DIGITAL SIGNAL PROCESSING

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1

Discrete-Time Signals and Systems

1.0 Introduction

A *signal* can be defined as a function that conveys information, generally about the state or behavior of a physical system. Although signals can be represented in many ways, in all cases the information is contained in a pattern of variations of some form. For example, the signal may take the form of a pattern of time variations or a spatially varying pattern. Signals are represented *mathematically* as functions of one or more independent variables. For example, a speech signal would be represented mathematically as a function of time and a picture would be represented as a brightness function of two spatial variables. It is a common convention, and one that will be followed in this book, to refer to the independent variable of the mathematical representation of a signal as time, although it may in fact not represent time.

The independent variable of the mathematical representation of a signal may be either continuous or discrete. *Continuous-time* signals are signals that are defined at a continuum of times and thus are represented by continuous variable functions. *Discrete-time signals* are defined at discrete times and thus the independent variable takes on only discrete values; i.e., discrete-time signals are represented as sequences of numbers. As we will see, signals such as speech or pictures may have either a continuous or a discrete variable representation, and if certain conditions hold, these representations are entirely equivalent.

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In addition to the fact that the independent variables can be either continuous or discrete, the signal amplitude may be either continuous or discrete. *Digital signals* are those for which both time and amplitude are discrete. Continuous-time, continuous-amplitude signals are sometimes called *analog signals*.

In almost every area of science and technology, signals must be processed to facilitate the extraction of information. Thus, the development of signal processing techniques and systems is of great importance. These techniques usually take the form of a transformation of a signal into another signal that is in some sense more desirable than the original. For example, we may wish to design transformations for separating two or more signals that have been combined in some way; we may wish to enhance some component or parameter of a signal; or we may wish to estimate one or more parameters of a signal. Signal processing systems may be classified along the same lines as signals. That is, *continuous-time systems* are systems for which both the input and output are continuous-time signals and *discrete-time systems* are those for which the input and output are discrete-time signals. Similarly *analog systems* are systems for which the input and output are analog signals and *digital systems* are those for which the input and output are digital signals. *Digital signal processing*, then, deals with transformations of signals that are discrete in both amplitude and time. This chapter, and in fact the major part of this book, deals with discrete-time rather than digital signals and systems. The effects of discrete amplitude are considered in detail in Chapter 9.

Discrete-time signals may arise by sampling a continuous-time signal or they may be generated directly by some discrete-time process. Whatever the origin of the discrete-time signals, digital signal processing systems have many attractive features. They can be realized with great flexibility using general-purpose digital computers, or they can be realized with digital hardware. They can, if necessary, be used to simulate analog systems or, more importantly, to realize signal transformations impossible to realize with analog hardware. Thus, digital representations of signals are often desirable when sophisticated signal processing is required.

In this chapter we consider the fundamental concepts of discrete-time signals and signal processing systems first for one-dimensional signals and then for two-dimensional signals. We shall place the most emphasis on the class of linear shift-invariant discrete-time systems. It will be true in this chapter and succeeding ones that many of the properties and results that we derive will be similar to properties and results for linear time-invariant continuous-time systems as presented in a variety of excellent texts [1–3]. In fact, it is possible to approach the discussion of discrete-time systems by treating sequences as analog signals that are impulse trains. This approach, if implemented carefully, can lead to correct results and in fact forms the basis for much of the classical discussion of sampled data systems (see, for example, [4–6]). In many present digital signal processing applications, however, not all sequences arise from sampling a continuous-time signal. Furthermore, many discrete-time systems are not simply approximations to corresponding analog systems. Therefore, rather than attempt to force results from analog system theory into a discrete framework, we shall derive similar results starting within a framework and with notation suitable to discrete-time systems. Discrete-time signals will be related to analog signals only when necessary.

1.1 Discrete-Time Signals—Sequences

In discrete-time system theory, we are concerned with processing signals that are represented by sequences. A sequence of numbers x, in which the nth number in the sequence is denoted $x(n)$, is formally written as

$$x = \{x(n)\}, \qquad -\infty < n < \infty \tag{1.1}$$

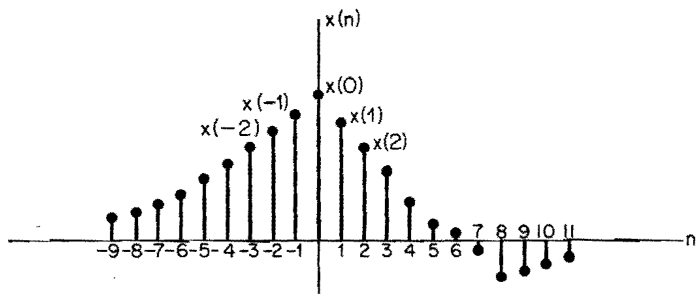

Fig. 1.1 Graphical representation of a discrete-time signal.

Although sequences do not always arise from sampling analog waveforms, for convenience we shall refer to $x(n)$ as the "nth sample" of the sequence. Also, although strictly speaking $x(n)$ denotes the nth number in the sequence, the notation of Eq. (1.1) is often unnecessarily cumbersome, and it is convenient and unambiguous to refer to "the sequence $x(n)$." Discrete-time signals (i.e., sequences) are often depicted graphically as shown in Fig. 1.1. Although the abscissa is drawn as a continuous line, it is important to recognize that $x(n)$ is only defined for integer values of n. It is *not* correct to think of $x(n)$ as being zero for n not an integer; $x(n)$ is simply undefined for non-integer values of n.

Some examples of sequences are shown in Fig. 1.2. The *unit-sample sequence*, $\delta(n)$, is defined as the sequence with values

$$\delta(n) = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

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