Laser and Gaussian Beam Propagation and Transformation

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INTRODUCTION

Optical engineers and researchers working on optics deal with laser beams and optical systems as usual tools in their specific areas. The knowledge of the special characteristics of the propagation of laser beams through optical systems has to be one of the keystones of their actual work, and the clear definition of their characteristic parameters has an important impact in the success of the applications of laser sources.[1–6] In this article, we will provide some basic hints about the characterization and transformation of laser beams that also deserve special attention in basic and specific text books (e.g., see Refs. [7–13]). The Gaussian beam case is treated in the first place because of its simplicity.[14,15] Besides, it allows to introduce some characteristic parameters whose definition and meaning will be extended along the following sections to treat any kind of laser beam. In between, we will show how the beam is transformed by linear optical systems. These systems are described by using the tools of matrix optics.[16–18]

In the following, we will assume that laser beams have transversal dimensions small enough to consider them as paraxial beams. What it means is that the angular spectrum of the amplitude distribution is located around the axis of propagation, allowing a parabolic approximation for the spherical wavefront of the laser beam. In the paraxial approach, the component of the electric field along the optical axis is neglected. The characterization of laser beams within the nonparaxial regime can be done, but it is beyond the scope of this presentation.[19–22] We will take the amplitudes of the beams as scalar quantities. This means that the polarization effects are not considered, and the beam is assumed to be complete and homogeneously polarized. A proper description of the polarization dependences needs an extension of the formalism that is not included here.[23–28] Pulsed laser beams also need a special adaptation[29–32] of the fundamental description presented here.

GAUSSIAN BEAMS

Gaussian beams are the simplest and often the most desirable type of beam provided by a laser source. As we will see in this section, they are well characterized and the evolution is smooth and easily predicted. The amplitude function representing a Gaussian beam can be deduced from the boundary conditions of the optical resonator where the laser radiation is produced.[7–9,33,34] The geometrical characteristics of the resonator determine the type of laser emission obtained. For stable resonators neglecting a small loss of energy, the amplitude distribution is self-reproduced in every round trip of the laser through the resonator. Unstable resonators produce an amplitude distribution more complicated than in the stable case. Besides, the energy leaks in large proportion for every round trip. For the sake of simplicity, we restrict this first analysis to those laser sources producing Gaussian beams. The curvature of the mirrors of the resonator and their axial distance determine the size and the location of the region showing the highest density of energy along the beam. The transversal characteristics of the resonator allow the existence of a set of amplitude distributions that are usually named as modes of the resonator. The Gaussian beam is the lowest-degree mode, and therefore it is the most commonly obtained from all stable optical resonators.

Although the actual case of the laser beam propagation is a 3-D problem (two transversal dimensions x,y, and one axial dimension z), it is easier to begin with the explanation and the analysis of a 2-D laser beam (one transversal dimension x, and one axial dimension z). The amplitude distribution of a Gaussian laser beam can be written as:[7,9,34]

$$\Psi(x,z) = \sqrt[4]{\frac{2}{\pi}}\sqrt{\frac{\exp\{-i[\phi(z)-\phi_0]\}}{\omega(z)}} \times \exp\left[-i\frac{kx^2}{2R(z)}-\frac{x^2}{\omega^2(z)}\right] \quad (1)$$

This expression describes the behavior of the laser beam amplitude as a function of the transversal coordinate x and the axial coordinate z. $k = 2\pi/\lambda$ is the wave number, where λ is the wavelength of the material where the beam propagates. The functions $R(z)$, $\omega(z)$, and $\phi(z)$ deserve special attention and are described in the following subsections. Before that, it is interesting to take a closer look at Fig. 1 where we plot the irradiance pattern in terms of x and z. This irradiance is the square modulus of the amplitude distribution presented above. We can see

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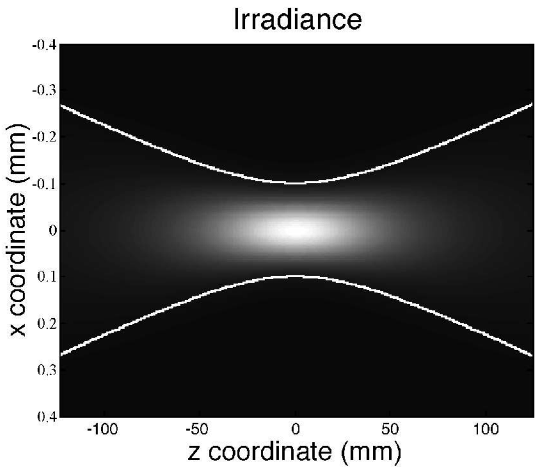

Fig. 1 Map of the irradiance distribution of a Gaussian beam. The bright spot corresponds with the beam waist. The hyperbolic white lines represent the evolution of the Gaussian width when the beam propagates through the beam waist position. The transversal Gaussian distribution of irradiance is preserved as the beam propagates along the z axis.

that the irradiance shows a maximum around a given point where the transversal size of the beam is minimum. This position belongs to a plane that is named as the beam waist plane. It represents a pseudo-focalization point with very interesting properties. Once this first graphical approach has been made, it lets us define and explain in more detail the terms involved in Eq. 1.

Width

This is probably one of the most interesting parameters from the designer point of view.[14,35,36] The popular approach of a laser beam as a ''laser ray'' has to be reviewed after looking at the transversal dependence of the amplitude. The ray becomes a beam and the width parameter characterizes this transversal extent. Practically, the question is to know how wide is the beam when it propagates through a given optical system. The exponential term of Eq. 1 shows a real and an imaginary part. The imaginary part will be related with the phase of the beam, and the real part will be connected with the transversal distribution of irradiance of the beam. Extracting this real portion, the following dependences of the amplitude and the irradiance are:

$$\Psi(x,z) \propto \exp\left[-\frac{x^2}{\omega^2(z)}\right] \tag{2}$$

$$I(x,z) = |\Psi(x,z)|^2 \propto \exp\left[-\frac{2x^2}{\omega^2(z)}\right] \tag{3}$$

where the function $\omega(z)$ describes the evolution along the propagation direction of the points having a decrease of $1/e$ in amplitude, or $1/e^2$ in irradiance with respect to the amplitude at the propagation axis. There exist some others definitions for the width of a beam related with some other fields.[36–38] For example, it is sometimes useful to have the width in terms of the full width at half the maximum (FWHM) values.[14] In Fig. 2, we see how the Gaussian width and the FWHM definitions are related. In Fig. 3, we calculate the portion of the total irradiance included inside the central part of the beam limited by those previous definitions. Both the 2-D and the 3-D cases are treated. For the 3-D case, we have assumed that the beam is rotationally symmetric with respect to the axis of propagation.

Another important issue in the study of the Gaussian beam width is to know its evolution along the direction of propagation z. This dependence is extracted from the evolution of the amplitude distribution. This calculation provides the following formula:

$$\omega(z) = \omega_0\sqrt{1+\left(\frac{z\lambda}{\pi\omega_0^2}\right)^2} \tag{4}$$

The graphical representation is plotted in Fig. 1 as a white line overimposed on the irradiance distribution. We can see that it reaches a minimum at $z = 0$, this being the minimum value of ω_0. This parameter, which governs the rest of the evolution, is usually named as the beam waist width. It should be noted that $\omega(z)$ depends on λ, where λ is the wavelength in the material where the beam is propagating. At each perpendicular plane, the z beam shows a Gaussian profile. The width reaches the minimum at the waist and then the beam expands. The same

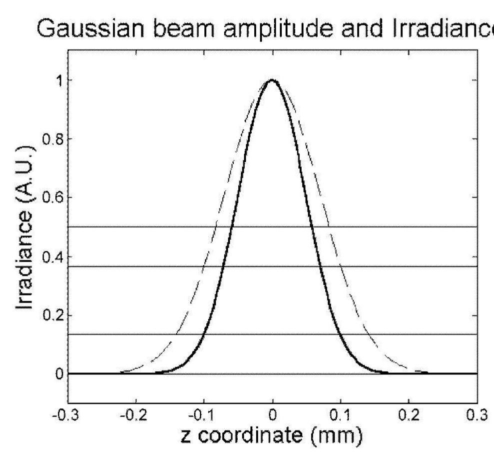

Fig. 2 Transversal profile of the Gaussian beam amplitude at the beam waist (dashed line) and irradiance (solid line). Both of them have been normalized to the maximum value. The value of the width of the beam waist ω_0 is 0.1 mm. The horizontal lines represent (in increasing value) the $1/e^2$ of the maximum irradiance, the $1/e$ of the maximum amplitude, and the 0.5 of the maximum irradiance and amplitude.

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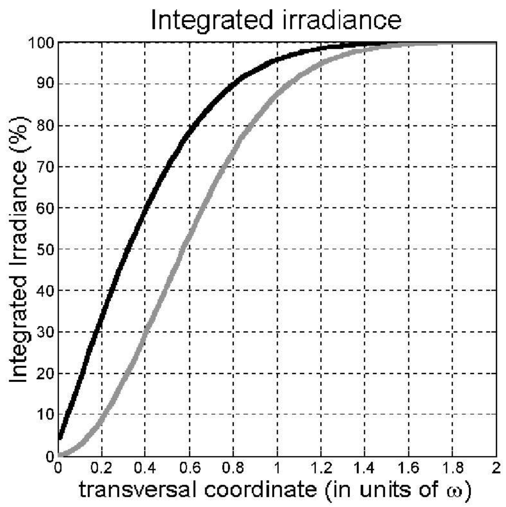

Fig. 3 Integrated irradiance for a 2-D Gaussian beam (black line) and for a rotationally symmetric 3-D Gaussian beam. The horizontal axis represents the width of a 1-D slit (for the 2-D beam) and the diameter of a circular aperture (for the 3-D beam) that is located in front of the beam. The center of the beam coincides with the center of the aperture. The size of the aperture is scaled in terms of the Gaussian width of the beam at the plane of the aperture.

amount of energy located at the beam waist plane needs to be distributed in each plane. As a consequence, the maximum of irradiance at each z plane drops from the beam waist very quickly, as it is expressed in Fig. 1.

Divergence

Eq. 4 has a very interesting behavior when z tends to ∞ (or $-\infty$). This width dependence shows an oblique asymptote having a slope of:

$$\theta_0 \cong \tan\,\theta_0 = \frac{\lambda}{\pi\omega_0} \tag{5}$$

where we have used the paraxial approach. This parameter is named divergence of the Gaussian beam. It describes the spreading of the beam when propagating towards infinity. From the previous equation, we see that the divergence and the width are reciprocal parameters. This means that larger values of the width mean lower values of the divergence, and vice versa. This relation has even deeper foundations, which we will show when the characterization of generalized beams is made in terms of the parameters already defined for the Gaussian beam case. Using this relation, we can conclude that a good collimation (very low value of the divergence) will be obtained when the beam is wide. On the contrary, a high focused beam will be obtained by allowing a large divergence angle.

Radius of Curvature

Following the analysis of the amplitude distribution of a Gaussian beam, we now focus on the imaginary part of the exponential function that depends on x:

$$\exp\left[-i\frac{kx^2}{2R(z)}\right] \tag{6}$$

where k is the wave number and $R(z)$ is a function of z. The previous dependence is quadratic with x. It is the paraxial approach of a spherical wavefront having a radius $R(z)$. Therefore this function is known as the radius of curvature of the wavefront of the Gaussian beam. Its dependence with z is as follows:

$$R(z) = z\left[1+\left(\frac{\pi\omega_0^2}{z\lambda}\right)^2\right] \tag{7}$$

When z tends to infinity, it shows a linear variation with z that is typical of a spherical wavefront that originated at $z=0$; i.e., coming from a point source. However, the radius of curvature is infinity at the beam waist position. This means that at the beam waist, the wavefront is plane. A detailed description of the previous equation is shown in Fig. 4. The absolute value of the radius of curvature is larger (flatter wavefront) than the corresponding point source located at the beam waist along the whole propagation.

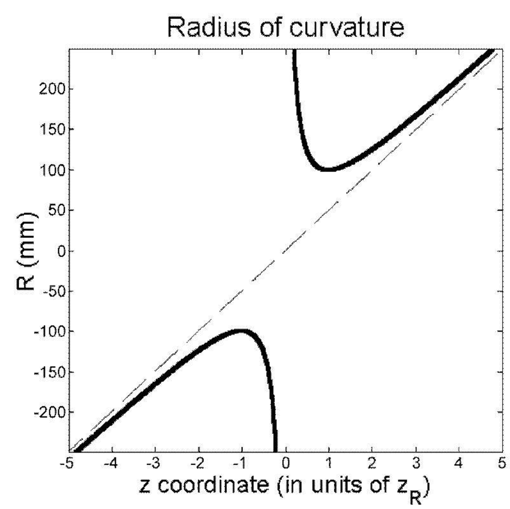

Fig. 4 Radius of curvature of a Gaussian beam around the beam waist position. The beam reaches a minimum of the absolute value of the radius at a distance of $+z_R$ and $-z_R$ from the beam waist. At the beam waist position, the radius of curvature is infinity, meaning that the wavefront is plane at the beam waist. The dashed line represents the radius of curvature of a spherical wavefront produced by a point source located at the point of maximum irradiance of the beam waist.

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Rayleigh Range

The width, the local divergence, and the radius of curvature contain a special dependence with ω_0 and λ. This dependence can be written in the form of length that is defined as:

$$z_R = \frac{\pi\omega_0^2}{\lambda} \tag{8}$$

This parameter is known as the Rayleigh range of the Gaussian beam. Its meaning is related to the behavior of the beam along the propagating distance. It is possible to say that the beam waist dimension along z is z_R. The width at $z = z_R$ is $\sqrt{2}$ larger than in the waist. The radius of curvature shows its minimum value (the largest curvature) at $z = z_R$. From the previous dependence, we see that the axial size of the waist is larger (with quadratic dependence) as the width is larger. Joining this dependence and the relation between the width and the divergence, we find that as the collimation becomes better, then the region of collimation becomes even larger because the axial extension of the beam waist is longer.

Guoy Phase Shift

There exists another phase term in Eq. 1. This term is $\phi(z)$. This is known as the Guoy phase shift. It describes a π phase shift when the wavefront crosses the beam waist region (see pp. 682–685 of Ref. [7]). Its dependence is:

$$\phi(z) = \tan^{-1}\left(\frac{z}{z_R}\right) \tag{9}$$

This factor should be taken into account any time the exact knowledge of the wavefront is needed for the involved applications.

3-D GAUSSIAN BEAMS

In ''Gaussian Beams,'' we have described a few parameters characterizing the propagation of a 2-D beam. Actually, these parameters can be extended to a rotationally symmetric beam assuming that the behavior is the same for any meridional plane containing the axis of propagation. Indeed, we did an easy calculation of the encircled energy for a circular beam by using these symmetry considerations (see Fig. 3). However, this is not the general case for a Gaussian beam.[39–41] For example, when a beam is transformed by a cylindrical lens, the waist on the plane along the focal power changes, and the other remains the same. If a toric, or astigmatic, lens is used, then two perpendicular directions can be defined. Each one would introduce a change in the beam that will be different from the other. Even more, for some laser sources, the geometry of the laser cavity produces an asymmetry that is transferred to a nonrotationally symmetric beam propagation. This is the case of edge-emitting semiconductor lasers, where the beam can be modeled by having two 2-D Gaussian beam propagations.[42] In all these cases, we can define two coordinate systems: the beam reference system linked to the beam symmetry and propagation properties, and the laboratory reference system.

The evolution of the simplest case of astigmatic Gaussian beams can be decoupled into two independent Gaussian evolutions along two orthogonal planes. The beams allowing this decoupling are named as orthogonal astigmatic Gaussian beams. Typically, these beams need some other parameters to characterize the astigmatism of the laser, besides the parameters describing the Gaussian evolution along the reference planes of the beam reference system. When the beam reaches the waist in the same plane for the two orthogonal planes defined within the beam reference system, we only need to provide the ellipticity parameter of the irradiance pattern at a given plane. In some other cases, both orthogonal planes describing a Gaussian evolution do not produce the waist at the same plane. In this case, another parameter describing this translation should be provided. This parameter is sometimes named as longitudinal astigmatism. Although the beam propagation is located in two orthogonal planes, it could be possible that these planes do not coincide with the orthogonal planes of the laboratory reference system. An angle should also be given to describe the rotation of the beam reference system with respect to the laboratory reference system.

When the planes of symmetry of the beam do not coincide with the planes of symmetry of the optical systems that the beam crosses, it is not possible to decouple the behavior of the resulting beam into two orthogonal planes. This lack of symmetry provides a new variety of situations that are usually named as general astigmatism case.[39]

Orthogonal Astigmatic Beams

In Fig. 5, we represent a 3-D Gaussian beam having the beam waist along the direction of x and the direction of y in the same z plane. In this case, the shape of the beam will be elliptic at every transversal plane along the propagation, except for two planes along the propagation that will show a circular beam pattern. The characteristic parameters are the Gaussian width along the x and y directions.

In Fig. 6a, we plot together the evolution of the Gaussian widths along the two orthogonal planes where the beam is decoupled. The intersection of those planes is

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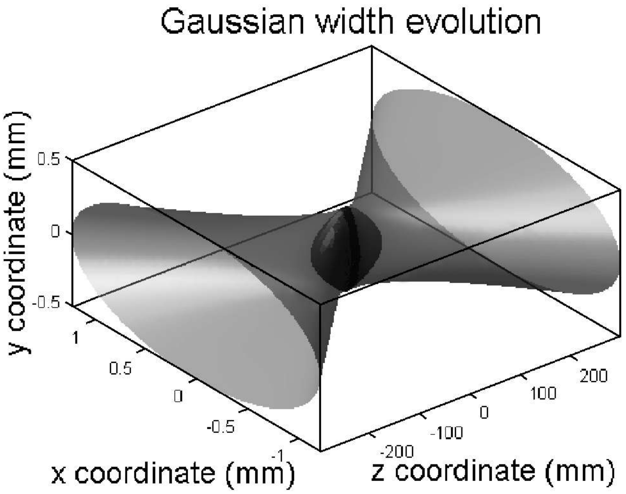

Fig. 5 A 3-D representation of the evolution of the Gaussian width for an orthogonal astigmatic Gaussian beam. The Gaussian beam waist coincides at the $z = 0$ plane and the beam reference directions coincide with the laboratory reference directions. The sizes of the waists are $\omega_{0x} = 0.07$ mm, $\omega_{0y} = 0.2$ mm, and $\lambda = 632.8$ nm. In the center of the beam, we have represented the volume of space defined by the surface where $1/e^2$ of the maximum irradiance is reached. It can be observed that the ellipticity of the irradiance pattern changes along the propagation and the larger semiaxis changes its direction: in the beam waist plane, the large semiaxis is along the y direction and at 300 mm, it has already changed toward the x direction.

the axis of propagation of the beam. The evolution of the beam along these two planes is described independently. There are two different beam waists (one for each plane) with different sizes. To properly describe the whole 3-D beam, it is necessary to provide the location and the size of these two beam waists. The locations of the intersection points for the Gaussian widths correspond with the planes showing a circular irradiance pattern. An interesting property of this type of beam is that the ellipticity of the irradiance profile changes every time a circular irradiance pattern is reached along the propagation, swapping the directions of the long and the short semiaxes.

In Fig. 6b, we represent the evolution of the two radii of curvature within the two orthogonal reference planes. The analytical description of the wavefront of the beam is given by a 3-D paraboloid having two planes of symmetry. The intersection of the two evolutions of the radii of curvature describes the location of a spherical wavefront. It is important to note that a circular irradiance pattern does not mean a spherical wavefront for this type of beams. To completely describe this beam, we need the values of the beam waists along the beam reference directions, and the distance between these two beam waists along the propagation direction. This parameter is sometimes known as longitudinal astigmatism. If the whole beam were rotated with respect to the laboratory reference system, an angle describing such a rotation would also be necessary.

Nonorthogonal Astigmatic Beams. General Astigmatic Beams

The variety of situations in the nonorthogonal case is richer than in the orthogonal one and provides a lot of information about the beam. General astigmatic Gaussian beams were described by Arnaud and Kogelnik[39] by

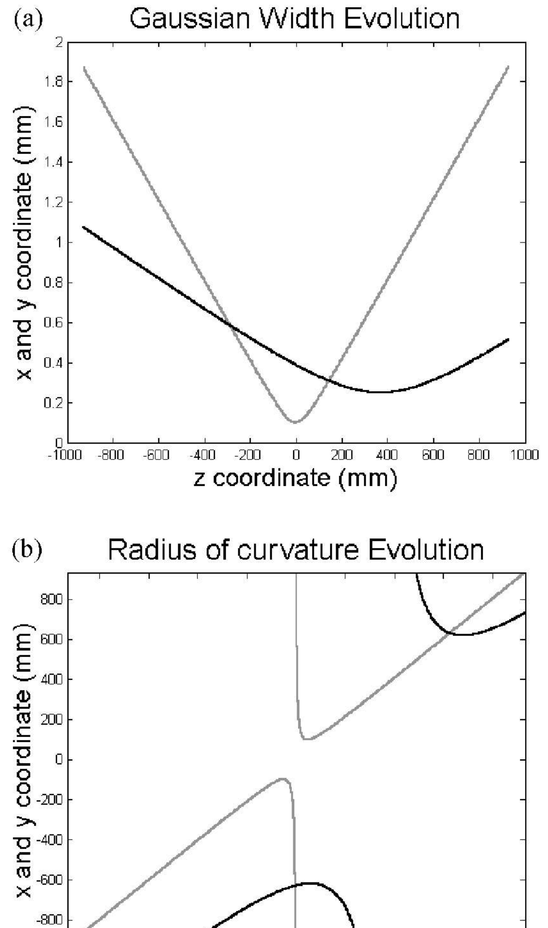

Fig. 6 Evolution of the Gaussian width (a) and the radius of curvature (b) for an orthogonal astigmatic Gaussian beam having the following parameters: $\omega_{0x} = 0.1$ mm, $\omega_{0y} = 0.25$ mm, $\lambda = 633$ nm, and 360 mm of distance between both beam waists. The black curve corresponds with the y direction and the gray curve is for the x direction. The intersection in (a) represents the position of the points having circular patterns of irradiance. The intersection in (b) represents those planes showing a spherical wavefront. The plots shows how both conditions cannot be fulfilled simultaneously.

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adding a complex nature to the rotation angle that relates the intrinsic beam axis (beam reference system) with the extrinsic (laboratory references system) coordinate system. One of the most interesting properties of these beams is that the elliptic irradiance pattern rotates along the propagation axis. To properly characterize these Gaussian beams, some more parameters are necessary to provide a complete description of this new behavior. The most relevant is the angle of rotation between the beam reference system and the laboratory reference system, which now should be provided with real and imaginary parts. In Fig. 7, we show the evolution of the Gaussian width for a beam showing a nonorthogonal astigmatic evolution. An important difference with respect to Fig. 5, besides the rotation of axis, is that the nonorthogonal astigmatic beams shows a twist of the $1/e^2$ envelope that makes possible the rotation of the elliptical irradiance pattern. It should be interesting to note that in the case of the orthogonal astigmatism, the elliptic irradiance pattern does not change the orientation of their semi-axes; it only swaps their role. However, in the general astigmatic case, or nonorthogonal astigmatism, the rotation is smooth and depends on the imaginary part of the rotation angle.

A simple way of obtaining these types of beams is by using a pair of cylindrical or toric lenses with their characteristic axes rotated by an angle different from zero or 90°. Although the input beam is circular, the resulting beam will exhibit a nonorthogonal astigmatic character. The reason for this behavior is related to the loss of symmetry between the input and the output beams along each one of the lenses.

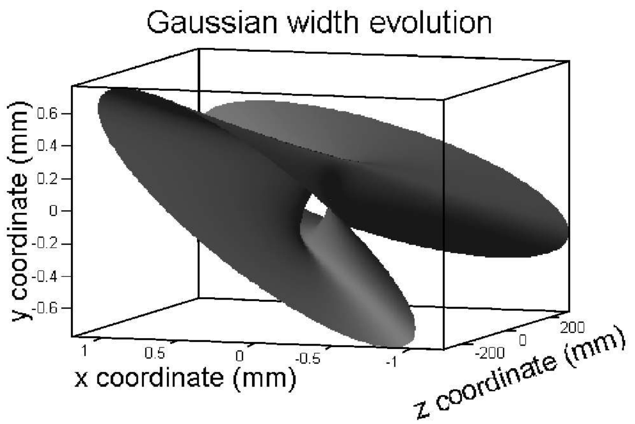

Fig. 7 Evolution of the Gaussian width for the case of a nonorthogonal astigmatic Gaussian beam. The parameters of this beam are: $\lambda = 633$ nm $\omega_{0x} = 0.07$ mm, $\omega_{0y} = 0.2$ mm, and the angle of rotation has a complex value of $\alpha = 25° - i15°$. The parameters, except for the angle, are the same as those of the beam plotted in Fig. 5. However, in this case, the beam shows a twist due to the nonorthogonal character of its evolution.

ABCD LAW FOR GAUSSIAN BEAMS

ABCD Matrix and *ABCD* Law

Matrix optics has been well established a long time ago.[16–18,43,44] Within the paraxial approach, it provides a modular transformation describing the effect of an optical system as the cascaded operation of its components. Then each simple optical system is given by its matrix representation.

Before presenting the results of the application of the matrix optics to the Gaussian beam transformation, we need to analyse the basis of this approach (e.g., see Chapter 15 of Ref. [7]). In paraxial optics, the light is presented as ray trajectories that are described, at a given meridional plane, by its height and its angle with respect to the optical axis of the system. These two parameters can be arranged as a column vector. The simplest mathematical object relating two vectors (besides a multiplication by a scalar quantity) is a matrix. In this case, the matrix is a 2×2 matrix that is usually called the *ABCD* matrix because its elements are labeled as *A*, *B*, *C*, and *D*. The relation can be written as:

$$\begin{pmatrix} x_2 \\ x'_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} \tag{10}$$

where the column vector with subindex 1 stands for the input ray, and the subindex 2 stands for the output ray.

An interesting result of this previous equation is obtained when a new magnitude is defined as the ratio between height and angle. From Fig. 8, this parameter coincides with the distance between the ray–optical axis intersection and the position of reference for the description of the ray. This distance is interpreted as the radius of curvature of a wavefront departing from that intersection point and arriving to the plane of interest where the column vector is described. When this radius of curvature is obtained by using the matrix relations, the following result is found:

$$R_2 = \frac{AR_1 + B}{CR_1 + D} \tag{11}$$

This expression is known as the *ABCD* law for the radius of curvature. It relates the input and output radii of curvature for an optical system described by its *ABCD* matrix.

The Complex Radius of Curvature, *q*

For a Gaussian beam, it is possible to define a radius of curvature describing both the curvature of the wavefront

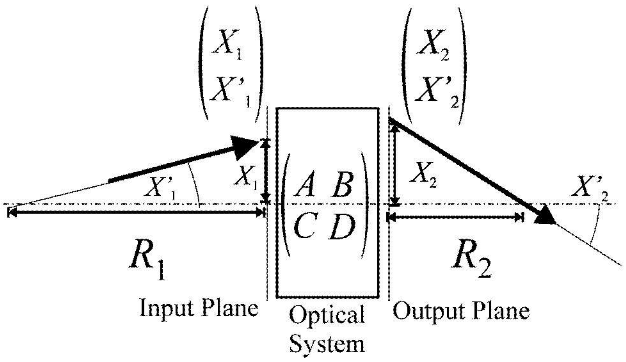

Fig. 8 The optical system is represented by the *ABCD* matrix. The input and the output rays are characterized by their height and their slope with respect to the optical axis. The radius of curvature is related to the distance between the intersection of the ray with the optical axis and the input or the output planes.

and the transversal size of the beam. The nature of this radius of curvature is complex. It is given by:[9,34]

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi\omega(z)^2} \tag{12}$$

If the definition and the dependences of $R(z)$ and $\omega(z)$ are used in this last equation, it is also possible to find another alternative expression for the complex radius of curvature as:

$$q(z) = z + iz_R \tag{13}$$

By using this complex radius of curvature, the phase dependence of the beam (without taking into account the Guoy phase shift) and its transversal variation is written as:

$$\exp\left[-i\frac{kx^2}{2q(z)}\right] \tag{14}$$

Once this complex radius of curvature is defined, the *ABCD* law can be proposed and be applied for the calculation of the change of the parameters of the beam. This is the so-called *ABCD* law for Gaussian beams (see Chapter 3 of Ref. [18]):

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad \text{or} \quad \frac{1}{q_2} = \frac{C + D\frac{1}{q_1}}{A + B\frac{1}{q_1}} \tag{15}$$

The results of the application of the *ABCD* law can be written in terms of the complex radius of curvature and the Gaussian width by properly taking the real and imaginary parts of the resulting complex radius of curvature.

Invariant Parameter

When a Gaussian beam propagates along an *ABCD* optical system, its complex radius of curvature changes according to the *ABCD* law. The new parameters of the beam are obtained from the value of the new complex radius of curvature. However, there exists an invariant parameter that remains the same throughout *ABCD* optical systems. This invariant parameter is defined as:

$$\theta_0\omega_0 = \frac{\lambda}{\pi} \tag{16}$$

Its meaning has been already described in ''Divergence.'' It will be used again when the quality parameter is defined for arbitrary laser beams.

Tensorial *ABCD* Law

The previous derivation of the *ABCD* law has been made for a beam along one meridional plane containing the optical axis of the system that coincides with the axis of propagation. In the general case, the optical system or the Gaussian beam cannot be considered as rotationally symmetric. Then the beam and the system need to be described in a 3-D frame. This is done by replacing each one of the elements of the *ABCD* matrix by a 2×2 matrix containing the characteristics of the optical system along two orthogonal directions in a transversal plane. In the general case, these 2×2 boxes may have nondiagonal elements that can be diagonalized after a given rotation. This rotation angle can be different in diagonalizing different boxes when nonorthogonal beams are treated. Then the *ABCD* matrix becomes an *ABCD* tensor in the form of:

$$P = \begin{pmatrix} A_{xx} & A_{xy} & B_{xx} & B_{xy} \\ A_{yx} & A_{yy} & B_{yx} & B_{yy} \\ C_{xx} & C_{xy} & D_{xx} & D_{xy} \\ C_{yx} & C_{yy} & D_{yx} & D_{yy} \end{pmatrix} \tag{17}$$

where, by symmetry considerations, $A_{xy} = A_{yx}$ and is the same for the B, C, and D, boxes.

For a Gaussian beam in the 3-D case, we will need to expand the definition of the complex radius of curvature to the tensorial domain.[45] The result is as follows:

$$Q^{-1} = \begin{pmatrix} \frac{\cos^2\theta}{q_x} + \frac{\sin^2\theta}{q_y} & \frac{1}{2}\sin 2\theta\left(\frac{1}{q_x} - \frac{1}{q_y}\right) \\ \frac{1}{2}\sin 2\theta\left(\frac{1}{q_x} - \frac{1}{q_y}\right) & \frac{\sin^2\theta}{qx} + \frac{\cos^2\theta}{q_y} \end{pmatrix} \tag{18}$$

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where θ is the angle between the laboratory reference coordinate system and the beam reference system. If the beam is not orthogonal, then the angle becomes a complex angle and the expression remains valid. By using this complex curvature tensor, the tensorial *ABCD* law (see Section 7.3 of Ref. [18]) can be written as:

$$Q_2^{-1} = \frac{\bar{C} + \bar{D}Q_1^{-1}}{\bar{A} + \bar{B}Q_1^{-1}} \tag{19}$$

where *A*, *B*, *C*, and *D* with bars are the 2×2 boxes of the *ABCD* tensor.

ARBITRARY LASER BEAMS

As we have seen in the previous sections, Gaussian beams behave in a very easy way. Its irradiance profile and its evolution are known and their characterization can be made with a few parameters. Unfortunately, there are a lot of applications and laser sources that produce laser beams with irradiance patterns different from those of the Gaussian beam case. The simplest cases of these non-Gaussian beams are the multimode laser beams. They have an analytical expression that can be used to know and to predict the irradiance at any point of the space for these types of beams. Moreover, the most common multimode laser beams contain a Gaussian function in the core of their analytical expression. However, some other more generalized types of irradiance distribution do not respond to simple analytical solution. In those cases, and even for multimode Gaussian beams, we can still be interested in knowing the transversal extension of the beam, its divergence in the far field, and its departure from the Gaussian beam case that is commonly taken as a desirable reference. Then the parameterization of arbitrary laser beams becomes an interesting topic for designing procedures because the figures obtained in this characterization can be of use for adjusting the optical parameters of the systems using them.[46,47]

Multimode Laser Beams

The simplest cases of these types of arbitrary beams are those corresponding to the multimode expansion of laser beams (see Chapters 16, 17, 19, 20, and 21 of Refs. [7,34]). These multimode expansions are well determined by their analytical expressions showing a predictable behavior. Besides, the shape of the irradiance distribution along the propagation distance remains the same. There exist two main families of multimode beams: Laguerre–Gaussian beams, and Hermite–Gaussian beams. They appear as solutions of higher order of the conditions of resonance of the laser cavity.

Their characteristic parameters can be written in terms of the order of the multimode beam.[48–53] When the beam is a monomode of higher-than-zero order, its width can be given by the following equation:

$$\omega_n = \omega_0\sqrt{2n+1} \tag{20}$$

for the Hermite–Gauss beam of n order, and

$$\omega_{pm} = \omega_0\sqrt{2p+m+1} \tag{21}$$

for the Laguerre–Gauss beam of p radial and m azimuthal orders, where ω_0 is the width of the corresponding zero order or pure Gaussian beam. For an actual multimode beam, the values of the width, the divergence, and the radius of curvature depend on the exact combination of modes, and need to be calculated by using the concepts defined in ''Generalized Laser Beams.'' In Fig. 9, we have plotted three Hermite–Gaussian modes containing the same Gaussian beam that has an elliptic shape. We have plotted them at different angles to show how, in the beam reference system, the modes are oriented along two orthogonal directions.

Generalized Laser Beams

When the irradiance distribution has no analytical solution, or when we are merely dealing with actual beams coming from actual sources showing diffraction, fluctuations, and noise, it is necessary to revise the definitions of the parameters characterizing the beam. For example, the definition of the width of the beam provided by the $1/e^2$ decay in irradiance may not be valid any longer. In the case of multimode laser beams, the irradiance falls below $1/e^2$ at several locations along the transversal plane. The same is applied to the other parameters. Then it is necessary to provide new definitions of the parameters. These definitions should be applied to any kind of laser beam, even in the case of partially coherent beams. Two different approaches have been made to this problem of analytical and generalized characterization of laser beams. One of them can be used on totally coherent laser beams. This is based on the knowledge of the map of the amplitude, and on the calculation of the moments of the irradiance distribution of the beam.[46] The other approach can be used on partially coherent laser beams, and is based on the properties of the cross-spectral density and the Wigner distribution.[47]

It is important to note that the parameters defined in this section must be taken as global parameters. They do not describe local variations of the irradiance distribution. On the other hand, the definitions involve integration, or summation, from $-\infty$ to $+\infty$. To carry out these integrations properly, the analytical expressions need to

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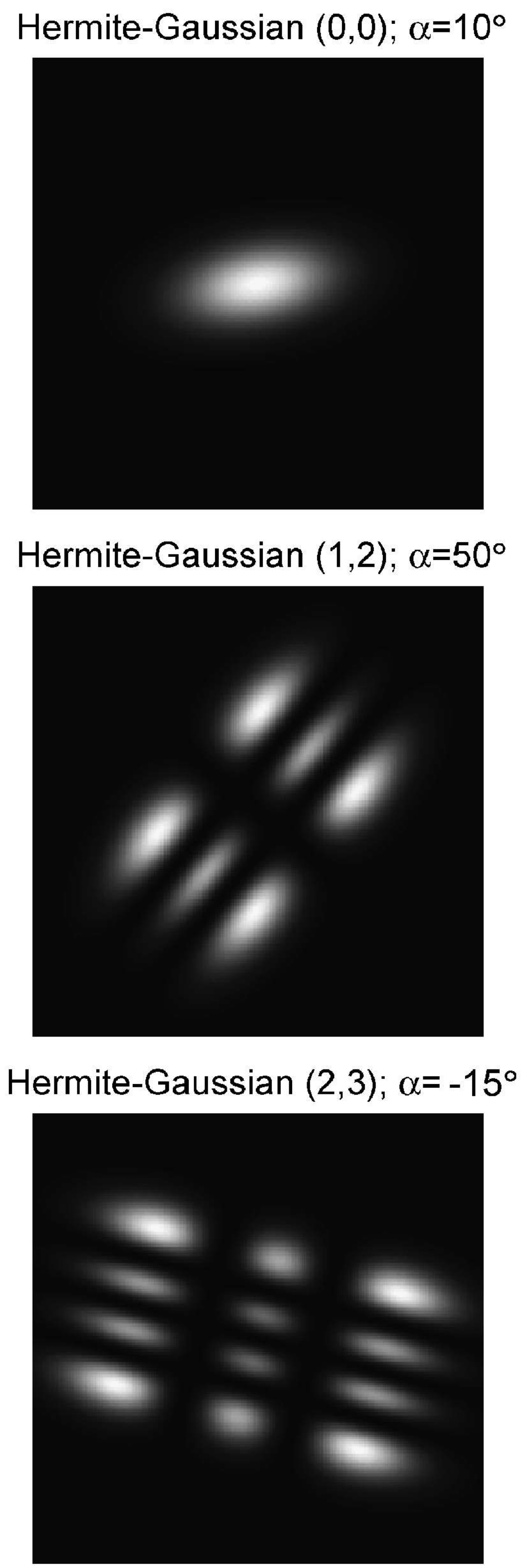

Fig. 9 Irradiance patterns for three multimode Hermite–Gaussian beams. The modes are represented at three rotations with respect to the laboratory reference system. The inner rectangular symmetry remains the same. The transversal size of the mode increases with the mode order.

be well defined and be integrable along those regions.[54] In an experimental setup, these limits are obviously not reached. The practical realizations of the definitions need to deal with important conditions about the diffraction, the noise, the image treatment, and some other experimental issues that are mostly solved through characterization devices currently used for the measurements of these parameters.

Totally Coherent Laser Beams in Two Dimensions

As we did with the Gaussian beams, we are going to introduce the most characteristic parameters for a 2-D beam[48,49,54–58] defined in terms of the moments of the irradiance distribution and its Fourier transform. After that, we will generalize the definitions to the 2-D case.

Generalized width

When the amplitude map $\Psi(x)$ of a laser source is accessible, it is possible to define the width of the beam in terms of the moments of the irradiance distribution as:

$$\omega(\Psi) = 2\sqrt{\frac{\int_{-\infty}^{\infty}|\Psi(x)|^2[x - x(\Psi)]^2\mathrm{d}x}{\int_{-\infty}^{\infty}|\Psi(x)|^2\mathrm{d}x}} = 2\sqrt{\frac{\int_{-\infty}^{\infty}|\Psi(x)|^2x^2\mathrm{d}x}{\int_{-\infty}^{\infty}|\Psi(x)|^2\mathrm{d}x} - x^2(\Psi)} \quad (22)$$

where the denominator is the total irradiance of the beam and $x(\Psi)$ is the position of the "center of mass" of the beam:

$$x(\Psi) = \frac{\int_{-\infty}^{\infty}|\Psi(x)|^2x\mathrm{d}x}{\int_{-\infty}^{\infty}|\Psi(x)|^2\mathrm{d}x} \quad (23)$$

The introduction of this parameter allows to apply the definition to a beam described in a decentered coordinate system. It is easy to check that in the case of a Gaussian distribution, the width is the Gaussian width defined in the previous sections.

Generalized divergence

As we saw in the definition of the divergence for Gaussian beams, the divergence is related to the spreading of the beam along its propagation. This concept is described analytically by the Fourier transform of the amplitude distribution, i.e., also named as the angular spectrum. The

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Fourier transform $\Phi(\xi)$ of the amplitude distribution $\Psi(x)$ is defined as:

$$\Phi(\xi) = \int_{-\infty}^{\infty} \Psi(x)\exp(-i2\pi x\xi)\mathrm{d}x \tag{24}$$

where ξ is the transverse spatial frequency that is related to the angle by means of the wavelength. The far-field distribution of irradiance is then given by the squared modulus of $\Phi(\xi)$. Once this irradiance distribution is obtained, it is possible to define an angular width that is taken as the divergence of the beam. This generalized divergence is defined as:

$$\theta_0(\Phi) = 2\lambda\sqrt{\frac{\int_{-\infty}^{\infty}|\Phi(\xi)|^2[\xi-\xi(\Phi)]^2\mathrm{d}\xi}{\int_{-\infty}^{\infty}|\Phi(\xi)|^2\mathrm{d}\xi}} = 2\lambda\sqrt{\frac{\int_{-\infty}^{\infty}|\Phi(\xi)|^2\xi^2\mathrm{d}\xi}{\int_{-\infty}^{\infty}|\Phi(\xi)|^2\mathrm{d}\xi} - \xi^2(\Phi)} \tag{25}$$

where $\xi(\Phi)$ is given by:

$$\xi(\Phi) = \frac{\int_{-\infty}^{\infty}|\Phi(\xi)|^2\xi\mathrm{d}\xi}{\int_{-\infty}^{\infty}|\Phi(\xi)|^2\mathrm{d}\xi} \tag{26}$$

This parameter is related with the misalignment, or tilt, of the beam that is the product of $\lambda\xi$.

Generalized radius of curvature

Another parameter defined for Gaussian beams was the radius of curvature.[54–56] For totally coherent laser beams, it is also possible to define an effective or generalized radius of curvature for arbitrary amplitude distributions. This radius of curvature is the radius of the spherical wavefront that best fits the actual wavefront of the beam. This fitting is made by weighting the departure from the spherical wavefront with the irradiance distribution. The analytical expression for this radius of curvature can be written as follows:

$$\frac{1}{R(\Psi)} = \frac{i\lambda}{\pi\omega^2(\Psi)\int_{-\infty}^{\infty}|\Psi(x)|^2\mathrm{d}x} \times \int_{-\infty}^{\infty}\left(\frac{\partial\Psi(x)}{\partial x}\Psi^*(x) - \Psi(x)\frac{\partial\Psi^*(x)}{\partial x}\right) \times [x - x(\Psi)]\mathrm{d}x \tag{27}$$

The integral containing the derivatives of the amplitude distribution can be written in different ways by using the properties of the Fourier transform. This integral is also related with the crossed moments (in x and ξ) of the beam. It will play an important role in the definition of the invariant parameter of the beam.

Generalized complex radius of curvature

By using the previous definitions, it is possible to describe a generalized complex radius of curvature as follows:

$$\frac{1}{q(\Psi)} = \frac{1}{R(\Psi)} - i\sqrt{\frac{\theta_0^2(\Phi)}{\omega^2(\Psi)} - \frac{1}{R^2(\Psi)}} \tag{28}$$

Now the transformation of the complex radius of curvature can be carried out by applying the *ABCD* law. It is important to note that there are three parameters involved in the calculation of the generalized complex radius of curvature: $\omega^2(\Psi)$, $\theta_0^2(\Psi)$, and $R(\Psi)$. The application of the *ABCD* law provides two equations: one for the real part, and one for the imaginary. Therefore we will need another relation involving these three parameters to solve the problem of the transformation of those beams by *ABCD* optical systems. This third relation is given by the invariant parameter, or quality factor.

Quality factor, M^2

For the Gaussian beam case, we have found a parameter that remains invariant through *ABCD* optical systems. Now in the case of totally coherent non-Gaussian beams, we can define a new parameter that will have the same properties. It will be constant along the propagation through *ABCD* optical systems. Its definition (see list of references in Ref. [59]) in terms of the previous characterizing parameters is:

$$M^2 = \frac{\pi}{\lambda}\omega(\Psi)\sqrt{\theta_0^2(\Phi) - \frac{\omega^2(\Psi)}{R^2(\Psi)}} \tag{29}$$

This invariance, along with the results obtained from the *ABCD* law applied to the generalized complex radius of curvature, allows to calculate the three resulting parameters for an *ABCD* transformation. The value of the square root of the M^2 parameter has an interesting meaning. It is related to the divergence that would be obtained if the beam having an amplitude distribution Ψ is collimated at the plane of interest. The collimation should be considered as having an effective, or generalized, radius of curvature equal to infinity. From the definition of $R(\Psi)$, this is an averaged collimation. The divergence of this collimated beam is the minimum obtainable for such a beam having a generalized width of $\omega(\Psi)$. Then the M^2 factor represents the product of the width defined as a second moment (a variance in the x coordinate) times the

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minimum angular width obtainable for a given beam by canceling the phase (a variance in the ξ coordinate).

The previous definition and the one based on the moments of the Wigner function of the M^2 parameter as a quality factor allow to compare between different types of beam structures and situations.[54,60–69] The Gaussian beam is the one considered as having the maximum quality. The value of M^2 for a Gaussian beam is 1. It is not possible to find a lower value of the M^2 for actual, realizable beams. This property, along with its definition in terms of the variance in x and ξ, resembles very well an uncertainty principle.

Usually, quality factors are parameters that increase when the quality grows; larger values usually mean better quality. This is not the case for M^2 that becomes larger as the beam becomes worse. However, the scientific and technical community involved in the introduction and the use of M^2 has accepted this parameter as a quality factor for laser beams.

Besides the interesting properties of invariance and bounded values, it is important to find the practical meaning of the beam quality factor. A beam showing better quality and lower value of M^2 will behave better for collimation and focalization purposes. It means that the minimum size of the spot obtainable with a given optical system will be smaller for a beam having a lower value of M^2. Analogously, a better beam can be better collimated; i.e., its divergence will be smaller than another beam showing a higher value of M^2 and collimated with the same optical system.

Totally Coherent Beams in Three Dimensions

Once these parameters have been defined for the 2-D case, where their meanings and definitions are clearer, we will describe the situation of a 3-D totally coherent laser beam. The parameters needed to describe globally the behavior of a 3-D beam will be an extension of the 2-D case adapted to this case, in the same way as that for 3-D Gaussian beams.[46,66]

The width and the divergence become tensorial parameters that are defined as 2×2 matrices. These matrices involve the calculation of the moments of the irradiance distribution, both in the plane of interest and in the Fourier-transformed plane (angular spectrum). In order to provide a compact definition, we first define the normalized moments used in the definitions:

$$\langle x^n y^m \rangle = \frac{\int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 x^n y^m \mathrm{d}x\mathrm{d}y}{\int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 \mathrm{d}x\mathrm{d}y} \tag{30}$$

$$\langle \xi^n \eta^m \rangle = \frac{\int\int_{-\infty}^{\infty} |\Phi(\xi,\eta)|^2 \xi^n \eta^m \mathrm{d}\xi\mathrm{d}\eta}{\int\int_{-\infty}^{\infty} |\Phi(\xi,\eta)|^2 \mathrm{d}\xi\mathrm{d}\eta} \tag{31}$$

Then the width (actually the square of the width) is defined as the following tensor:

$$W^2 = 4\left[\begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} - \begin{pmatrix} \langle x \rangle \\ \langle y \rangle \end{pmatrix} \begin{pmatrix} \langle x \rangle & \langle y \rangle \end{pmatrix}\right] \tag{32}$$

The vector $(\langle x \rangle, \langle y \rangle)$ describes any possible decentering of the beam. The term containing this vector can be cancelled by properly displacing the center of the coordinate system where the beam is described. It should be noted that there exists a coordinate system where this matrix is diagonal.

The tensor of divergences is also represented as a 2×2 matrix defined as:

$$\Theta^2 = 4\lambda^2\left[\begin{pmatrix} \langle \xi^2 \rangle & \langle \xi\eta \rangle \\ \langle \xi\eta \rangle & \langle \eta^2 \rangle \end{pmatrix} - \begin{pmatrix} \langle \xi \rangle \\ \langle \eta \rangle \end{pmatrix} \begin{pmatrix} \langle \xi \rangle & \langle \eta \rangle \end{pmatrix}\right] \tag{33}$$

As in the width tensor, the second term of this definition accounts for the tilting of the beam with respect to the direction of propagation established by the coordinate system. Again, an appropriate rotation (it may be different from the one for diagonalizing W^2) and a displacement of the coordinate system can produce a diagonal form of the divergence and the cancellation of the second term of the divergence tensor. It should be noted that, in general, the angle of rotation that diagonalizes the width tensor may be different from the angle of rotation diagonalizing the divergence tensor. This is the case for nonorthogonal, general–astigmatic, 3-D beams.

The definition of the radius of curvature needs the definition of the following tensor: (see Eq. 34 below) where ϕ is the phase of the amplitude distribution, and

$$\Psi(x,y) = |\Psi(x,y)| \exp[i\phi(x,y)] \tag{35}$$

$$S = -\frac{2\lambda}{\pi\int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 \mathrm{d}x\mathrm{d}y} \begin{pmatrix} \int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 (x - \langle x \rangle) \frac{\partial\phi(x,y)}{\partial x} \mathrm{d}x\mathrm{d}y & \int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 (x - \langle x \rangle) \frac{\partial\phi(x,y)}{\partial y} \mathrm{d}x\mathrm{d}y \\ \int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 (y - \langle y \rangle) \frac{\partial\phi(x,y)}{\partial x} \mathrm{d}x\mathrm{d}y & \int\int_{-\infty}^{\infty} |\Psi(x,y)|^2 (y - \langle y \rangle) \frac{\partial\phi(x,y)}{\partial y} \mathrm{d}x\mathrm{d}y \end{pmatrix} \tag{34}$$

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By using this tensor, the radius of curvature can be calculated as the following 2×2 matrix that represents the reciprocal of the radius of curvature:

$$R^{-1} = (W^2)^{-1}S + \frac{1}{\mathrm{Tr}[W^2]}[S^{\mathrm{T}} - (W^2)^{-1}SW^2] \quad (36)$$

where superscript T means transposition.

The transformation of these parameters by an *ABCD* optical system (see Section 7.3.6 of Ref. [18]) can use the definition of the complex radius of curvature for arbitrary laser beams. Alternatively, it is obtained by defining the following matrix that describes the beam:

$$B = \begin{pmatrix} W^2 & S \\ S^{\mathrm{T}} & \Theta^2 \end{pmatrix} \quad (37)$$

where the elements are the 2×2 matrix defined previously. This beam matrix is transformed by the following relation:

$$B_2 = PB_1P^{\mathrm{T}} \quad (38)$$

where P is the *ABCD* tensor defined previously, and superscript T means transposition.

3-D quality factor

The quality factor of a laser beam has been defined in the 2-D case as an invariant parameter of the beam when it propagates along *ABCD* optical systems. The extension of the formalism to the 3-D case requires the definition of a quality tensor as follows:

$$M^4 = \frac{\pi^2}{\lambda^2}(W^2\Theta^2 - S^2) \quad (39)$$

where W^2, Θ^2, and S have been defined previously. It can be shown that the trace of this M^4 tensor remains invariant after transformation along *ABCD* optical systems. Therefore a good quality factor, defined as a single number, is given as:

$$J = \frac{1}{2}(M_{xx}^4 + M_{yy}^4) \quad (40)$$

where M_{xx}^4 and M_{yy}^4 are the diagonal elements of the quality tensor.[60,66] Its minimum value is again equal to one, and it is only reached for Gaussian beams.

Partially Coherent Laser Beams

A partially coherent light beam is better described by its second-order functions correlating the amplitude distributions along the space and time. One of these functions is the cross-spectral density function that, in the 2-D case, can be written as:

$$\Gamma(x, s, z) = \{\Psi(x + s/2, z)\Psi^*(x - s/2, z)\} \quad (41)$$

where * means complex conjugation and { } stands for an ensemble average. The Wigner distribution is defined as the Fourier transform of the cross-spectral density:

$$h(x, \xi, z) = \int \Gamma(x, s, z)\exp(-i2\pi\xi s)\mathrm{d}s \quad (42)$$

This Wigner distribution contains information about the spatial irradiance distribution and its angular spectrum. The use of the Wigner distribution in optics has been deeply studied and it seems to be very well adapted to the analysis of partially coherent beam, along with the cross-spectral density function.[47,60,61,70–78] For a centered and an aligned partially coherent beam, it is possible to define both the width and the divergence as:

$$\omega_{\mathrm{W}}^2 = 4\frac{\int\int_{-\infty}^{\infty} x^2 h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi}{\int\int_{-\infty}^{\infty} h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi} \quad (43)$$

$$\theta_{0,\mathrm{W}}^2 = 4\frac{\int\int_{-\infty}^{\infty} \xi^2 h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi}{\int\int_{-\infty}^{\infty} h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi} \quad (44)$$

where the subindex w means that we are dealing with the Wigner distribution. The radius of curvature is defined as:

$$\frac{1}{R_{\mathrm{W}}} = \frac{\int\int_{-\infty}^{\infty} x\xi h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi}{\int\int_{-\infty}^{\infty} x^2 h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi} \quad (45)$$

As we can see, all the parameters are based on the calculation of the moments of the Wigner distribution.[78] By using all these moments, it is also possible to define the following quality factor for partially coherent beams:[60]

$$M_{\mathrm{W}}^4 = \frac{\pi^2}{\lambda^2}\left[\left(\int\int_{-\infty}^{\infty} x^2 h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi\right) \times \left(\int\int_{-\infty}^{\infty} \xi^2 h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi\right) - \left(\int\int_{-\infty}^{\infty} x\xi h(x, \xi, z)\mathrm{d}x\mathrm{d}\xi\right)^2\right] \quad (46)$$

The evolution of the parameters of these partially coherent beams can be obtained by using the transformation properties of the Wigner distribution.[75,76]

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Partially coherent laser beams in three dimensions

For a 3-D partially coherent beam, it is necessary again to transform the scalar parameters into tensorial ones. Their definitions resemble very well those definitions obtained in the case of totally coherent beams. The width for this partially coherent beam is:

$$W_{\mathrm{W}}^2 = 4\begin{pmatrix} \langle x^2\rangle_{\mathrm{W}} & \langle xy\rangle_{\mathrm{W}} \\ \langle xy\rangle_{\mathrm{W}} & \langle y^2\rangle_{\mathrm{W}} \end{pmatrix} \tag{47}$$

where the subindex W in the calculation of the moments stands for the moments of the Wigner distribution:

$$\langle p\rangle_{\mathrm{W}} = \int\int_{-\infty}^{\infty} p(x,y,\xi,\eta,z)h(x,y,\xi,\eta,z)\mathrm{d}x\mathrm{d}y\mathrm{d}\xi d\eta \tag{48}$$

where p is any product of x, y, ξ, η, and their powers. The divergence for centered and aligned beams becomes:

$$\Theta_{0,\mathrm{W}}^2 = 4\begin{pmatrix} \langle \xi^2\rangle_{\mathrm{W}} & \langle \xi\eta\rangle_{\mathrm{W}} \\ \langle \xi\eta\rangle_{\mathrm{W}} & \langle \eta^2\rangle_{\mathrm{W}} \end{pmatrix} \tag{49}$$

The crossed moment tensor S_{W} is also defined as:

$$S_{\mathrm{W}} = \begin{pmatrix} \langle x\xi\rangle_{\mathrm{W}} & \langle x\eta\rangle_{\mathrm{W}} \\ \langle y\xi\rangle_{\mathrm{W}} & \langle y\eta\rangle_{\mathrm{W}} \end{pmatrix} \tag{50}$$

All these three tensors can be grouped in a 4×4 matrix containing the whole information about the beam.[79] This matrix is built as follows:

$$B_{\mathrm{W}} = \begin{pmatrix} \langle x^2\rangle_{\mathrm{W}} & \langle xy\rangle_{\mathrm{W}} & \langle x\xi\rangle_{\mathrm{W}} & \langle x\eta\rangle_{\mathrm{W}} \\ \langle xy\rangle_{\mathrm{W}} & \langle y^2\rangle_{\mathrm{W}} & \langle y\xi\rangle_{\mathrm{W}} & \langle y\eta\rangle_{\mathrm{W}} \\ \langle x\xi\rangle_{\mathrm{W}} & \langle y\xi\rangle_{\mathrm{W}} & \langle \xi^2\rangle_{\mathrm{W}} & \langle \xi\eta\rangle_{\mathrm{W}} \\ \langle x\eta\rangle_{\mathrm{W}} & \langle y\eta\rangle_{\mathrm{W}} & \langle \xi\eta\rangle_{\mathrm{W}} & \langle \eta^2\rangle_{\mathrm{W}} \end{pmatrix} \tag{51}$$

Now the transformation of the beam by a 3-D optical system is performed as the following matricial product:

$$B_{\mathrm{W},2} = PB_{\mathrm{W},1}P^{\mathrm{T}} \tag{52}$$

where the matrix P is the one already defined in the description of 3-D *ABCD* systems.

Within this formalism, it is also possible to define a quality factor,[61] invariant under *ABCD* 3-D transformations, in the following form:

$$J = \mathrm{Tr}\left[W_{\mathrm{W}}^2\Theta_{0,\mathrm{W}}^2 - S_{\mathrm{W}}^2\right] \tag{53}$$

where Tr means the trace of the matrix inside the floors.

CONCLUSION

The Gaussian beam is the simplest case of laser beams actually appearing in practical optical systems. The parameters defined for Gaussian beams are: the width, which informs about the transversal extension of the beam; the divergence, which describes the spreading of the beam in the far field; and the radius of curvature, which explains the curvature of the associated wavefront. There also exist some other derived parameters, such as the Rayleigh range, which explains the extension of the beam waist along the propagation axis, and the Guoy phase shift, which describes how the phase includes an extra π phase shift after crossing the beam waist region. Although simple, Gaussian beams exhibit a great variety of realizations when 3-D beams are studied. They can be rotated, displaced, and twisted. To properly evaluate such effects, some other parameters have been defined by accounting for the ellipticity of the irradiance pattern, the longitudinal astigmatism, and the twisting of the irradiance profile.

Some other types of beams include the Gaussian beam as the core of their amplitude profile. This is the case of multimode laser beams. When the beam is totally coherent, it can be successfully described by extending the definitions of the Gaussian beam case by means of the calculation of the moments of their irradiance distribution (both in the plane of interest and in the far field). The definition of a quality factor M^2 has provided a figure for comparing different types of beams with respect to the best quality beam: the Gaussian beam. Another extension of the characteristic parameters of the Gaussian beams to partially coherent beam can be accomplished by using the cross-spectral density and the Wigner distribution and their associated moments.

Summarizing, Gaussian laser beams are a reference of quality for a laser source. The description of other types of generalized, non-Gaussian, nonspherical, nonorthogonal, laser beams is referred to the same type of parameters describing the Gaussian case.

REFERENCES

1. *Power and Energy Measuring Detectors, Instruments, and Equipment for Laser Radiation*; International Electrotechnical Commission, 1990, IEC 61040, Ed. 1.0.
2. *Laser and Laser-Related Equipment—Test Methods for Laser Beam Parameters—Test Methods for Laser Beam Power, Energy, and Temporal Characteristics*; International Organization for Standardization, 1998. ISO 11554.
3. *Laser and Laser-Related Equipment—Test Methods for Laser Beam Parameters—Beam Widths, Divergence Angle and Beam Propagation Factor*; International Organization for Standardization, 1999. ISO 11146.
4. *Laser and Laser-Related Equipment—Test Methods for*

Laser Beam Parameters—Beam Positional Stability; International Organization for Standardization, 1999. ISO 11670.
5. *Laser and Laser-Related Equipment—Test Methods for Laser Beam Parameters—Polarization*; International Organization for Standardization, 1999. ISO 12005.
6. *Optics and Optical Instruments—Laser and Laser-Related Equipment—Test Methods for Laser Beam Power (Energy) Density Distribution*; International Organization for Standardization, 2000. ISO 13694.
7. Siegman, A.E. *Lasers*; Oxford University Press: Mill Valley, CA, 1986.
8. Svelto, O. *Principles of Lasers,* 3rd Ed.; Plenum Press: New York, 1989.
9. Kogelnik, H. Propagation of Laser Beams. In *Applied Optics and Optical Engineering*; Shannon, R., Wyant, J.C., Eds.; Academic Press: San Diego, 1979; Vol. VII, 155–190.
10. O'Shea, D.C. Gaussian Beams. In *Elements of Modern Optical Design*; John Wiley & Sons: New York, 1985; 230–269.
11. Saleh, B.E.A.; Teich, M.C. Beam Optics. In *Fundamentals of Photonics*; John Wiley and Sons: New York, 1991; 80–107.
12. Pedrotti, F.L.; Pedrotti, L.S. Characterization of Laser Beams. In *Introduction to Optics*; Prentice-Hall: Englewood Cliffs, NJ, 1993; 456–483.
13. Guenther, R. Diffraction and Gaussian Beams. In *Modern Optic*; John Wiley and Sons: New York, 1990; 323–360.
14. Marshall, G.F. Gaussian Laser Beam Diameters. In *Laser Beam Scanning*; Optical Engineering Series, Marcel Dekker: New York, 1985; Vol. 9, 289–301.
15. Mansuripur, M. Gaussian beam optics. Opt. Photon News **2001**, *1*, 44–47. January.
16. Brower, W. *Matrix Methods of Optical Instrument Design*; Benjamin: New York, 1964.
17. Gerrad, A.; Burch, J.M. *Introduction to Matrix Method in Optics*; John Wiley and Sons: New York, 1975.
18. Wang, S.; Zhao, D. *Matrix Optics*; Springer-Verlag: Berlin, 2000.
19. Agrawal, G.P.; Pattanayak, D.N. Gaussian beam propagation beyond the paraxial approximation. J. Opt. Soc. Am. **1979**, *69*, 575–578.
20. Porras, M.A. The best quality optical beam beyond the paraxial approach. Opt. Commun. **1994**, *111*, 338–349.
21. Porras, M.A. Non-paraxial vectorial moment theory of light beam propagation. Opt. Commun. **1996**, *127*, 79–95.
22. Borghi, R.; Santarsiero, M.; Porras, M.A. A non-paraxial Bessel–Gauss beams. J. Opt. Soc. Am. A. **2001**, *18*, 1618–1626.
23. Hillion, P. Gaussian beam at a dielectric interface. J. Opt. **1994**, *25*, 155–164.
24. Porras, M.A. Nonspecular reflection of general light beams at a dielectric interface. Opt. Commun. **1997**, *135*, 369–377.
25. Zhao, D.; Wang, S. Comparison of transformation characteristics of linearly polarized and azimuthally polarized Bessel–Gauss beams. Opt. Commun. **1996**, *131*, 8–12.
26. Movilla, J.M.; Piquero, G.; Martínez-Herrero, R.; Mejías, P.M. Parametric characterization of non-uniformly polarized beams. Opt. Commun. **1998**, *149*, 230–234.
27. Alda, J. Transverse angular shift in the reflection of light beams. Opt. Commun. **2000**, *182*, 1–10.
28. Nasalski, W. Three-dimensional beam reflection at dielectric interfaces. Opt. Commun. **2001**, *197*, 217–233.
29. Dijiali, S.P.; Dienes, A.; Smith, J.S. *ABCD* matrices for dispersive pulse propagation. IEEE J. Quantum Electron. **1990**, *26*, 1158–1164.
30. Kostenbauder, A.G. Ray–pulse matrices: A rational treatment for dispersive optical systems. IEEE J. Quantum Electron. **1990**, *26*, 1147–1157.
31. Lin, Q.; Wang, S.; Alda, J.; Bernabéu, E. Transformation of pulses nonideal beams in a four-dimension domain. Opt. Lett. **1993**, *18*, 1–3.
32. Martínez, C.; Encinas, F.; Serna, J.; Mejías, P.M.; Martínez-Herrero, R. On the parametric characterization of the transversal spatial structure of laser pulses. Opt. Commun. **1997**, *139*, 299–305.
33. Fox, A.G.; Li, T. Resonant modes in a maser interferometer. Bell Syst. Tech. J. **1961**, *40*, 453–488.
34. Kogelnik, H.; Li, T. Laser beams and resonators. Proc. IEEE **1966**, *54*, 1312–1329.
35. Self, S.A. Focusing of spherical Gaussian beams. Appl. Opt. **1983**, *22*, 658–661.
36. Siegman, A.E. Defining and Measuring Laser Beam Parameters. In *Laser Beam Characterization*; Mejías, P.M., Weber, H., Martínez-Herrero, R., González-Ureña, A., Eds.; Sociedad Española de Òptica: Madrid, 1993; 1–21.
37. Ronchi, L.; Porras, M.A. The relationship between the second order moment width and the caustic surface radius of laser beams. Opt. Commun. **1993**, *103*, 201.
38. Porras, M.A.; Medina, R. Entropy-based definition of laser beam spot size. Appl. Opt. **1995**, *34*, 8247–8251.
39. Arnaud, J.; Kogelnik, H. Gaussian beams with general astigmatism. Appl. Opt. **1969**, *8*, 1687–1693.
40. Turunen, J. Astigmatism in laser beam optical systems. Appl. Opt. **1986**, *25*, 2908–2911.
41. Serna, J.; Nemes, G. Decoupling of coherent Gaussian beams with general astigmatism. Opt. Lett. **1993**, *18*, 1774–1776.
42. Alda, J.; Vázquez, D.; Bernabéu, E. Wavefront and amplitude profile for astigmatic beams in semiconductor lasers: Analytical and graphical treatment. J. Opt. **1988**, *19*, 201–206.
43. Nazarathy, M.; Shamir, J. First-order optics—a canonical operator representation: Lossless systems. J. Opt. Soc. Am. **1982**, *72*, 356–364.
44. Macukow, B.; Arsenault, H.H. Matrix decomposition for nonsymmetrical optical systems. J. Opt. Soc. Am. **1983**, *73*, 1360–1366.
45. Alda, J.; Wang, S.; Bernabéu, E. Analytical expression for the complex radius of curvature tensor Q for generalized Gaussian beams. Opt. Commun. **1991**, *80*, 350–352.

46. Porras, M.A. Leyes de Propagación y Transformación de Haces Láser por Sistemas Ópticos ABCD. PhD Dissertation; University Complutense of Madrid: Spain, 1992, (in Spanish).
47. Serna, J. Caracterización Espacial de Haces Luminosos Bajo Propagación a Través de Sistemas Ópticos de Primer Orden. PhD Dissertation; University Complutense of Madrid: Spain, 1993, (in Spanish).
48. Carter, W.H. Spot size and divergence for Hermite Gaussian beams of any order. Appl. Opt. **1980**, *19*, 1027–1029.
49. Phillips, R.L.; Andrews, L.C. Spot size and divergence for Laguerre Gaussian beams of any order. Appl. Opt. **1983**, *22*, 643–644.
50. Luxon, J.T.; Parker, D.E.; Karkheck, J. Waist location and Rayleigh range for higher-order mode laser beams. Appl. Opt. **1984**, *23*, 2088–2090.
51. Taché, J.P. Derivation of the *ABCD* law for Laguerre–Gaussian beams. Appl. Opt. **1987**, *26*, 2698–2700.
52. Luxon, J.T.; Parker, D.E. Practical spot size definition for single higher-order rectangular-mode laser beams. Appl. Opt. **1981**, *20*, 1728–1729.
53. Lü, B.; Ma, H. A comparative study of elegant and standard Hermite–Gaussian beams. Opt. Commun. **1999**, *174*, 99–104.
54. Porras, M.A.; Alda, J.; Bernabéu, E. Complex beam parameter and ABCD law for non-Gaussian and non-spherical light beams. Appl. Opt. **1992**, *31*, 6389–6402.
55. Bélanger, P.A. Beam propagation and the ABCD ray matrices. Opt. Lett. **1991**, *16*, 196–198.
56. Siegman, A.E. Defining the effective radius of curvature for a nonideal optical beam. IEEE J. Quantum Electron. **1991**, *27*, 1146–1148.
57. Champagne, Y. A second-moment approach to the time-averaged spatial characterization of multiple-transverse-mode laser beams. J. Opt. Soc. Am. **1995**, *12*, 1707–1714.
58. Morin, M.; Bernard, P.; Galarneau, P. Moment definition of the pointing stability of a laser beam. Opt. Lett. **1994**, *19*, 1379–1381.
59. http://www-ee.stanford.edu/~siegman/beam_quality_refs.html (accessed November 2002).
60. Serna, J.; Martínez-Herrero, R.; Mejías, P.M. Parametric characterization of general partially coherent beams propagating through ABCD optical systems. J. Opt. Soc. Am. A **1991**, *8*, 1094–1098.
61. Serna, J.; Mejías, P.M.; Martínez-Herrero, R. Beam quality dependence on the coherence length of Gaussian Schell-model fields propagating through ABCD optical systems. J. Mod. Opt. **1992**, *39*, 625–635.
62. Siegman, A.E. Analysis of laser beam quality degradation caused by quartic phase aberrations. Appl. Opt. **1993**, *32*, 5893–5901.
63. Siegman, A.E. Binary phase plates cannot improve laser beam quality. Opt. Lett. **1993**, *18*, 675–677.
64. Ruff, J.A.; Siegman, A.E. Measurement of beam quality degradation due to spherical aberration in a simple lens. Opt. Quantum Electron. **1994**, *26*, 629–632.
65. Martínez-Herrero, R.; Piquero, G.; Mejías, P.M. Beam quality changes produced by quartic phase transmittances. Opt. Quantum Electron. **1995**, *27*, 173–183.
66. Alda, J.; Alonso, J.; Bernabéu, E. Characterization of aberrated laser beams. J. Opt. Soc. Am. A **1997**, *14*, 2737–2747.
67. Piquero, G.; Movilla, J.M.; Mejías, P.M.; Martínez-Herrero, R. Beam quality of partially polarized beams propagating through lenslike birefringent elements. J. Opt. Soc. Am. A **1999**, *16*, 2666–2668.
68. Ramee, S.; Simon, R. Effects of holes and vortices on beam quality. J. Opt. Soc. Am. A **2000**, *17*, 84–94.
69. Alda, J. Quality improvement of a coherent and aberrated laser beam by using an optimum and smooth pure phase filter. Opt. Commun. **2001**, *192*, 199–204.
70. Bastiaans, M.J. Wigner distribution function and its application to first-order optics. J. Opt. Soc. Am. **1979**, *69*, 1710–1716.
71. Simon, R.; Sudarshan, E.C.G.; Mukunda, N. Generalized rays in first order optics. Transformation properties of Gaussian Schell model fields. Phys. Rev., A **1984**, *29*, 3273–3279.
72. Bastiaans, M.J. Application of the Wigner distribution function to partially coherent light. J. Opt. Soc. Am. A **1986**, *3*, 1227–1246.
73. Lavi, S.; Prochaska, R.; Keren, E. Generalized beam parameters and transformations laws for partially coherent light. Appl. Opt. **1988**, *27*, 3696–3703.
74. Simon, R.; Mukunda, N.; Sudarshan, E.C.G. Partially coherent beams and a generalized *ABCD* law. Opt. Commun. **1988**, *65*, 322–328.
75. Bastiaans, M.J. Propagation laws for the second-order moments of the Wigner distribution function in first-order optical systems. Optik **1989**, *82*, 173–181.
76. Bastiaans, M.J. Second-order moments of the Wigner distribution function in first order optical systems. Optik **1991**, *88*, 163–168.
77. Gori, F.; Santarsiero, M.; Sona, A. The change of the width for a partially coherent beam on paraxial propagation. Opt. Commun. **1991**, *82*, 197–203.
78. Martínez-Herrero, R.; Mejías, P.M.; Weber, H. On the different definitions of laser beam moments. Opt. Quantum Electron. **1993**, *25*, 423–428.
79. Nemes, G.; Siegman, A.E. Measurement of all ten second-order moments of an astigmatic beam by the use of rotating simple astigmatic (anamorphic) optics. J. Opt. Soc. Am. A **1994**, *11*, 2257–2264.

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