Encyclopedia of Sparse Graph Codes

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Abstract

Evaluation of Gallager codes for low error tolerance, short block length and high rate applications.

Sparse graph codes include Gallager codes, Tanner codes, MN codes Repeat–Accumulate codes (RA codes), and turbo codes, all of which have near–Shannon limit performance.

This paper (which is still in preparation) describes empirical properties of a wide selection of these codes, comparing in particular the codes' block error rates with an emphasis on undetected versus detected errors. We explore the dependence of block error rate on block length and other code construction parameters. Histograms of decoding time are also shown.

Draft 1 concentrates on small block lengths.

Summary

1 Regular Gallager codes

For each construction and block length I generated three random codes. Their performances are shown in figures 1–4 to give an idea of the variability within one construction.

The program GHG.p was used to make these codes. This program ensures there are no four–cycles in the code's graph. In subsequent figures, one representative (the best) of these three is selected.

1.1 Dependence on transmitted block length N

Figures 5 and 6.

1.2 Dependence on column weight t

Figures 7 and 8.

2 High rate Gallager codes

This section includes a collection of codes with rates $191/273 = 0.7$ and $813/1057 = 0.77$, and a collection of codes with rates greater than 0.89 and block lengths around 2000 and 4000.

2.1 Rates 0.7 and 0.77

2.2 Codes with rates above 0.89

More results on these codes can be found in another publication [3].

2.3 Discussion

In practical systems such as disc drives, people concatenate an outer ECC with an inner run-length-limiting code (RLL code), the latter being small and non-linear. This doesn't seem ideal, since it means that the errors confronting the ECC are rather complex. So, what if we could make an ECC that *is* an RLL code?

An example of such an ECC is (surprise!) a Gallager code. If I make a Gallager code whose top rows go like this:

```
1111100000000000000000000
0000011111000000000000000
0000000000111110000000000
0000000000000001111100000  ....
                 .
                 .
                 .
```

where the row weight, k, is 5 in this example, then the maximum possible run length of 1s in a codeword is 2(k-1) = 8.

If the row weight is even instead of odd, we can get more bang for our buck: if k=6, we can modify every codeword in the code by adding the vector `100000100000100000100000100000100000.` to it. Then all codewords will have a maximum possible run length of 1s equal to $2(k-1)$, *and* a maximum possible run length of 0s equal to $2(k-1)$.

With a minor tweak of the Gallager code, I can reduce $2(k-1)$ above to $2k-3$.

Thus it is easy to make, for example, a good Gallager code with rate $R = 1/2$ that

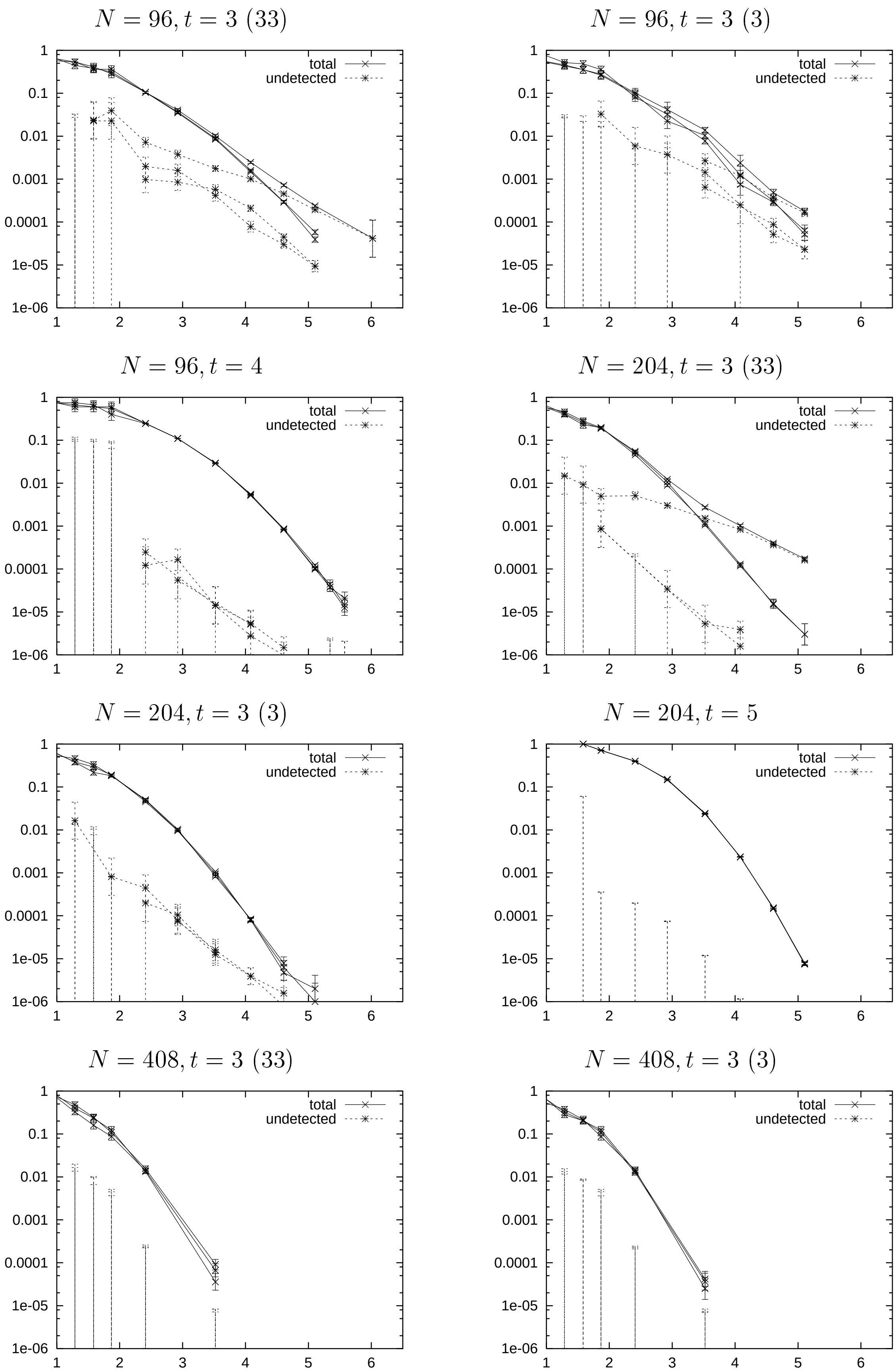

Figure 1: Regular Gallager codes with rate $R = 1/2$. Variability within one construction. Dependence of block error rate on signal to noise ratio, weight per column t and transmitted block length N. Vertical axis: block error rate. Horizontal axis: E_b/N

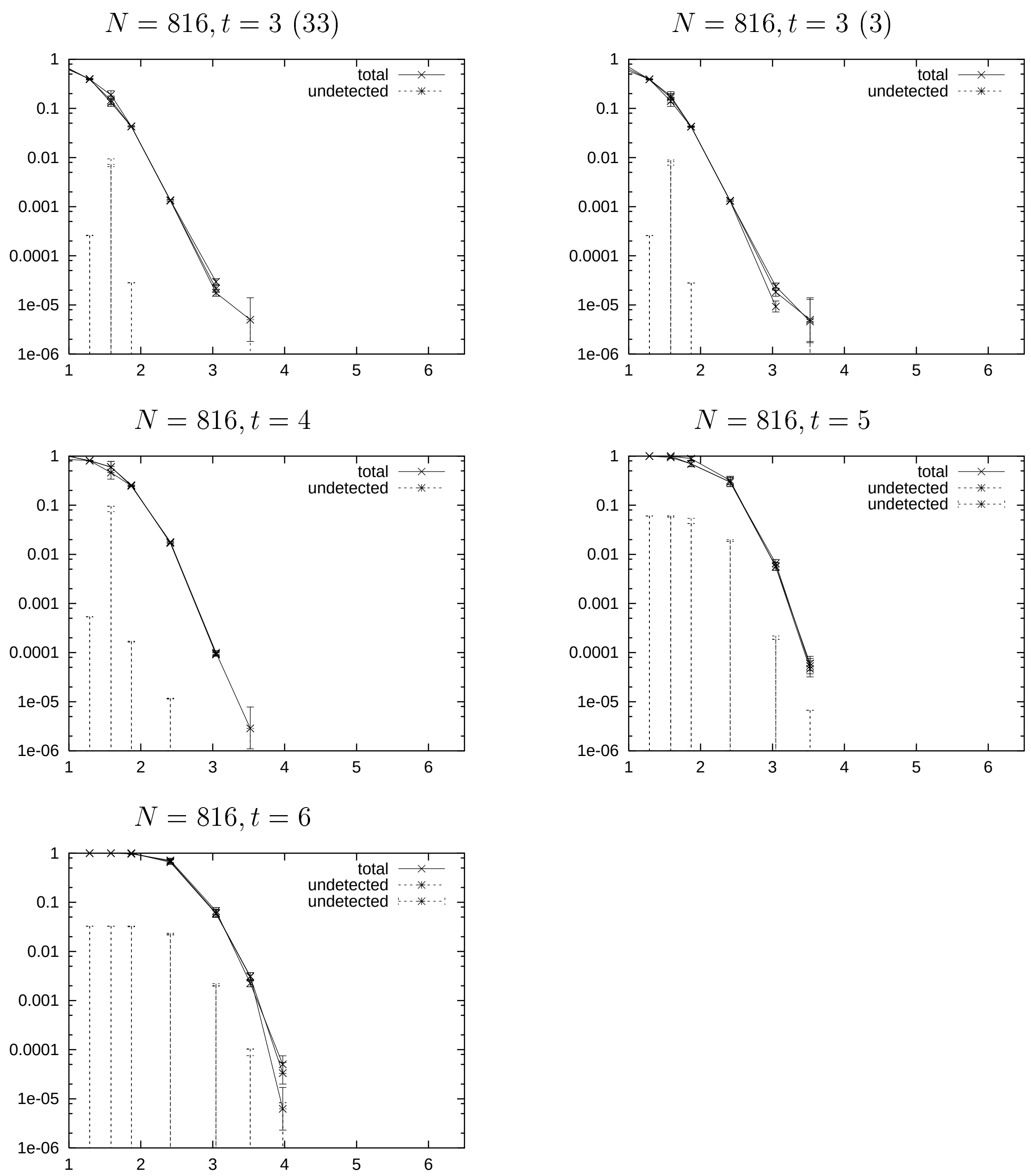

Figure 2: Regular Gallager codes with rate $R = 1/2$. (continued)

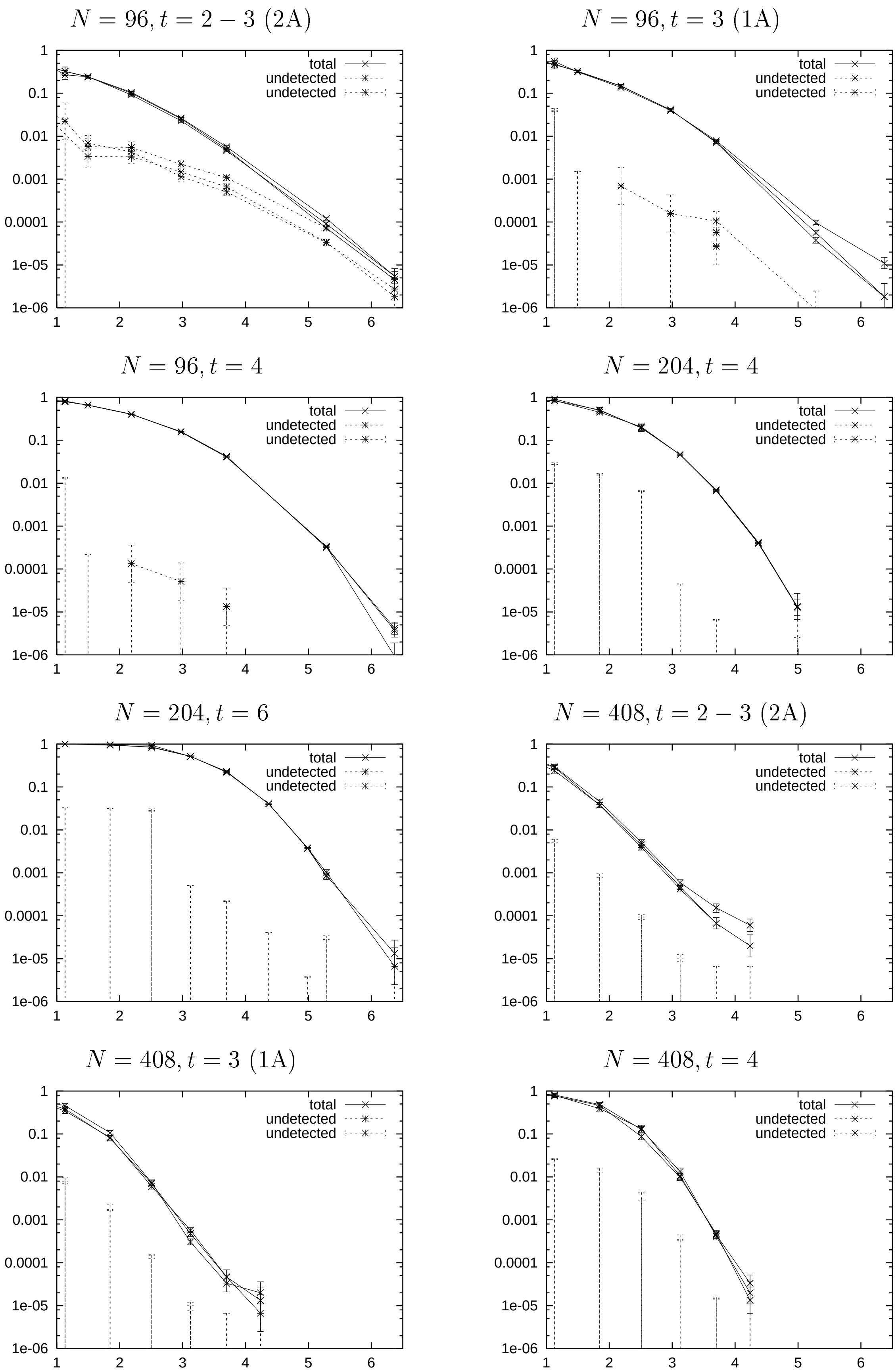

Figure 3: Regular Gallager codes with rate $R = 1/3$. Variability within one construction. Dependence of block error rate on signal to noise ratio, weight per column t and transmitted block length N. Vertical axis: block error rate. Horizontal axis: E_b/N_0

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