shifted in phase relative to the field, the phase shift increasing from 0 to $\pi/2$ as the relative role of collisons ν_m/ω increases from 0 to ∞.

The oscillation displacement and velocity (3.4) can always be resolved into two components, one proportional to the magnitude of the field $E = E_0 \sin \omega t$, and the other to its rate of change, $\dot{E} = \omega E_0 \cos \omega t$:

$$r = \frac{eE_0}{m(\omega^2 + \nu_m^2)} \sin \omega t + \frac{\nu_m}{\omega} \frac{eE_0}{m(\omega^2 + \nu_m^2)} \cos \omega t ,$$

$$v = \frac{\omega eE_0}{m(\omega^2 + \nu_m^2)} \cos \omega t - \frac{\nu_m eE_0}{m(\omega^2 + \nu_m^2)} \sin \omega t . \tag{3.5}$$

The ratio of the components is determined by the relative role of collisions and is unambiguously related to the phase shift φ. This form of presenting the solution adds visual clarity to the results of the subsequent sections.

Expressions (3.4,5) show that the role of collisions is characterized by the ratio of the effective frequency ν_m and the circular frequency of the field $\omega = 2\pi f$, which is greater than the frequency f by nearly an order of magnitude.[1] In the limit $\nu_m^2 \ll \omega^2$, formulas (3.4,5) are close to (3.1) for free oscillations. To illustrate numerical values, consider an example of microwave radiation at frequency $f = 3\,\text{GHz}$; $\lambda = 10\,\text{cm}$, $\omega = 1.9 \times 10^{10}\,\text{s}^{-1}$. Let $p \approx 1\,\text{Torr}$, then $\nu_m \approx 3 \times 10^9\,\text{s}^{-1} \ll \omega$; $E_0 = 500\,\text{V/cm}$, roughly corresponding to the threshold

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[1] When the degree of spatial uniformity of the field is evaluated, the displacement amplitude must be compared not with wavelength $\lambda = c/f$, but with $\lambdabar = \lambda/2\pi$: $a/\lambdabar = eE_0/m\omega^2 \lambdabar = u/c$.

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of microwave breakdown at such pressures. Formulas (3.2) show that $a = 2.5 \times 10^{-3}$ cm, $u = 4.7 \times 10^7$ cm/s. We find that $a \ll \lambda = 1.6$ cm, that is, the field in the electromagnetic wave is "uniform".

3.1.3 Drift Oscillations

In the limit of very frequent collisions or relatively low frequencies, $\nu_m^2 \gg \omega^2$, the oscillation velocity drops to

$$v \approx -\frac{eE_0}{m\nu_m} \sin \omega t = -\frac{eE(t)}{m\nu_m} = -\mu_e E(t) = v_d(t) \ . \tag{3.6}$$

At each moment of time, the oscillation velocity coincides with the drift velocity that corresponds to the field vector at this moment. For brevity, we refer to such oscillations in the *mobility regime* as *drift oscillations*.

An electron behaves as it would in a constant field, responding to relatively slow changes of the field. Its displacement,

$$r \approx A \cos \omega t \ , \quad A = \frac{eE_0}{m\nu_m\omega} = \frac{\mu_e E_0}{\omega} \ , \tag{3.7}$$

has an amplitude A less than that of free oscillations in the same field by a factor of $\nu_m/\omega \gg 1$.

The oscillations of electrons in rf fields (and of course, at lower frequencies) are of drift type. For example, the collision frequency at $f \approx 10$ MHz, $\nu_m \approx 3 \times 10^9 \, p \, \text{s}^{-1}$, exceeds $\omega \sim 10^8 \, \text{s}^{-1}$ even at fairly low pressures of $p \sim 0.03$ Torr. In order to maintain a low-pressure, weakly ionized plasma by an rf field, one usually needs the values of E_0/p of the same order as E/p in a constant field. Therefore, at $f \approx 10$ MHz and $E_0/p \approx 10$ V/(cm · Torr), we have $A \sim 0.1$ cm regardless of pressure.

3.2 Electron Energy

3.2.1 Collisionless Motion

If collisions do not occur, the field does no work, on the average, on an electron; indeed, (3.1) implies that

$$\langle -eE \cdot v \rangle = -\frac{eE_0^2}{m\omega} \langle \sin \omega t \cos \omega t \rangle - eE_0 \cdot v_0 \langle \sin \omega t \rangle = 0 \ ,$$

where angle brackets denote time averaging.

The electric field pumps up the motion of the electron only once, when it is switched on; then the electron's energy $mv^2/2$ pulsates but remains unchanged on the average. The time averaged energy $\langle mv^2/2 \rangle$ is made up of the energy of translational motion $mv_0^2/2$, corresponding to the mean velocity $v_0 = \langle v(t) \rangle$, and that of oscillations. In the case of free oscillations, the latter energy is

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