



IEEE TRANSACTIONS ON

10/25/99

COMMUNICATIONS

A PUBLICATION OF THE IEEE COMMUNICATIONS SOCIETY



OCTOBER 1999

VOLUME 47

NUMBER 10

IECMBT

(ISSN 0090-6778)

TRANSACTIONS LETTERS

Coding

Comparison of Constructions of Irregular Gallager Codes..... *D. J. C. MacKay, S. T. Wilson, and M. C. Davey* 1449

Data Compression

Level-Compressed Huffman Decoding *K.-L. Chung and J.-G. Wu* 1455

Digital Communications

Maximum Ratio Transmission *T. K. Y. Lo* 1458

Modulation/Detection

The Constellation-Shaping Algorithm Using Closed-Form Expressions for the Number of Ring Combinations..... *C. E. D. Sterian* 1462

Optimum Precompensation Filters for IQ Modulation Systems *J. Tuthill and A. Cantoni* 1466

Personal Communication Systems

Optimal Adaptive Spectrum Utilization in Cellular Communications Systems *M. G. Kazantzakis, P. P. Demestichas, E. C. Tzifa, and M. E. Anagnostou* 1469

Synchronization

About the Asymptotic Performance of MMSE MIMO DFE for Filter-Bank Based Multicarrier Transmission *L. Vandendorpe, J. Louveaux, B. Maison, and A. Chevreuil* 1472

TRANSACTIONS PAPERS

Coding

Very Low Rate Trellis/Reed-Muller (TRM) Codes *J.-P. Chaib and H. Leib* 1476

On Decoding of Both Errors and Erasures of a Reed-Solomon Code Using an Inverse-Free Berlekamp-Massey Algorithm *J.-H. Jeng and T.-K. Truong* 1488

The Union Bound for Turbo-Coded Modulation Systems over Fading Channels *T. M. Duman and M. Salehi* 1495

Fading/Equalization

Performance Analysis of Optimum Combining with Multiple Interferers in Flat Rayleigh Fading *E. Villier* 1503

Reuse Within a Cell—Interference Rejection or Multiuser Detection? *C. Tiestav, M. Sternad, and A. Ahlén* 1511

Multiple Access

Time-Varying Narrow-Band Interference Rejection in Asynchronous Multiuser DS/CDMA Systems over Frequency-Selective Fading Channels *S. Buzzi, M. Lops, and A. M. Tulino* 1523

Personal Communication Systems

Latency Probability of a Retransmission Scheme for Error Control on a Two-State Markov Channel..... *M. Zorzi and R. R. Rao* 1537

Spread Spectrum

Performance Enhancement of DSSS Systems: Two-Dimensional Interference Suppression *S. G. Glisic, Z. B. Nikolic, D. Pokrajac, and P. A. Leppänen* 1549

Channel Estimation for DS-CDMA Downlink with Aperiodic Spreading Codes *A. J. Weiss and B. Friedlander* 1561

Convolutionally Coded Multicarrier DS-CDMA Systems in a Multipath Fading Channel— Part I: Performance Analysis *D. N. Rowitch and L. B. Milstein* 1570

Optimal Reception, Performance Bound, and Cutoff Rate Analysis of References-Assisted Coherent CDMA Communications with Applications *F. Ling* 1583

(Contents Continued on Back Cover)



IEEE COMMUNICATIONS SOCIETY

The field of interest of the Communications Society consists of all telecommunications including telephone, telegraphy, facsimile, and point-to-point television, by electromagnetic propagation including radio; wire, aerial; underground, coaxial, and submarine cables; waveguides, communication satellites, and lasers; in marine, aeronautical, space and fixed station services; repeaters, radio relaying, signal storage, and regeneration; telecommunication error detection and correction; multiplexing and carrier techniques; communication switching systems; data communications; and communication theory.

In addition to the above, this TRANSACTIONS contains papers pertaining to analog and digital signal processing and modulation, audio and video encoding techniques, the theory and design of transmitters, receivers, and repeaters for communication via optical and sonic media, the design and analysis of computer communication systems, and the development of communication software. Contributions of theory enhancing the understanding of communication systems and techniques are included, as are discussions of the social implications of the development of communication technology. All members of the IEEE are eligible for membership in the Society upon payment of the annual Society membership fee of \$23.00. Members may receive this TRANSACTIONS upon payment of an additional \$27.00 (\$50.00 total), or the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS upon payment of an additional \$32.00 (\$55.00 total), or both publications upon payment of an additional \$59.00 (\$82.00 total). For information on joining, write to the IEEE at the address below. Member copies of Transactions/Journals are for personal use only.

IEEE COMMUNICATIONS SOCIETY TRANSACTIONS Editorial Board

W. H. TRANTER, *Director of Journals*
Dept. Elec. & Comput. Eng.
Virginia Polytechnic Inst.
Blacksburg, VA 24061

D. P. TAYLOR, *Editor-in-Chief*
Dept. Elec. Electron. Eng.
Univ. Canterbury
Christchurch, New Zealand

D. G. DAUT, *Publications Editor*
Dept. Elec. & Comput. Eng.
Rutgers Univ.
Piscataway, NJ 08855

I. JACOBS, *Senior Advisor*
Dept. Elec. Eng.
Virginia Polytechnic Inst.
Blacksburg, VA 24061

Modulation & Signal Design

A. AHLEN, *Demodulation & Equalization*
Signals and Systems, Uppsala Univ.
Uppsala, Sweden
O. ANDRISANO, *Modulation for Fading Channels*
DEIS/CSITE, Bologna, Italy
S. BENEDETTO, *Area Editor*
Dept. Elec., Politec. di Torino, Italy
M. BRANDT-PEARCE, *CDMA Systems*
EE Dept., Univ. Virginia
Charlottesville, VA
G. CAIRE, *Multisuser Detection & CDMA*
Inst. Eurocom, Cedex, France
G. CHERUBINI, *CDMA Systems*
IBM Research Lab., Zurich, Switzerland
G. CORAZZA, *Spread Spectrum*
DEIS, Univ. of Bologna, Bologna, Italy
B. L. HUGHES, *Theory & Systems*
ECE Dept.
North Carolina State Univ., Raleigh, NC
R. A. KENNEDY, *Data Commun.*
Res. School Info. Sci. Eng.
Australian Nat'l. Univ., Canberra, Australia
R. KOHNO, *Spread Spectrum Theory & Appl.*
Div. ECE, Yokohama Univ., Yokohama, Japan
U. MITRA, *Spread Spectrum/Equalization*
EE Dept., Ohio State Univ., Columbus, OH
C. ROBERTSON, *Spread Spectrum Syst.*
ECE Dept.
Naval Postgraduate School, Monterey, CA
S. ROY, *Theory/Systems & Electronic Processes*
EE Dept., Univ. Washington, Seattle, WA
W. E. RYAN, *Modulation, Coding & Equalization*
ECE Dept., Univ. Arizona, Tucson, AZ
C. TELLAMBURA, *Multicarrier Systems*
Information Tech., Monash Univ.,
Clayton, Australia

Wireless Communication

S. ARYAVISITAKUL, *Wireless Techniques & Fading*
Home Wireless Networks, Norcross, GA
N. C. BEAULIEU, *Wireless Commun. Theory*
EE Dept., Queen's Univ., Kingston, Canada

K. C. CHEN, *Wireless Data Commun.*
National Taiwan Univ., Taipei, Taiwan
J. C.-I. CHUANG, *Area Editor*
AT&T Labs.—Research, Red Bank, NJ

A. GOLDSMITH, *Multisuser and Adaptive Systems*
EE Dept., Stanford Univ., Stanford, CA

B. JABBARI, *Wireless Multiple Access*
ECE Dept., George Mason Univ., Fairfax, VA

P. Y. KAM, *Modulation & Detection*
Electrical Engineering
National Univ. Singapore, Singapore

Z. KOSTIC, *Wireless Systems*
AT&T Labs.—Research, Red Bank, NJ

K. B. LETAIEF, *Wireless Systems*
EEE Dept., HKUST, Hong Kong

K. K. LEUNG, *Wireless Network Access & Performance*
AT&T Labs.—Research, Red Bank, NJ

Y. LI, *Wireless Communication Theory*
AT&T Labs.—Research, Red Bank, NJ

P. T. MATHIOPOULOS, *Wireless Personal Commun.*
EE Dept., Univ. Brit. Columbia
Vancouver, BC, Canada

E. SOUSA, *CDMA Systems*
EE Dept., Univ. Toronto, Toronto, Canada

R. A. VALENZUELA, *Wireless Systems*
Lucent Technologies, Bell Labs., Holmdel NJ

J. WANG, *Wireless Spread Spectrum*
EEE Dept., Univ. Hong Kong, Hong Kong

Optical Communication

D. K. HUNTER, *Photonic Networks*
Dept. Electron. Elect. Eng.
Univ. Strathclyde, Glasgow, U.K.

J. J. O'REILLY, *Opt. Commun. Syst.*
Dept. Elec. Electron. Eng.
Univ. College, London, U.K.

(Information for Authors, ComSoc Board of Governors, Departments, and Committees appear on Cover III)

Editors

P. PRUCNAL, *Area Editor & Light. Networks*
EE Dept., Princeton Univ., Princeton, NJ
O. TONGUZ, *Optical Transmission Systems*
ECE Dept., SUNY, Buffalo, NY

Speech, Image, Video, & Signal Process.

M. R. CIVANLAR, *Image Commun. Syst.*
AT&T Labs.—Research, Holmdel, NJ
B. GIROD, *Image Processing, Area Editor*
EE Dept., Univ. Erlangen-Nuremberg
Germany

K. ILLGNER, *Image Processing and Transmission*
Texas Instruments, Inc., Dallas, TX

K.-K. MA, *Video & Signal Processing*
Elect. & Electron. Eng.
Nanyang Tech. Univ., Singapore

C. S. RAVISHANKAR, *Speech Process.*
Hughes Network Systems
Germantown, MD

A. H. WATANABE, *Video Image*
NTT Human Interface Labs., Commun.
Yokosuka, Japan

Transmission Systems

S. GELFAND, *Equalization*
ECE Dept., Purdue Univ., West Lafayette, IN

H. LUI, *Synchronization & Equalization*
EE Dept., Univ. Washington, Seattle, WA

R. RAHELI, *Detection, Equalization & Coding*
Information Eng., Univ. Parma, Parma, Italy

J. K. TOWNSEND, *CAD Commun. Syst.*
ECE Dept., NC State Univ., Raleigh, NC

L. VANDENDORPE, *Synchronization & Equalization*
Univ. Catholique de Louvain,
Louvain-la-Neuve, Belgium

C.-L. WANG, *Equalization*
EE Dept., National Tsing Hua Univ., Taiwan

M. Z. WIN, *Equalization & Diversity*
AT&T Labs.—Research, Red Bank, NJ

J. WINTERS, *Area Editor & Equalization*
AT&T Bell Labs., Holmdel, NJ

Network Architecture

G. S. KUO, *Commun. Architect.*
Dept. Inf. Mgt., National Central Univ.
Chung-Li, Taiwan

N. MCKEOWN, *Switching & Routing*
EECS Dept., Stanford Univ., Stanford, CA

G. O'REILLY, *Commun. Switch.*
AT&T Labs., Lincroft, NJ

A. PATTAVINA, *Switching Arch. Perform.*
Dept. Elec. & Inform.
Politec. di Milano, Italy

R. RAO, *Packet Multiple Access*
Elect. and Computer Eng.
Univ. California, San Diego
La Jolla, CA

P. E. RYNES, *Switch. Syst.*
AT&T Bell Labs., Naperville, IL

E. G. SABLE, *Area Editor*
AT&T Bell Labs., Holmdel, NJ

Coding & Commun. Theory

T. AULN, *Coding & Commun. Theory Appl.*
Chalmers Univ. Technol., Goteborg, Sweden
E. AYANÖGLU, *Commun. Theory & Coding Appl.*
Bell Labs., Lucent Tech., Holmdel, NJ

M. FOSSORIER, *Coding & Commun. Theory*
EE Dept., Univ. Hawaii, Honolulu, HI

P. HOEHER, *Coding & Iterative Processing*
Information & Coding Theory Lab.
Kiel Univ., Kiel, Germany

R. KOETTER, *Coding Theory & Techniques*
CSL, Univ. Illinois, Urbana, IL

C. SCHLEGEL, *Coding Theory & Techniques*
EE Dept., Univ. Utah, Salt Lake City, UT

R. D. WESEL, *Coding & Coded Modulation*
EE Dept., UCLA, Los Angeles, CA

S. G. WILSON, *Area Editor & Coding*
Theory & Appl.
EE Dept., Univ. Virginia, Charlottesville, VA

D. DIVSALAR
Y. BAR-NESS

THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC. Officers

KENNETH R. LAKER, *President*
BRUCE A. EISENSTEIN, *President-Elect*
MAURICE PAPO, *Secretary*
DAVID A. CONNER, *Treasurer*
ARTHUR W. WINSTON, *Vice President, Educational Activities*

LYOUD A. MORLEY, *Vice President, Publication Activities*
DANIEL R. BENIGNI, *Vice President, Regional Activities*
DONALD C. LOUGHRY, *Vice President, Standards Association*
MICHAEL S. ADLER, *Vice President, Technical Activities*
PAUL J. KOSTEK, *President, IEEE USA*

DAVID G. DAUT, *Director, Division III—Communications Technology Division*

Executive Staff

DANIEL J. SESENE, *Executive Director*

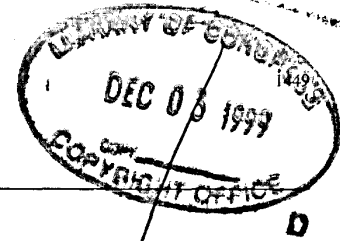
DONALD CURTIS, *Human Resources*
ANTHONY DURNIAK, *Publications*
JUDITH GORMAN, *Standards Activities*
CELIA JANKOWSKI, *Regional Activities*
PETER A. LEWIS, *Educational Activities*

RICHARD D. SCHWARTZ, *Business Administration*
W. THOMAS SUTTLE, *Professional Activities*
MARY WARD-CALLAN, *Technical Activities*
JOHN WITSKEN, *Information Technology*

IEEE Periodicals

Transactions/Journals Department
Staff Director: FRAN ZAPPULLA
Editorial Director: VALERIE CAMMARATA
Production Director: ROBERT SMREK
Transactions Manager: GAIL S. FERENC
Electronic Publishing Manager: STEPHEN COHEN
Managing Editor: MONA MITTRA
Associate Editor: LISA M. PERRY

IEEE TRANSACTIONS ON COMMUNICATIONS (ISSN 0090-6778) is published monthly by The Institute of Electrical and Electronics Engineers, Inc. Responsibility for the contents rests upon the authors and not upon the IEEE, the Society, or its members. **IEEE Corporate Office:** 3 Park Avenue, 17th Floor, NY 10016-5997. **IEEE Operations Center:** 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331. **NJ Telephone:** 732-981-0060. **Price/Publication Information:** Individual copies: IEEE Members \$10.00 (first copy only), nonmembers \$20.00 per copy. (Note: Add \$4.00 postage and handling charge to any order from \$1.00 to \$50.00, including prepaid orders.) Member and nonmember subscription prices available upon request. Available in microfiche and microfilm. **Copyright and Reprint Permissions:** Abstracting is permitted with credit to the source. Libraries are permitted to photocopy for private use of patrons, provided the per-copy fee indicated in the code at the bottom of the first page is paid through the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923. For all other copying, reprint, or republication permission, write to Copyrights and Permissions Department, IEEE Publications Administration, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331. Copyright © 1999 by The Institute of Electrical and Electronics Engineers, Inc. All rights reserved. Periodicals Postage Paid at New York, NY and at additional mailing offices. **Postmaster:** Send address changes to IEEE TRANSACTIONS ON COMMUNICATIONS, IEEE, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331. GST Registration No. 125634188. Printed in U.S.A.



Transactions Letters

Comparison of Constructions of Irregular Gallager Codes

David J. C. MacKay, Simon T. Wilson, *Associate Member, IEEE*, and Matthew C. Davey

Abstract—The low-density parity check codes whose performance is closest to the Shannon limit are “Gallager codes” based on irregular graphs. We compare alternative methods for constructing these graphs and present two results. First, we find a “super-Poisson” construction which gives a small improvement in empirical performance over a random construction. Second, whereas Gallager codes normally take N^2 time to encode, we investigate constructions of regular and irregular Gallager codes that allow more rapid encoding and have smaller memory requirements in the encoder. We find that these “fast encoding” Gallager codes have equally good performance.

Index Terms—Channel coding, error correction coding, Gaussian channels, graph theory, iterative probabilistic decoding, random codes.

I. INTRODUCTION

GALLAGER codes [3], [4] are low-density parity check codes constructed at random subject to constraints on the weight of each row and of each column. The original *regular* Gallager codes have very sparse random parity check matrices with uniform weight t per column and t_r per row. (We will also use the term “regular” for codes that have nearly uniform weight columns and rows—for example, codes which have some weight 2 columns and some weight 3 columns.) These codes are asymptotically good and can be practically decoded with Gallager’s sum-product algorithm giving near Shannon limit performance when large block lengths are used [6]–[8]. Regular Gallager codes have also been found to be competitive codes for short block-length code-division multiple-access (CDMA) applications [10].

Recent advances in the performance of Gallager codes are summarized in Fig. 1. The rightmost curve shows the performance of a regular binary Gallager code with rate 1/4. The best known binary Gallager codes are *irregular* codes whose parity check matrices have *nonuniform* weight per column [5]; the performance of one such code is shown by the second curve from the right. The best known Gallager codes of all are Gallager codes defined over finite fields $GF(q)$ [1], [2]. The remaining two solid curves in Fig. 1 show the performance of a regular Gallager code over $GF(16)$ [2] and

Paper approved by S. B. Wicker, the Editor for Coding Theory and Techniques of the IEEE Communications Society. Manuscript received August 11, 1998; revised January 27, 1999. This paper was presented in part at the 1998 Allerton Conference on Communications, Control, and Computing, Allerton House, IL, September 1998.

The authors are with the Department of Physics, University of Cambridge, Cambridge CB3 0HE, U.K. (e-mail: mackay@mrao.cam.ac.uk; stw11@mrao.cam.ac.uk; mcdavey@mrao.cam.ac.uk).

Publisher Item Identifier S 0090-6778(99)07784-3.

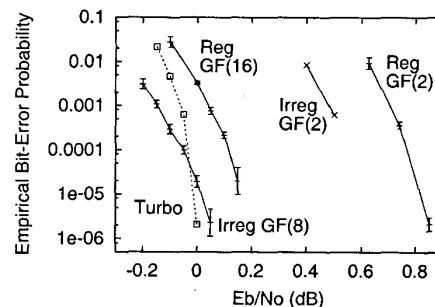


Fig. 1. Empirical results for Gaussian channel, rate 1/4 left-right: irregular LDPC, $GF(8)$ blocklength 24 000 bits; JPL Turbo, blocklength 65 536 bits; regular LDPC, $GF(16)$, blocklength 24 448 bits; irregular LDPC, $GF(2)$, blocklength 64 000 bits; regular LDPC, $GF(2)$, blocklength 40 000 bits. (Reproduced from [1].)

an irregular code over $GF(8)$ with bit-error probability of 10^{-4} at $E_b/N_0 = -0.05$ dB [1]. In comparing this code with the rate 1/4 turbo-code shown by the dotted line, the following points should be noted. 1) The transmitted blocklength of the irregular Gallager code is only 24 000 bits, whereas that of the turbo-code is 65 536 bits. 2) The errors made by the Gallager codes were all detected errors, whereas turbo-codes make undetected errors at high signal-to-noise ratio. This difference is not caused by a difference in the decoding algorithm: both codes are decoded by the sum-product algorithm [9]. Turbo-codes make undetected errors because they have *low-weight codewords*. For Gallager codes, the rate of occurrence of undetected errors is extremely small because they have good distance properties (the minimum distance scales linearly with the blocklength) [4]. In all our experiments with Gallager codes of block length greater than 1000 and column weight at least 3, undetected errors have never occurred.

The excellent performance of irregular Gallager codes is the motivation for this paper, in which we explore ways of further enhancing these codes.

The irregular codes of Luby, Mitzenmacher, Shokrollahi, and Spielman [5] have parity check matrices with both nonuniform weight per row and nonuniform weight per column. It has not yet been established whether both of these nonuniformities are desirable. In our experience with codes for noisy channels, performance is more sensitive to the distribution of column weights. In this paper, we concentrate on irregular codes with the weight per row as uniform as possible.

We can define an irregular Gallager code in two steps. First, we select a *profile* that describes the desired number of columns of each weight and the desired number of rows of

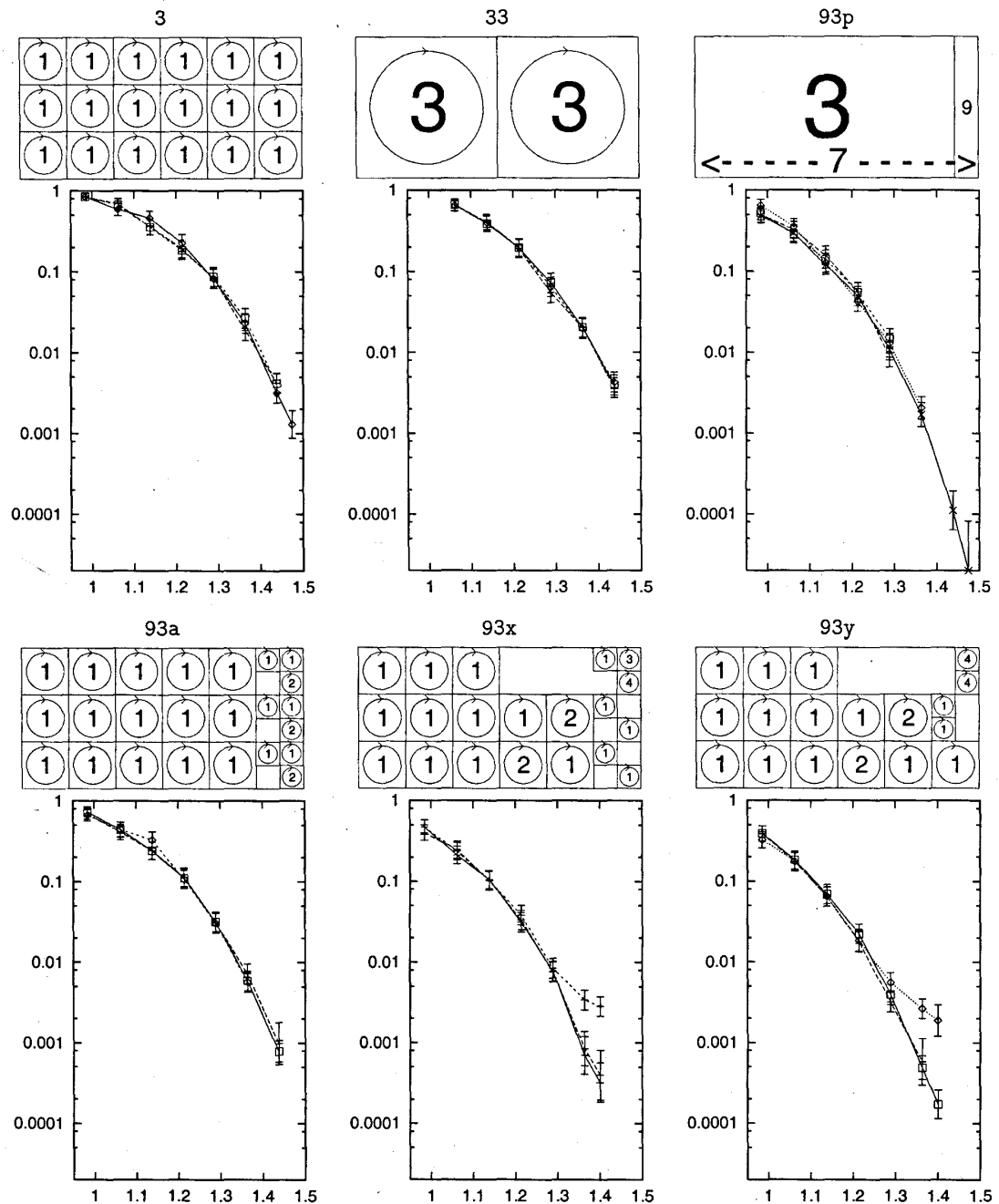


Fig. 2. Upper panels: constructions of regular and irregular codes. Lower panels: performance of these codes. The construction types shown are regular, (3, 33), Poisson (93p), sub-Poisson (93a), super-Poisson (93x), and super-Poisson (93y). Notation for upper panels for all constructions except 93p: an integer represents a number of permutation matrices superposed on the surrounding square. Horizontal and vertical lines indicate the boundaries of the permutation blocks. Notation for the Poisson construction 93p: integers "3" and "9" represent column weights. The integer "7" represents the row weight. Lower panels show the performance of several random codes of each construction. Vertical axis: block error probability. Horizontal axis: E_b/N_0 in dB. All codes have $N = 9972$ and $K = M = 4986$. All errors were detected errors, as is usual with Gallager codes.

each weight. The parity check matrix of a code can be viewed as defining a bipartite graph with "bit" vertices corresponding to the columns and "check" vertices corresponding to the rows. Each nonzero entry in the matrix corresponds to an edge connecting a bit to a check. The profile specifies the degrees of the vertices in this graph.

Second, we choose a *construction method*, that is, a pseudo-random algorithm for putting edges between the vertices in a

way that satisfies the constraints. (In the case of nonbinary Gallager codes, we also need to choose an algorithm for assigning values to the nonzero entries in the matrix.)

This paper has two parts. In the first part (Section III), we compare alternative construction methods for a fixed profile in order to find out whether the construction method matters. In the second part (Section IV), we examine regular and irregular constructions which lend themselves to rapid encoding. One

motivation for this second study is that the only drawback of regular Gallager codes compared to turbo-codes for CDMA applications appears to be their greater encoding complexity [10].

In the experiments presented here, we study binary codes with rate 1/2 and blocklength about $N = 10\,000$. We simulate an additive white Gaussian noise channel in the usual way [2] and examine the block error probability as a function of the signal-to-noise ratio. The error bars we show are one standard deviation error bars on the estimate of the logarithm of the block error probability \hat{p} defined. Thus, when we observe r failures out of n trials, $p_{\pm} = \hat{p} \exp(\pm \sigma_{\log p})$ where $\sigma_{\log p} = \sqrt{(n-r)/(rn)}$.

II. CONSTRUCTIONS

We compare the following methods.

Poisson: The edges are placed “completely at random,” subject to the profile constraints and the rule that you cannot put two edges between one pair of vertices, which would correspond to a double entry in the parity check matrix. One way to implement a Poisson construction is to make a list of all the columns in the matrix, with each column appearing in the list a number of times equal to its weight, then make a similar list of all the rows in the matrix, each row appearing with multiplicity equal to its weight, and then map one list onto the other by a random permutation, taking care not to create duplicate entries [5].

A variation of this construction is to require that no two columns in the parity check matrix have an overlap greater than one, i.e., forbid cycles of length 4 in the graph. (Similar to construction 1A in [8].) A second variation requires that the graph have no cycles of length less than some l . (Similar to construction 1B in [8].) This constraint can be quite hard to enforce if the profile includes high weight rows or columns.

Permutations: We can build parity check matrices by superposing random permutation matrices [4]. The convenience of this method depends on the profile. There are many ways of laying out these permutation matrices to satisfy a given profile. We will distinguish “super-Poisson” and “sub-Poisson” constructions.

- In a super-Poisson construction, the distribution of high weight columns per row has greater variance than a Poisson distribution.
- In a sub-Poisson construction, the distribution of high weight columns per row has smaller variance than a Poisson distribution.

III. COMPARING POISSON, SUPER-POISSON, AND SUB-POISSON CONSTRUCTIONS

A. Profiles and Constructions Studied in this Paper

1) **Regular Codes—3 and 33:** As our baseline, we study regular Gallager codes with weight per column exactly $t = 3$ and weight per row exactly $t_r = 6$. We construct parity check matrices satisfying this profile from permutation matrices in two ways, labeled “3” and “33,” shown diagrammatically in the upper panels of Fig. 2. In the figure, a square containing

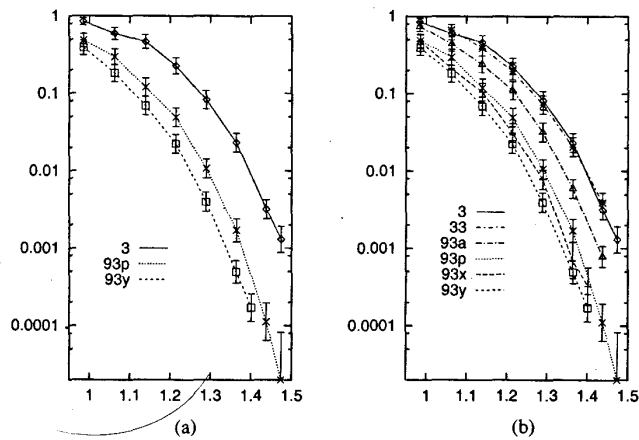


Fig. 3. (a) Comparison of one representative of each of the constructions: 3 (regular), 93p (Poisson) and 93y (super-Poisson). (b) Representatives of all six constructions in Fig. 2. Vertical axis: block error probability. Horizontal axis: E_b/N_0 in dB.

TABLE I
THE TWO PROFILES STUDIED IN THIS PAPER

Profile 3	Column weight	Fraction of columns	Row weight	Fraction
	3	1	6	1
Profile 93	Column weight	Fraction of columns	Row weight	Fraction
	3	11/12	7	1
	9	1/12		

an integer (for example, “3”) denotes the superposition inside that square of that number of random permutation matrices. The matrices are generated at random subject to the constraint that no two nonzero entries coincide.

2) **Irregular Codes—93p, 93a, 93x, and 93y:** We chose the profile “93” shown in Table I. It has columns of weight 9 and of weight 3; all rows have weight 7. Note that this profile only differs from the regular profile “3” in that some extra 1’s are added to 1/12 of the columns. We emphasize that this profile has not been carefully optimized, so the results of this paper should not be taken as describing the best that can be done with irregular binary Gallager codes. We chose this profile because it lends itself to interesting experiments.

We will refer to the bits that connect to nine checks as “elite” bits. We use four different constructions that match this profile, named as follows. These constructions are depicted diagrammatically in the upper panels of Fig. 2.

Poisson—93p: In this construction, while most checks will connect to one or two elite bits, a fraction of them will connect to more than two elite bits, and some will connect to none.

Sub-Poisson—93a: This construction allocates exactly one or two elite bits to each check.

Super-Poisson: 93x and 93y are, respectively, moderately and very super-Poisson. In 93y, one third of the checks are connected to four elite bits, one third are connected to one, and one third are connected to none.

B. Results

1) **Variability Within Each Construction:** For each construction, we created several codes in order to assess the variability of performance within each ensemble. All codes

Explore Litigation Insights

Docket Alarm provides insights to develop a more informed litigation strategy and the peace of mind of knowing you're on top of things.

Real-Time Litigation Alerts



Keep your litigation team up-to-date with **real-time alerts** and advanced team management tools built for the enterprise, all while greatly reducing PACER spend.

Our comprehensive service means we can handle Federal, State, and Administrative courts across the country.

Advanced Docket Research



With over 230 million records, Docket Alarm's cloud-native docket research platform finds what other services can't. Coverage includes Federal, State, plus PTAB, TTAB, ITC and NLRB decisions, all in one place.

Identify arguments that have been successful in the past with full text, pinpoint searching. Link to case law cited within any court document via Fastcase.

Analytics At Your Fingertips



Learn what happened the last time a particular judge, opposing counsel or company faced cases similar to yours.

Advanced out-of-the-box PTAB and TTAB analytics are always at your fingertips.

API

Docket Alarm offers a powerful API (application programming interface) to developers that want to integrate case filings into their apps.

LAW FIRMS

Build custom dashboards for your attorneys and clients with live data direct from the court.

Automate many repetitive legal tasks like conflict checks, document management, and marketing.

FINANCIAL INSTITUTIONS

Litigation and bankruptcy checks for companies and debtors.

E-DISCOVERY AND LEGAL VENDORS

Sync your system to PACER to automate legal marketing.