

# Principles of Optics

*Electromagnetic Theory of Propagation,  
Interference and Diffraction of Light*

*by*

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SIXTH (CORRECTED) EDITION



PERGAMON PRESS

OXFORD · NEW YORK · BEIJING · FRANKFURT  
SÃO PAULO · SYDNEY · TOKYO · TORONTO

U.K.	Pergamon Press, Headington Hill Hall, Oxford OX3 0BW, England
U.S.A.	Pergamon Press, Maxwell House, Fairview Park, Elmsford, New York 10523, U.S.A.
PEOPLE'S REPUBLIC OF CHINA	Pergamon Press, Qianmen Hotel, Beijing, People's Republic of China
FEDERAL REPUBLIC OF GERMANY	Pergamon Press, Hammerweg 6, D-6242 Kronberg, Federal Republic of Germany
BRAZIL	Pergamon Editora, Rua Eça de Queiros, 346, CEP 04011, São Paulo, Brazil
AUSTRALIA	Pergamon Press Australia, P.O. Box 544, Potts Point, N.S.W. 2011, Australia
JAPAN	Pergamon Press, 8th Floor, Matsuoka Central Building, 1-7-1 Nishishinjuku, Shinjuku-ku, Tokyo 160, Japan
CANADA	Pergamon Press Canada, Suite 104, 150 Consumers Road, Willowdale, Ontario M2J 1P9, Canada

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First edition 1959  
 Second (revised) edition 1964  
 Third (revised) edition 1965  
 Fourth (revised) edition 1970  
 Fifth (revised) edition 1975  
 Reprinted 1975, 1977  
 Sixth edition 1980  
 Reprinted (with corrections) 1983  
 Reprinted 1984  
 Reprinted (with corrections) 1986  
 Reprinted 1987

**Library of Congress Cataloging in Publication Data**

Born, Max  
 Principles of optics—6th ed. (with corrections).  
 I. Optics  
 I. Title II. Wolf, Emil  
 535 QC351 80-41470  
 ISBN 0-08-026482-4 Hardcover  
 ISBN 0-08-026481-6 Flexicover

*Printed in Great Britain by A. Wheaton & Co. Ltd., Exeter*

TABLE XXVI

The "penetration depth"  $d$  for copper for radiation in three familiar regions of the spectrum, calculated with the static conductivity  $\sigma \sim 5.14 \cdot 10^{17} \text{ sec}^{-1}$  and  $\mu = 1$ .

Radiation	Infra-red	Microwaves	Long radio waves
$\lambda_0$	$10^{-3} \text{ cm}$	$10 \text{ cm}$	$1000 \text{ m} = 10^5 \text{ cm}$
$d$	$6.1 \cdot 10^{-7} \text{ cm}$	$6.1 \cdot 10^{-5} \text{ cm}$	$6.1 \cdot 10^{-3} \text{ cm}$

A perfect conductor is characterized by infinitely large conductivity ( $\sigma \rightarrow \infty$ ). Since according to (16),  $\epsilon/\sigma = (1 - \kappa^2)/\nu\kappa$ , we have in this limiting case  $\kappa^2 \rightarrow 1$ , or by (16a),  $n \rightarrow \infty$ . Such a conductor would not permit the penetration of an electromagnetic wave to any depth at all and would reflect all the incident light (cf. § 13.2 below).

Whilst the refractive index of transparent substances may easily be measured from the angle of refraction, such measurements are extremely difficult to carry out for metals, because a specimen of the metal which transmits any appreciable fraction of incident light has to be exceedingly thin. Nevertheless KUNDT\* succeeded in constructing metal prisms that enabled direct measurements of the real and imaginary parts of the complex refractive index to be made. Usually, however, the optical constants of metals are determined by means of katoptric rather than dioptric experiments, i.e. by studying the changes which light undergoes on reflection from a metal, rather than by means of measurements on the light transmitted through it.

### 13.2. REFRACTION AND REFLECTION AT A METAL SURFACE

We have seen that the basic equations relating to the propagation of a plane time-harmonic wave in a conducting medium differ from those relating to propagation in a transparent dielectric only in that the real constants  $\epsilon$  and  $k$  are replaced by complex constants  $\hat{\epsilon}$  and  $\hat{k}$ . It follows that the formulae derived in Chapter I, as far as they involve only linear relations between the components of the field vectors of plane monochromatic waves, apply also in the present case. In particular, the boundary conditions for the propagation of a wave across a surface of discontinuity and hence also the formulae of § 1.5 relating to refraction and reflection remain valid.

Consider first the propagation of a plane wave from a dielectric into a conductor, both media being assumed to be of infinite extent, the surface of contact between them being the plane  $z = 0$ . By analogy with § 1.5 (8) the law of refraction is

$$\sin \theta_t = \frac{1}{\hat{n}} \sin \theta_i. \quad (1)$$

Since  $\hat{n}$  is complex, so is  $\theta_t$ , and this quantity therefore no longer has the simple significance of an angle of refraction.

Let the plane of incidence be the  $xz$ -plane. The space-dependent part of the phase of the wave in the conductor is given by  $\hat{k}(\mathbf{r} \cdot \mathbf{s}^{(t)})$  where (cf. § 1.5 (4))

$$s_x^{(t)} = \sin \theta_t, \quad s_y^{(t)} = 0, \quad s_z^{(t)} = \cos \theta_t. \quad (2)$$

\* A. KUNDT, *Ann. d. Physik*, **34** (1888). 469.



From (1) and (2) and § 13.1 (15)

$$s_x^{(t)} = \sin \theta_i = \frac{\sin \theta_i}{n(1 + i\kappa)} = \frac{1 - i\kappa}{n(1 + \kappa^2)} \sin \theta_i, \quad (3a)$$

$$s_z^{(t)} = \cos \theta_i = \sqrt{1 - \sin^2 \theta_i} \\ = \sqrt{1 - \frac{(1 - \kappa^2)}{n^2(1 + \kappa^2)^2} \sin^2 \theta_i + i \frac{2\kappa}{n^2(1 + \kappa^2)^2} \sin^2 \theta_i}. \quad (3b)$$

It is convenient to express  $s_z^{(t)}$  in the form

$$s_z^{(t)} = \cos \theta_i = qe^{i\gamma} \quad (4)$$

( $q, \gamma$  real). Expressions for  $q$  and  $\gamma$  in terms of  $n, \kappa$  and  $\sin \theta_i$  are immediately obtained on squaring (3b) and (4) and equating real and imaginary parts. This gives

$$\left. \begin{aligned} q^2 \cos 2\gamma &= 1 - \frac{1 - \kappa^2}{n^2(1 + \kappa^2)^2} \sin^2 \theta_i, \\ q^2 \sin 2\gamma &= \frac{2\kappa}{n^2(1 + \kappa^2)^2} \sin^2 \theta_i. \end{aligned} \right\} \quad (5)$$

It follows that

$$\begin{aligned} \hat{k}(\mathbf{r} \cdot \mathbf{s}^{(t)}) &= \frac{\omega}{c} n(1 + i\kappa)(xs_x^{(t)} + zs_z^{(t)}) \\ &= \frac{\omega}{c} n(1 + i\kappa) \left[ \frac{x(1 - i\kappa)}{n(1 + \kappa^2)} \sin \theta_i + z(q \cos \gamma + iq \sin \gamma) \right] \\ &= \frac{\omega}{c} [x \sin \theta_i + znq (\cos \gamma - \kappa \sin \gamma) + inzq(\kappa \cos \gamma + \sin \gamma)]. \end{aligned} \quad (6)$$

We see that the surfaces of constant amplitude are given by

$$z = \text{constant}, \quad (7)$$

and are, therefore, planes parallel to the boundary. The surfaces of constant real phase are given by

$$x \sin \theta_i + znq (\cos \gamma - \kappa \sin \gamma) = \text{constant}, \quad (8)$$

and are planes whose normals make an angle  $\theta'_i$  with the normal to the boundary, where

$$\left. \begin{aligned} \cos \theta'_i &= \frac{ng(\cos \gamma - \kappa \sin \gamma)}{\sqrt{\sin^2 \theta_i + n^2g^2(\cos \gamma - \kappa \sin \gamma)^2}}, \\ \sin \theta'_i &= \frac{\sin \theta_i}{\sqrt{\sin^2 \theta_i + n^2g^2(\cos \gamma - \kappa \sin \gamma)^2}}. \end{aligned} \right\} \quad (9)$$

Since the surfaces of constant amplitude and the surfaces of constant phase do not in general coincide with each other, the wave in the metal is an *inhomogeneous wave*.

If we denote the square root in (9) by  $n'$ , the equation for  $\sin \theta'_i$  may be written in the form  $\sin \theta' = \sin \theta_i/n'$ , i.e. it has the form of SNELL'S law. However,  $n'$  depends

now not only on the quantities that specify the medium, but also on the angle of incidence  $\theta_i$ .

We may also derive expressions for the amplitude and the phase of the refracted and reflected waves by substituting for  $\theta_t$  the complex value given by (1) in the FRESNEL formulae (§ 1.5.2). The explicit expressions will be given in § 13.4.1 in connection with the theory of stratified conducting media. Here we shall consider how the optical constants of the metal may be deduced from observation of the reflected wave.

Since we assumed that the first medium is a dielectric, the reflected wave is an ordinary (homogeneous) wave with a real phase factor. As in § 1.5 (21a) the amplitude components  $A_{\parallel}$ ,  $A_{\perp}$  of the incident wave and the corresponding components  $R_{\parallel}$ ,  $R_{\perp}$  of the reflected wave are related by

$$\left. \begin{aligned} R_{\parallel} &= \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} A_{\parallel}, \\ R_{\perp} &= -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} A_{\perp}. \end{aligned} \right\} \quad (10)$$

Since  $\theta_t$  is now complex, so are the ratios  $R_{\parallel}/A_{\parallel}$  and  $R_{\perp}/A_{\perp}$ , i.e. characteristic phase changes occur on reflection; thus incident linearly polarized light will in general become elliptically polarized on reflection at the metal surface. Let  $\phi_{\parallel}$  and  $\phi_{\perp}$  be the phase changes, and  $\rho_{\parallel}$  and  $\rho_{\perp}$  the absolute values of the reflection coefficients, i.e.

$$r_{\parallel} = \frac{R_{\parallel}}{A_{\parallel}} = \rho_{\parallel} e^{i\phi_{\parallel}}, \quad r_{\perp} = \frac{R_{\perp}}{A_{\perp}} = \rho_{\perp} e^{i\phi_{\perp}}. \quad (11)$$

Suppose that the incident light is *linearly polarized* in the azimuth  $\alpha_i$  i.e.

$$\tan \alpha_i = \frac{A_{\perp}}{A_{\parallel}}, \quad (12)$$

and let  $\alpha_r$  be the azimuthal angle (generally complex) of the light that is reflected. Then\*

$$\tan \alpha_r = \frac{R_{\perp}}{R_{\parallel}} = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_t)} \tan \alpha_i = P e^{-i\Delta} \tan \alpha_i, \quad (13)$$

where

$$P = \frac{\rho_{\perp}}{\rho_{\parallel}}, \quad \Delta = \phi_{\perp} - \phi_{\parallel}. \quad (14)$$

We note that  $\alpha_r$  is real in the following two cases:

- (1) For normal incidence ( $\theta_i = 0$ ); then  $P = 1$  and  $\Delta = -\pi$ , so that  $\tan \alpha_r = -\tan \alpha_i$ .
- (2) For grazing incidence ( $\theta_i = \pi/2$ ); then  $P = 1$  and  $\Delta = 0$ , so that  $\tan \alpha_r = \tan \alpha_i$ .

It should be remembered that in the case of normal incidence the directions of the incident and reflected rays are opposed; thus the negative sign implies that the

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\* We write  $-i\Delta$  rather than  $+i\Delta$  in the exponent on the right-hand side of (13) to facilitate comparison with certain results of § 1.5.

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