Design of fan-out kinoforms in the entire scalar diffraction regime with an optimal-rotation-angle method

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An algorithm for the design of diffractive optical phase elements (kinoforms) that give rise to fan-out (i.e., spot) patterns was developed and tested. The algorithm is based on the Helmholtz–Kirchhoff rigorous scalar diffraction integral for the evaluation of the electric field behind the kinoform. The optimization of the kinoform phase modulation is performed with an efficient optimal-rotation-angle method. The algorithm permits any spatial configuration of the locations of the desired spots. For example, the spots (all or some) can be located at large angles to the optical axis (nonparaxial case) or they can be located in the near near field of the kinoform, i.e., where the Fresnel approximation is no longer valid. Two examples of fabricated kinoforms designed with this algorithm are presented. © 1997 Optical Society of America

Key words: Diffractive optics design, kinoforms, fan-out diffraction pattern, rigorous scalar diffraction, optimal rotation angle.

1. Introduction

A kinoform (or diffractive optical element) is a thin component with a depth relief on one surface. The relief modulates the phase of a laser beam that passes through the kinoform. The kinoform relief can be designed so that the distribution of light behind the kinoform closely resembles some desired one.

There are two principal types of desired light distributions. One is the even distribution of light over some area in a plane behind the kinoform. This is often called beam shaping after its most common application.[1–3] At least as important is the distribution of light to small (diffraction-limited) spots in space. This is sometimes called beam splitting, as it is often used for dividing the energy of a laser beam into many beams, or, to put it differently, for sending a signal from one channel (optical fiber or semiconductor laser emitter) into many. This paper treats the design of kinoforms performing this latter task. They are often called fan-out kinoforms.

The design algorithm described in this paper is based on scalar diffraction theory. For this theory to be valid, the feature size of the kinoform must not be too small. It is still debated as to what "too small" should mean.[4–6] Fan-out diffractive elements designed with scalar diffraction theory with a smallest feature size of only approximately two wavelengths have been fabricated, and they work fairly well.[7] In the scalar diffraction regime, several methods for designing kinoforms exist. The methods are often used when the diffraction pattern is in the Fresnel or Fraunhofer region of the kinoform. Then the field in the diffraction plane is given as the Fourier transform of the product of the field in the kinoform plane and the Fresnel (near-field) phase factor. Since the Fourier transform directly relates a spatial field distribution (in the kinoform plane) to another (in the diffraction plane), this will be referred to as a spatial Fourier transform. (This relation is as opposed to a different way of evaluating the diffracted field, the so-called propagation of the angular spectrum, which also utilizes Fourier transforms but in which the transforms relate fields to angular spectra.)

The Fourier transform is often evaluated with the efficient fast Fourier transform (FFT) algorithm. The so-called direct-search methods, like direct binary search and simulated annealing,[8–11] are generally demanding in terms of computer capacity. Less demanding are the iterative Fourier transform algorithms (IFTA's).[12–14] One iterates with the FFT between the kinoform plane and the diffraction plane. In both planes manipulations of the phase, ampli-

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tude, or both, of the fields are performed to force the algorithm to give a good solution.

There are also methods based on the spatial Fourier transform that use ways of optimizing the phase other than that of the IFTA's.[15,16] The use of the efficient FFT algorithm makes the IFTA's the fastest of the design methods. But they also have serious disadvantages: First, they cannot readily be used to calculate a kinoform whose spot pattern is three dimensional; instead they require that all spots be confined to a single plane at a specified distance behind the kinoform. Second, the position of the spots in this plane (the diffraction plane) cannot be chosen fully arbitrarily. This is because the FFT works with matrices, or grids, and the grid spacing in the diffraction plane is given after the grid size in the kinoform plane is specified. A spot in the diffraction plane will then always be in a grid position, and the distance along the axes of the grid between two spots will always be an integer times the grid size. Third, spatial Fourier transform methods require that the Fresnel approximation (of which the Fraunhofer approximation is a special case) be valid. This approximation is not valid if light is diffracted at large angles to the optical axis (nonparaxial case), nor is it valid if the desired spots are to be produced in the near near field of the kinoform (i.e., so close behind the kinoform that the Fresnel phase factor is no longer appropriate).

The design method described in this paper has none of these disadvantages. It can be used for a general configuration of spots. The accuracy in the evaluation of the electric field is in principle the same as that for the method of angular-spectrum propagation since, in both cases, the full scalar wave equation is used. The accuracy should be good as long as scalar theory can be used. Thus it cannot be used for extremely nonparaxial situations in which the longitudinal component of the field cannot be neglected.

The use of an optimal-rotation-angle (ORA) optimization, a technique used earlier for a spatial Fourier transform method,[17] also makes the method reasonably efficient. It has been tested successfully on an ordinary workstation for a kinoform with 250,000 elements, each allowed to impose a continuous phase modulation in the interval $[0, 2\pi]$ and with 60 spots in the diffraction pattern. Of course, it still requires more computer time than do the IFTA's. The algorithm gives the phase modulation in the kinoform plane as output. Normally, this can be interpreted directly as the relief depth; the depth and phase modulation of a pixel are simply proportional to each other. Only when the light is obliquely incident at a large angle does the relation between phase modulation and relief depth get more complicated. However, for these extreme cases the validity of the scalar theory itself is often questionable.

The design method in this paper has two major parts. In the first part, described in Section 2, the contribution to the electric field at some point in space (which will be a point where a spot should be) from a small portion of the kinoform (called a pixel) is

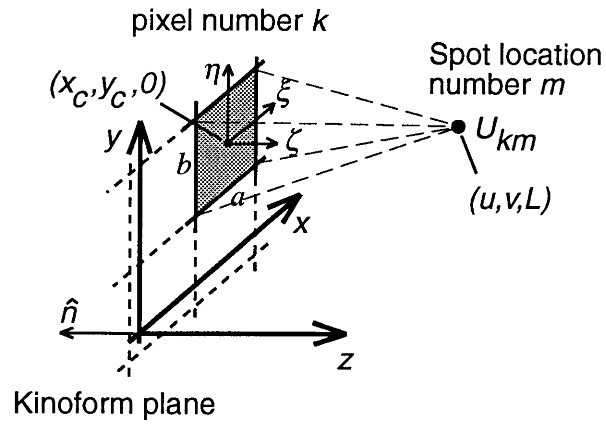

Fig. 1. Kinoform plane with pixel k indicated and one of the locations m in space.

determined. The Helmholtz–Kirchhoff integral and a minimum of approximations are used to make the calculations as accurate as possible but still efficient. The contribution to the electric field from a pixel depends on, for example, the obliquity of the incident field, the distance between the pixel and the point in space, and the amount of phase modulation of the pixel. The amount of phase modulation is the variable that is to be optimized. This optimization is the second major part of the method and is described in Section 3. The optimization is performed with an ORA method. With this method, the amount of phase modulation of the pixel is chosen so that the total field amplitude in the desired spot locations in space is maximized under the constraint that the relative intensities in the spots are the desired ones.

2. Contribution from One Pixel to the Field at a Spot Location

The kinoform is made up of small, rectangular segments called pixels. In this section we find the contribution to the electric field at some point m in space from pixel number k (see Fig. 1). This contribution is called U_{km}, and we want to write it as

$$U_{km} = A_{km} \exp(j\varphi_{km})\exp[j(\varphi_{\text{inc}} + \varphi_k)], \qquad (1)$$

where φ_{inc} is the known phase value of the field in the center of the pixel without phase modulation, i.e., as if the kinoform had no surface relief. The value of φ_{inc} depends on only the incident wave; for a plane wave it can be set to zero for all pixels. The term φ_k is the amount of phase modulation imposed by the surface relief. It is the variable that is optimized in the design algorithm. The phase modulation φ_k is assumed to be constant in the whole pixel, i.e., we assume that every pixel is a plateau at a constant depth. It should also be a good approximation for a continuous relief, provided the depth does not vary too much within one pixel.

We must now find the complex transfer function $A_{km} \exp(j\varphi_{km})$ from pixel k to spot m. In Goodman's book,[18] the integral theorem developed by Helmholtz and Kirchhoff with the Kirchhoff boundary conditions is stated as

$$U_{km} = \frac{1}{4\pi} \int\int_{\text{pixel } k} \frac{\partial U_k}{\partial n} G - U_k \frac{\partial G}{\partial n} \, dx dy, \qquad (2)$$

where G is the scalar free-space Green's function (or, to put it more simply, the scalar field from a point source) from point m to a position in the pixel:

$$G(x, y, z, u, v, L)|_{z=0} = \frac{\exp(jkr_{01})}{r_{01}}, \qquad (3)$$

where r_{01} is the distance from point m with coordinates (u, v, L) to a position in the pixel with coordinates (x, y, z):

$$r_{01} = [(u-x)^2 + (v-y)^2 + (L-z)^2]^{1/2}|_{z=0}, \qquad (4)$$

and k is the magnitude of the free-space wave vector (the k-vector)

$$k = \frac{2\pi}{\lambda_0}. \qquad (5)$$

Further, U_k is the complex electric field in pixel k. We assume that this field within each pixel can be approximated with a plane wave with a constant amplitude A_k and a direction of propagation (i.e., the direction of the k-vector) that is the direction of propagation in the center of that pixel. If we use coordinates ξ, η, and ζ with the origin in the pixel center (*cf.* Fig. 1) and also use our definition of φ_{inc} as the value of the phase in the pixel center without modulation (i.e., if $\varphi_k = 0$), then we have

$$U_k(\xi, \eta, \zeta)|_{\zeta=0} = A_k \exp[j(\varphi_{\text{inc}} + \varphi_k)] \times \exp[j(k_x\xi + k_y\eta + k_z\zeta)]|_{\zeta=0}. \qquad (6)$$

We start by calculating the normal derivative in the kinoform plane of U_k in Eq. (6). From the construction of Eq. (2) from the Green's theorems the normal vector of the kinoform plane points in the negative z direction (or ζ direction), as indicated in Fig. 1. Then

$$\frac{\partial U_k}{\partial n} = -\frac{\partial U_k}{\partial \zeta} = -jk_z U_k. \qquad (7)$$

Before taking the normal derivative of G, let us calculate the components of the k-vector for the plane wave in the pixel (*cf.* Fig. 2) expressed in terms of the radius of the outgoing wave R, in the absence of a phase relief, and the pixel-center positions x_c and y_c. From Fig. 2 it can be seen that

$$\frac{\bar{k}}{k} = \frac{\bar{R}_k}{|\bar{R}_k|} = \frac{x_c\hat{x} + y_c\hat{y} + R\hat{z}}{(x_c^2 + y_c^2 + R^2)^{1/2}}. \qquad (8)$$

But the k-vector components are defined as

$$\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}, \qquad (9)$$

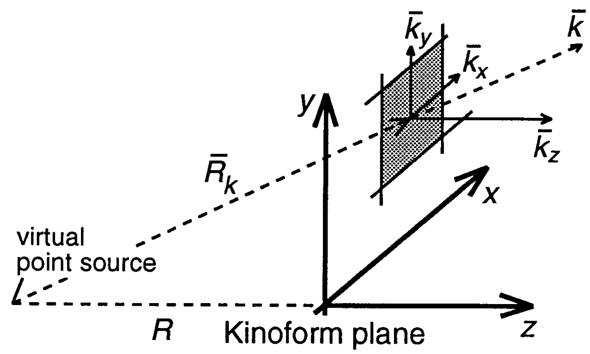

Fig. 2. Construction of the components of the k-vector in the center of pixel k.

and so the components k_x, k_y, and k_z can be identified directly from Eq. (8). If R happens to be negative, i.e., if the incident wave converges, all components should be *minus* the value obtained with Eqs. (8) and (9).

Now we calculate the normal derivative of G in the kinoform plane:

$$\frac{\partial G}{\partial n} = -\frac{\partial G}{\partial z} = \frac{L}{r_{01}} \left\{ jk - \frac{1}{r_{01}} \right\} G, \qquad (10)$$

and so we have, from Eq. (2),

$$U_{km} = \frac{1}{4\pi} \int\int_{\text{pixel } k} -jk_z U_k G - U_k \frac{L}{r_{01}} \left\{ jk - \frac{1}{r_{01}} \right\} G \, d\xi d\eta$$
$$\approx \frac{1}{4\pi} \left(-jk_z - \frac{L}{r_{01}^{\,c}} \left\{ jk - \frac{1}{r_{01}^{\,c}} \right\} \right) \int\int_{\text{pixel } k} U_k G \, d\xi d\eta. \qquad (11)$$

In expression (11) we approximated the two explicitly written r_{01} values by their value in the pixel center:

$$r_{01}^{\,c} = [(u - x_c)^2 + (v - y_c)^2 + L^2]^{1/2}. \qquad (12)$$

We will be a little more careful with the r_{01} value in the phase factor of G.

The integral in expression (11) is evaluated by the insertion of Eqs. (6) and (3):

$$\int\int_{\text{pixel } k} U_k G \, d\xi d\eta = \frac{A_k \exp[j(\varphi_{\text{inc}} + \varphi_k)]}{r_{01}^{\,c}} \times \int\int_{\text{pixel } k} \exp[j(k_x\xi + k_y\eta)] \times \exp(jkr_{01}) \, d\xi d\eta, \qquad (13)$$

where r_{01} in the denominator of G was approximated by its value in the pixel center. To obtain a simple analytical solution of the integral, we expand the remaining r_{01} in a Maclaurin series about the pixel-center coordinates:

$$r_{01} = \{(u - [x_c + \xi])^2 + (v - [y_c + \eta])^2 + L^2\}^{1/2}$$
$$\approx r_{01}^{\ c} + \frac{x_c\xi - u\xi + y_c\eta - v\eta}{r_{01}^{\ c}}, \qquad (14)$$

where only the terms linear in ξ and η are retained.

The integral is now solved readily. With the substitutions

$$\tilde{k}_x = k_x + \frac{k(x_c - u)}{r_{01}^{\ c}},$$
$$\tilde{k}_y = k_y + \frac{k(y_c - v)}{r_{01}^{\ c}}, \qquad (15)$$

the integral becomes

$$\int\int_{\text{pixel } k} U_k G \mathrm{d}\xi \mathrm{d}\eta = \frac{4A_k \exp[j(\varphi_{\text{inc}} + \varphi_k)]}{r_{01}^{\ c}} \times \exp(jkr_{01}^{\ c}) \frac{\sin\left(\tilde{k}_x \frac{a}{2}\right)}{\tilde{k}_x} \frac{\sin\left(\tilde{k}_y \frac{b}{2}\right)}{\tilde{k}_y}. \qquad (16)$$

Finally, we insert the factor from expression (11) before the integral to obtain the expression for the contribution from pixel k to the field in point m, U_{km}. The transfer function in Eq. (1) has now been found:

$$A_{km} \exp(j\varphi_{km}) = \frac{1}{4\pi}\left(-jk_z - \frac{L}{r_{01}^{\ c}}\left\{jk - \frac{1}{r_{01}^{\ c}}\right\}\right) \times \frac{4A_k}{r_{01}^{\ c}} \exp(jkr_{01}^{\ c}) \frac{\sin\left(\tilde{k}_x \frac{a}{2}\right)}{\tilde{k}_x} \frac{\sin\left(\tilde{k}_y \frac{b}{2}\right)}{\tilde{k}_y}, \qquad (17)$$

where A_{km} and φ_{km} are the absolute value and the argument, respectively, of this complex number. They do not change with the modulation φ_k and can thus be calculated for all (k, m) at the start of the design algorithm and stored in the computer memory.

One might worry about the approximations we have made to arrive at the desired result, Eq. (17). However, the small size of a pixel, typically a factor of a hundred (length scale) or more smaller than the whole kinoform, makes these approximations perfectly justified for virtually any situation; if they are not, one can simply increase the number of pixels.

3. Optimization of the Pixel Phase Modulation

The optimization of the phase modulation of a pixel is performed with an ORA[17] method. It gives an analytical expression for how much the phase modulation of a pixel should change, $\Delta\varphi_k$. One pixel at a time is optimized. The information needed to optimize a pixel is the current value of the phase modulation φ_k, the values A_{km} and φ_{km}, representing the transfer function between that pixel and all the spot locations, and the phase of the total field φ_m from all pixels in these spots. The influence of the other pixels enters through only the values of φ_m. In practice, to save time all the pixels in the kinoform are optimized before calculation of the fields in the spots, and thus φ_m, again. Then a new optimization of all the pixels is performed. The procedure is repeated until the kinoform performance is the desired one. The ORA method is described in the following.

Figure 3 shows the complex-number plane where the complex amplitude U_m (absolute value and phase) of one spot is shown. Also shown is the contribution to U_m from pixel k, U_{km}. It can be seen that, if the phase modulation of that pixel is changed by $\Delta\varphi_k$, thus rotating the contribution by the same angle, the length of the U_m vector (i.e., the absolute value of the field at point m) is changed by Δl. With ϕ_{km} defined as

$$\phi_{km} = \varphi_m - \{\varphi_{km} + \varphi_{\text{inc}} + \varphi_k\}, \qquad (18)$$

we can see from Fig. 3 that

$$\Delta l = A_{km} \cos(\phi_{km} - \Delta\varphi_k) - A_{km} \cos \phi_{km}. \qquad (19)$$

For the particular choice of $\Delta\varphi_k$ shown in Fig. 3, Δl is positive, which means that the light intensity increases in spot m. What about in the other spot locations? Will the intensity increase there as well? No, not in general, of course. However, there does exist a value of $\Delta\varphi_k$, the ORA, for which the *sum* of the changes Δl for all spot locations is maximized. Changing the phase modulation of pixel k by the ORA thus increases the kinoform efficiency.

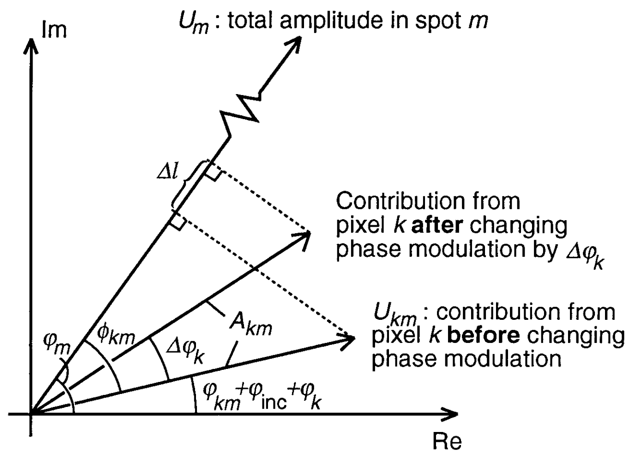

Fig. 3. Complex-number plane showing the amplitude change Δl of the field in spot m from changing the phase modulation of pixel k by $\Delta\varphi_k$. Re, real; Im, imaginary.

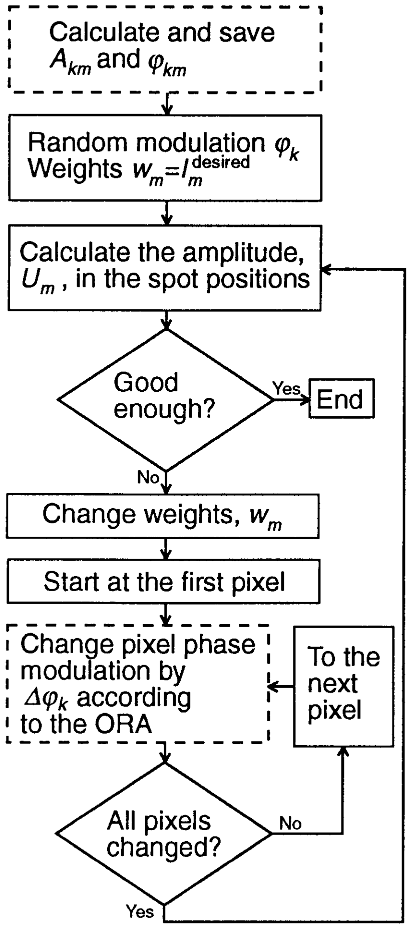

Fig. 4. Flowchart of the complete design algorithm.

To find the ORA we calculate the sum of the Δl values for all spots for a given pixel k:

$$\sum_m \Delta l = \sum_m [A_{km} \cos(\phi_{km} - \Delta\varphi_k) - A_{km} \cos \phi_{km}]$$
$$= S_1 \cos \Delta\varphi_k + S_2 \sin \Delta\varphi_k - S_1$$
$$= \begin{cases} S_3 \cos(\Delta\varphi_k - \alpha_k) - S_1 & \text{if } S_1 > 0 \\ -S_3 \cos(\Delta\varphi_k - \alpha_k) - S_1 & \text{if } S_1 < 0' \end{cases} \quad (20)$$

where

$$S_1 = \sum_m A_{km} \cos \phi_{km},$$
$$S_2 = \sum_m A_{km} \sin \phi_{km},$$
$$S_3 = ({S_1}^2 + {S_2}^2)^{1/2},$$
$$\alpha_k = \arctan\left(\frac{S_2}{S_1}\right). \quad (21)$$

It is directly seen from Eq. (20) that this sum is maximized if the phase modulation is changed by

$$\begin{aligned} \Delta\varphi_k &= \alpha_k, && \text{if} \quad S_1 > 0, \\ \Delta\varphi_k &= \alpha_k + \pi, && \text{if} \quad S_1 < 0, \\ \Delta\varphi_k &= \pi/2, && \text{if} \quad S_1 = 0 \text{ and } S_2 > 0, \\ \Delta\varphi_k &= -\pi/2, && \text{if} \quad S_1 = 0 \text{ and } S_2 < 0. \end{aligned} \quad (22)$$

The phase modulation φ_k is now changed by this value of $\Delta\varphi_k$, called the ORA. One then proceeds to the next pixel and performs a similar optimization.

To obtain exactly the desired intensity in each spot, we use modified forms of the sums S_1 and S_2:

$$S_1 = \sum_m w_m A_{km} \cos \phi_{km},$$
$$S_2 = \sum_m w_m A_{km} \sin \phi_{km}. \quad (23)$$

The only difference between Eqs. (23) and the sums in Eqs. (21) is the introduction of the real, positive numbers w_m that are weights for each of the spot locations. If the intensity in spot m is found to be too low, w_m is increased, and so the corresponding term becomes more important in the sum and vice versa if the intensity is too high. The weights are changed according to

$$w_m{}^{\text{new}} = w_m{}^{\text{old}} \left(\frac{I_m{}^{\text{desired}}}{I_m}\right)^{0.35}. \quad (24)$$

Here I_m is the intensity (the square of the absolute value of the field) in spot location m. The exact value of the exponent, here 0.35, is not important but should be low enough to avoid unstable behavior.

The procedure described above assumes that the phase modulation is continuous, i.e., φ_k can take any value in the interval $[0, 2\pi]$. It is not difficult, however, to modify the method for any kind of phase restriction. For instance, if the phase modulation is quantized, one simply changes φ_k by that integer multiple of the phase-quantization step that is closest to the value of $\Delta\varphi_k$ obtained with the ORA method.

A flowchart describing the whole algorithm is shown in Fig. 4. In the boxes with a dashed border are the two main procedures described in this section and in Section 2. The algorithm starts with the calculation of the transfer functions (A_{km} and φ_{km}) from all pixels to the desired spot locations. If the num-

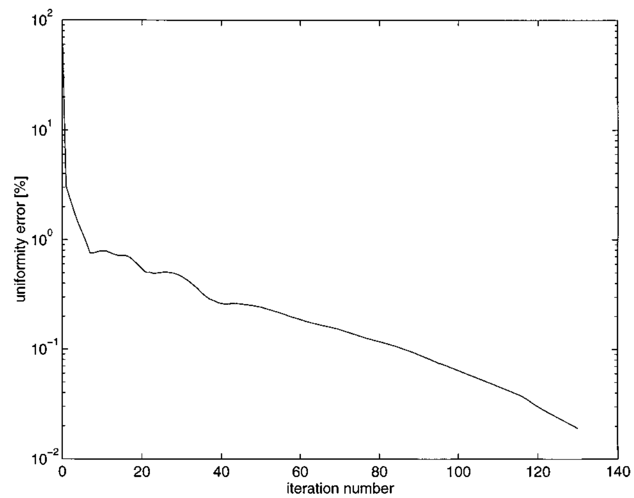

Fig. 5. Uniformity error versus the number of iterations for the design of the near near-field kinoform.

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