Irregular Turbocodes

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Abstract — **We construct irregular turbocodes with systematic bits that participate in varying numbers of trellis sections. By making the original rate 1/2 turbocode of Berrou *et al.* slightly irregular, we obtain a coding gain of 0.15 dB at BER = 10^{-4}.**

I. Irregular Turbocodes

Recently, significant coding gains have been obtained by making the codeword bits of low density parity check codes participate in varying numbers of parity checks (c.f. [1, 2]).

What we call an *irregular turbocode* [3] has the form shown in Fig. 1, which is a type of "trellis-constrained code" [3]. One way to describe the code is by a *degree profile*, $f_d \in [0, 1], d \in \{1, 2, \ldots, D\}$, where f_d is the fraction of codeword bits that have degree d and D is the maximum degree. Each codeword bit with degree d is repeated d times before being permuted and connected to the trellis for a convolutional code. If the bits in the convolutional code are partitioned into "systematic bits" and "parity bits", then by connecting each parity bit to a degree 1 codeword bit, we can encode in linear time by copying, permuting and encoding the systematic bits.

The overall rate R of an irregular turbocode is related to the rate R' of the convolutional code and the average degree $\bar{d}$ by $\bar{d}(1 - R') = 1 - R$. So, if the average degree is increased, the rate of the convolutional code must also be increased (*e.g.*, by puncturing or redesign) to keep the overall rate constant.

II. Decoding irregular turbocodes

Fig. 1 can be interpreted as the graphical model (factor graph, Bayesian network, *etc.*) [4, 5] for the irregular turbocode. Decoding consists of applying the sum-product algorithm (a generalized form of turbodecoding) in this graph.

The decoder first computes the N channel output log-likelihood ratios $L_1^0, \ldots, L_N^0$, and then repeats each log-likelihood ratio appropriately. For bit i with degree d_i, set $L_{i,1} \leftarrow L_i^0, \ldots, L_{i,d} \leftarrow L_i^0$. Next, the log-likelihood ratios are permuted and fed into the BCJR algorithm for the convolutional code, which, for bit i, produces d *a posteriori* log-probability ratios, $L'_{i,1}, \ldots, L'_{i,d}$. The current estimate of the log-probability ratio for bit i is $\hat{L}_i \leftarrow L_i^0 + \sum_{k=1}^{d}(L'_{i,k} - L_{i,k})$. The inputs to the BCJR algorithm for the next iteration, are computed by subtracting off the corresponding outputs from the BCJR algorithm produced by the previous iteration: $L_{i,k} \leftarrow \hat{L}_i - L'_{i,k}$.

III. Discussion

Fig. 2 shows the simulated BER-E_b/N_0 curves for the original regular turbocode and an irregular turbocode that we came up with by making 5% of the codeword bits in the original turbocode have degree 10 The irregular turbocode clearly performs better than the regular turbocode for BER > 10^{-4}.

For high E_b/N_0, most of the errors for the irregular turbocode were due to low-weight codewords. Our permuter was drawn from a uniform distribution over permuters, but we expect the BER "flattening" effect can be significantly reduced by carefully designing the permuter and the convolutional code, possibly by extending the method of "density evolution" to convolutional codes. We are also studying ways of constraining the degree 1 "parity" bits (*i.e.*, increasing their degree) to eliminate low-weight codewords.

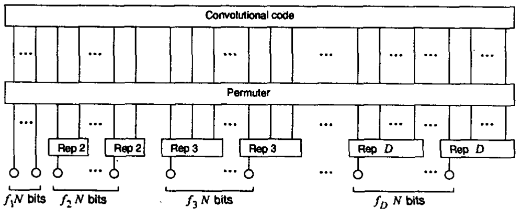

Figure 1: A general *irregular turbocode*. For $d = 1, \ldots, D$, fraction f_d of the codeword bits are repeated d times, permuted and connected to a convolutional code.

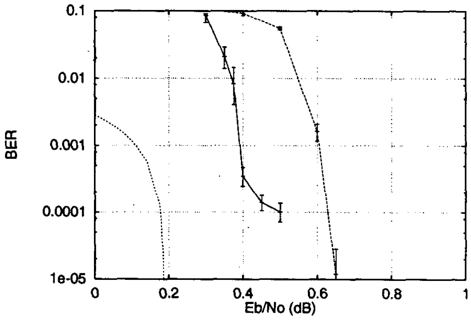

Figure 2: Performances of the original block length $N = 131{,}072$ turbocode (dashed line) and one of its irregular cousins (solid line).

For BER > 10^{-4} this irregular turbocode performs in the same regime as the best known irregular Gallager code [2]. We expect the improvement in performance to be even more significant for lower-rate codes, since the constituent convolutional code can have lower-rate, thus eliminating many low-weight codewords while retaining the benefit of irregularity.

References

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