Principles of Optics

Electromagnetic Theory of Propagation, Interference and Diffraction of Light

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TABLE XXVI

The "penetration depth" d for copper for radiation in three familiar regions of the spectrum, calculated with the static conductivity $\sigma \sim 5{\cdot}14 \,.\, 10^{17}\ \text{sec}^{-1}$ *and* $\mu = 1$.

Radiation	Infra-red	Microwaves	Long radio waves
λ_0	10^{-3} cm	10 cm	1000 m $= 10^5$ cm
d	$6{\cdot}1 \,.\, 10^{-7}$ cm	$6{\cdot}1 \,.\, 10^{-5}$ cm	$6{\cdot}1 \,.\, 10^{-3}$ cm

A perfect conductor is characterized by infinitely large conductivity ($\sigma \to \infty$). Since according to (16), $\varepsilon/\sigma = (1 - \kappa^2)/\nu\kappa$, we have in this limiting case $\kappa^2 \to 1$, or by (16a), $n \to \infty$. Such a conductor would not permit the penetration of an electromagnetic wave to any depth at all and would reflect all the incident light (cf. § 13.2 below).

Whilst the refractive index of transparent substances may easily be measured from the angle of refraction, such measurements are extremely difficult to carry out for metals, because a specimen of the metal which transmits any appreciable fraction of incident light has to be exceedingly thin. Nevertheless KUNDT* succeeded in constructing metal prisms that enabled direct measurements of the real and imaginary parts of the complex refractive index to be made. Usually, however, the optical constants of metals are determined by means of katoptric rather than dioptric experiments, i.e. by studying the changes which light undergoes on reflection from a metal, rather than by means of measurements on the light transmitted through it.

13.2. REFRACTION AND REFLECTION AT A METAL SURFACE

We have seen that the basic equations relating to the propagation of a plane time-harmonic wave in a conducting medium differ from those relating to propagation in a transparent dielectric only in that the real constants ε and k are replaced by complex constants $\hat{\varepsilon}$ and $\hat{k}$. It follows that the formulae derived in Chapter I, as far as they involve only linear relations between the components of the field vectors of plane monochromatic waves, apply also in the present case. In particular, the boundary conditions for the propagation of a wave across a surface of discontinuity and hence also the formulae of § 1.5 relating to refraction and reflection remain valid.

Consider first the propagation of a plane wave from a dielectric into a conductor, both media being assumed to be of infinite extent, the surface of contact between them being the plane $z = 0$. By analogy with § 1.5 (8) the law of refraction is

$$\sin \theta_t = \frac{1}{\hat{n}} \sin \theta_i. \tag{1}$$

Since $\hat{n}$ is complex, so is θ_t, and this quantity therefore no longer has the simple significance of an angle of refraction.

Let the plane of incidence be the xz-plane. The space-dependent part of the phase of the wave in the conductor is given by $\hat{k}(\mathbf{r}.\mathbf{s}^{(t)})$ where (cf. § 1.5 (4))

$$s_x^{(t)} = \sin \theta_t, \qquad s_y^{(t)} = 0, \qquad s_z^{(t)} = \cos \theta_t. \tag{2}$$

* A. KUNDT, *Ann. d. Physik*, **34** (1888). 469.

From (1) and (2) and § 13.1 (15)

$$s_x^{(t)} = \sin\theta_t = \frac{\sin\theta_i}{n(1+i\kappa)} = \frac{1-i\kappa}{n(1+\kappa^2)}\sin\theta_i, \tag{3a}$$

$$s_z^{(t)} = \cos\theta_t = \sqrt{1-\sin^2\theta_t}$$
$$= \sqrt{1 - \frac{(1-\kappa^2)}{n^2(1+\kappa^2)^2}\sin^2\theta_i + i\frac{2\kappa}{n^2(1+\kappa^2)^2}\sin^2\theta_i}. \tag{3b}$$

It is convenient to express $s_z^{(t)}$ in the form

$$s_z^{(t)} = \cos\theta_t = qe^{i\gamma} \tag{4}$$

(q, γ real). Expressions for q and γ in terms of n, κ and $\sin\theta_i$ are immediately obtained on squaring (3b) and (4) and equating real and imaginary parts. This gives

$$\left.\begin{aligned} q^2\cos 2\gamma &= 1 - \frac{1-\kappa^2}{n^2(1+\kappa^2)^2}\sin^2\theta_i,\\ q^2\sin 2\gamma &= \frac{2\kappa}{n^2(1+\kappa^2)^2}\sin^2\theta_i. \end{aligned}\right\} \tag{5}$$

It follows that

$$\begin{aligned} \hat{k}(\mathbf{r}\,.\,\mathbf{s}^{(t)}) &= \frac{\omega}{c}n(1+i\kappa)(xs_x^{(t)} + zs_z^{(t)})\\ &= \frac{\omega}{c}n(1+i\kappa)\left[\frac{x(1-i\kappa)}{n(1+\kappa^2)}\sin\theta_i + z(q\cos\gamma + iq\sin\gamma)\right]\\ &= \frac{\omega}{c}[x\sin\theta_i + znq(\cos\gamma - \kappa\sin\gamma) + inzq(\kappa\cos\gamma + \sin\gamma)]. \end{aligned} \tag{6}$$

We see that the surfaces of constant amplitude are given by

$$z = \text{constant}, \tag{7}$$

and are, therefore, planes parallel to the boundary. The surfaces of constant real phase are given by

$$x\sin\theta_i + znq(\cos\gamma - \kappa\sin\gamma) = \text{constant}, \tag{8}$$

and are planes whose normals make an angle θ_t' with the normal to the boundary, where

$$\left.\begin{aligned} \cos\theta_t' &= \frac{nq(\cos\gamma - \kappa\sin\gamma)}{\sqrt{\sin^2\theta_i + n^2q^2(\cos\gamma-\kappa\sin\gamma)^2}},\\ \sin\theta_t' &= \frac{\sin\theta_i}{\sqrt{\sin^2\theta_i + n^2q^2(\cos\gamma-\kappa\sin\gamma)^2}}. \end{aligned}\right\} \tag{9}$$

Since the surfaces of constant amplitude and the surfaces of constant phase do not in general coincide with each other, the wave in the metal is an *inhomogeneous wave*.

If we denote the square root in (9) by n', the equation for $\sin\theta_t'$ may be written in the form $\sin\theta' = \sin\theta_i/n'$, i.e. it has the form of SNELL's law. However, n' depends now not only on the quantities that specify the medium, but also on the angle of incidence θ_i.

We may also derive expressions for the amplitude and the phase of the refracted and reflected waves by substituting for θ_t the complex value given by (1) in the FRESNEL formulae (§ 1.5.2). The explicit expressions will be given in § 13.4.1 in connection with the theory of stratified conducting media. Here we shall consider how the optical constants of the metal may be deduced from observation of the reflected wave.

Since we assumed that the first medium is a dielectric, the reflected wave is an ordinary (homogeneous) wave with a real phase factor. As in § 1.5 (21a) the amplitude components $A_\parallel$, $A_\perp$ of the incident wave and the corresponding components $R_\parallel$, $R_\perp$ of the reflected wave are related by

$$\left.\begin{aligned} R_\parallel &= \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} A_\parallel, \\ R_\perp &= -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} A_\perp. \end{aligned}\right\} \tag{10}$$

Since θ_t is now complex, so are the ratios $R_\parallel/A_\parallel$ and $R_\perp/A_\perp$, i.e. characteristic phase changes occur on reflection; thus incident linearly polarized light will in general become elliptically polarized on reflection at the metal surface. Let $\phi_\parallel$ and $\phi_\perp$ be the phase changes, and $\rho_\parallel$ and $\rho_\perp$ the absolute values of the reflection coefficients, i.e.

$$r_\parallel = \frac{R_\parallel}{A_\parallel} = \rho_\parallel e^{i\phi_\parallel}, \qquad r_\perp = \frac{R_\perp}{A_\perp} = \rho_\perp e^{i\phi_\perp}. \tag{11}$$

Suppose that the incident light is *linearly polarized* in the azimuth α_i i.e.

$$\tan \alpha_i = \frac{A_\perp}{A_\parallel}, \tag{12}$$

and let α_r be the azimuthal angle (generally complex) of the light that is reflected. Then*

$$\tan \alpha_r = \frac{R_\perp}{R_\parallel} = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_t)} \tan \alpha_i = Pe^{-i\Delta} \tan \alpha_i, \tag{13}$$

where

$$P = \frac{\rho_\perp}{\rho_\parallel}, \qquad \Delta = \phi_\parallel - \phi_\perp. \tag{14}$$

We note that α_r is real in the following two cases:

(1) For normal incidence ($\theta_i = 0$); then $P = 1$ and $\Delta = -\pi$, so that $\tan \alpha_r = -\tan \alpha_i$.

(2) For grazing incidence ($\theta_i = \pi/2$); then $P = 1$ and $\Delta = 0$, so that $\tan \alpha_r = \tan \alpha_i$.

It should be remembered that in the case of normal incidence the directions of the incident and reflected rays are opposed; thus the negative sign implies that the

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* We write $-i\Delta$ rather than $+i\Delta$ in the exponent on the right-hand side of (13) to facilitate comparison with certain results of § 1.5.

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