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1. Sequence Design for Communications Applications  
**Pingzhi Fan and Michael Darnell**
2. Communications Coding and Signal Processing  
THIRD VOLUME ON COMMUNICATION THEORY AND APPLICATIONS  
*Edited by Bahram Honary, Michael Darnell and Paddy Farrell*
3. Coding, Communications and Broadcasting  
FOURTH VOLUME ON COMMUNICATION THEORY AND APPLICATIONS  
*Edited by Paddy Farrell, Michael Darnell and Bahram Honary*

# Coding, Com Broadcasting

FOURTH VOLUME  
THEORY AND APPL

*Edited by*

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## **Editorial Preface**

It is with great pleasure that I introduce the third book in the Series on Communications Applications. The aim of the series is to provide a forum for the work of researchers who have interacted beneficially. The Series has been successful in this regard. Most chapters in the three sections of the Series, Communications Techniques, and Applications, are concerned with topics of mutual interest and of the beneficial and close synergy between them.

This third book in the Series is a direct result of the work that has emerged from the International Symposium on Communications Applications, held regularly at the University of Exeter. The Symposium held in July 1999 achieved a most successful programme, from which key presentations were selected for this volume, for the benefit of all those who attend the event! The Editors are to be congratulated for their selection of material which will inform and benefit the research community. On behalf of Research Studies Press, it is a pleasure to include this outstanding contribution.

## Gallager Codes –

David J. C. MacKay (m  
Cavendish Laboratory, Cambri

### Abs

In 1948, Claude Shannon posed problems of information theory. The to communicate reliably over noisy He defined a theoretical limit, not to which communication is possible is not possible. Since 1948, coding error-correcting codes capable of g

In the last decade remarkable p that are defined in terms of sparse coded by a simple probability-base

This paper reviews low-density repeat-accumulate codes, and turbo Some previously unpublished result experiments on Gallager codes with rule for decoding of repeat-accum time and allows block decoding err (c) the empirical power-laws obeye codes.

### 1 Introduction

The central problem of communication a decoding system that make it possible channel. The encoding system uses from a set of codewords. The decoded output of the channel, which codeword been transmitted; for an appropriate d codeword to the received signal. A go are well spaced apart, so that codewo

Designing a good and practical en (a) it is hard to find an explicit set o generic code, decoding, *i.e.*, finding th is intractable.

However, a simple method for desi (1962), has recently been rediscovere

fined in terms of sparse random graphs. Because the graphs are constructed randomly, the codes are likely to have well-spaced codewords. And because the codes' constraints are defined by a sparse graph, the decoding problem can be solved — almost optimally — by message-passing on the graph. The practical performance of Gallager's codes and their modern cousins is vastly better than the performance of the codes with which textbooks have been filled in the intervening years.

## 2 Sparse Graph Codes

In a **sparse graph code**, the nodes in the graph represent the transmitted bits and the constraints they satisfy. For a linear code with a codeword length  $N$  and rate  $R = K/N$ , the number of constraints is of order  $M = N - K$ . [There could be more constraints, if they happen to be redundant.] Any linear code can be described by a graph, but what makes a sparse graph code special is that each constraint involves only a small number of variables in the graph: the number of edges in the graph scales roughly linearly with  $N$ , rather than as  $N^2$ .

The graph defining a **low-density parity-check code**, or Gallager code (Gallager 1962; Gallager 1963; MacKay 1999), contains two types of node: codeword bits, and parity constraints. In a regular  $(j, k)$  Gallager code (figure 1a), each codeword bit is connected to  $j$  parity constraints and each constraint is connected to  $k$  bits. The connections in the graph are made at random.

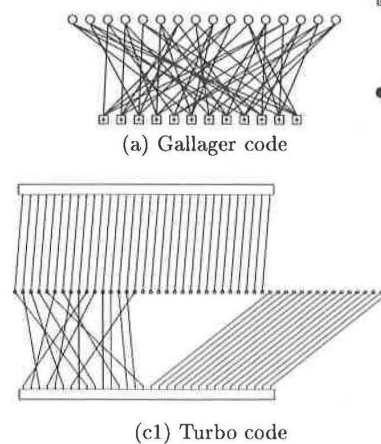
**Repeat-accumulate codes** (Divsalar *et al.* 1998) can be represented by a graph with four types of node (figure 1b): **equality constraints**  $\square$ , intermediate binary variables (black circles), **parity constraints**  $\oplus$ , and the transmitted bits (white circles). The encoder sets each group of intermediate bits to values read from the source. These bits are put through a fixed random permutation. The transmitted stream is the accumulated sum (modulo 2) of the permuted intermediate bits.

In a **turbo code** (Berrou and Glavieux 1996), the  $K$  source bits drive two linear feedback shift registers, which emit parity bits (figure 1c).

All these codes can be decoded by a local message-passing algorithm on the graph, the sum-product algorithm (MacKay and Neal 1996; McEliece *et al.* 1998), and, while this algorithm is not the optimal decoder, the empirical results are record-breaking. Figure 2 shows the performance of various sparse graph codes on a Gaussian channel. In figure 2(a) turbo codes with rate 1/4 are compared with regular and irregular Gallager codes over GF(2), GF(8) and GF(16). In figure 2(b) the performance of repeat-accumulate codes of various blocklengths and rate 1/3 is shown.

### THE BEST SPARSE GRAPH CODES

Which of the three types of sparse graph code is 'best' depends on the chosen rate and blocklength, the permitted encoding and decoding complexity, and



**Figure 1.** Graphs of three sparse graph codes. (a) A rate 1/4 low-density parity-check code with blocklength  $N = 16$ , and  $M = 4$  parity constraints. Each  $\circ$  represents a transmitted bit. Each  $\square$  is represented by  $\oplus$  squares. Each  $\square$  is connected to  $k = 4$  bits to which it is connected. (b) A repeat-accumulate code with blocklength  $N = 16$ . Each  $\circ$  represents a transmitted bit. Each  $\square$  represents an equality constraint. Each  $\square$  is connected to be equal. (c) A turbo code with rate 1/4 and blocklength  $N = 16$ . The two rectangles represent the systematic bits  $\{t^{(1)}\}$  and the parity bits  $\{t^{(2)}\}$ .

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