Spreading Sequences for Uplink and Downlink MC–CDMA Systems: PAPR and MAI Minimization

STÉPHANE NOBILET, JEAN-FRANÇOIS HÉLARD
LCST/INSA, 20 Av. des Buttes de Coësmes, 35043 Rennes Cedex, France
{stephane.nobilet, jean-francois.helard}@insa-rennes.fr

DAVID MOTTIER
Mitsubishi Electric ITE, 80 Av. des Buttes de Coësmes, 35700 Rennes, France
mottier@tcl.ite.mee.com

Abstract. This paper deals with spreading sequences selection for downlink and uplink Multi-Carrier Code Division Multiple Access (MC-CDMA) systems with the aim of minimizing the dynamic range of the transmitted multicarrier signal envelope and the multiple access interference. The crest factor of orthogonal and non-orthogonal sequences are compared analytically and by simulation for downlink and uplink phase shift keying MC-CDMA transmissions. Then, in order to minimize the multiple access interference produced by frequency selective channels, an optimized spreading sequence allocation procedure is presented. Finally, a selection of the spreading codes which jointly reduces the multiple access interference and the crest factor is proposed for downlink MC-CDMA systems.

1 INTRODUCTION

In recent years, Multi-Carrier Code Division Multiple Access (MC-CDMA) has been receiving widespread interests for wireless broadband multimedia applications. Combining Orthogonal Frequency Division Multiplex (OFDM) modulation and CDMA, this scheme benefits from the main advantages of both techniques [1]: high spectral efficiency, multiple access capability, robustness in case of frequency selective channels, high flexibility, narrow-band interference rejection, simple one-tap equalization, etc. In general, to reduce the Multiple Access Interference (MAI) in a synchronous system like the downlink mobile radio communication channel, the spreading sequences or codes, are chosen orthogonal. Besides, spreading sequences have to be selected in order to limit the dynamic range of the OFDM transmitted signal envelope, and therefore to mitigate the nonlinear distortions introduced by the high power amplifier.

This paper deals with the selection of spreading sequences for the downlink and uplink of high rate cellular networks with the aim of jointly minimizing the MAI and the nonlinear distortions. The peak-to-average power ratio and the crest factor are used for the evaluation of the dynamic range of the transmitted Phase Shift Keying (PSK) modulated multicarrier signal envelope for various orthogonal and non-orthogonal spreading codes. Furthermore, in order to minimize the MAI, an optimized allocation procedure of the spreading sequences is described. Finally, a selection of the spreading codes, which jointly reduces the MAI and the non-linear distortions, is proposed.

The paper is organized as follows. In Section 2, the considered MC-CDMA system is briefly described. Section 3 presents the studied spreading sequences and the different selection criteria. In Section 4, crest factor analytical results for uplink and downlink contexts are developed. Section 5 presents simulation results on crest factors and performance evaluation in terms of bit error rate for a simulation environment similar to the ETSI BRAN HIPERLAN/2 physical layer. Conclusions are drawn in Section 6.

2 SYSTEM DESCRIPTION

In a MC-CDMA transmitter, as represented on Figure 1, the data symbol $D_j(t)$, assigned to user j, is multiplied in the frequency domain by the spreading code $SC_j = [c_{1,j}, c_{2,j}, \ldots, c_{k,j}, \ldots, c_{L,j}]$. In this figure, the length L of the spreading code is equal to the number N_c of subcarriers, but this study is not limited to this particular case. However, the different results presented in this paper are given for $L = N_c$. After the multicarrier modulation, easily carried out by IFFT operation and the insertion of a guard interval, the signal $S_j(t) = \Re\left(\sum_{k=1}^{N_c} D_j(t)c_{k,j}e^{2i\pi f_k t}\right)$ is transmitted through a high power amplifier which has a limited peak output power [2].

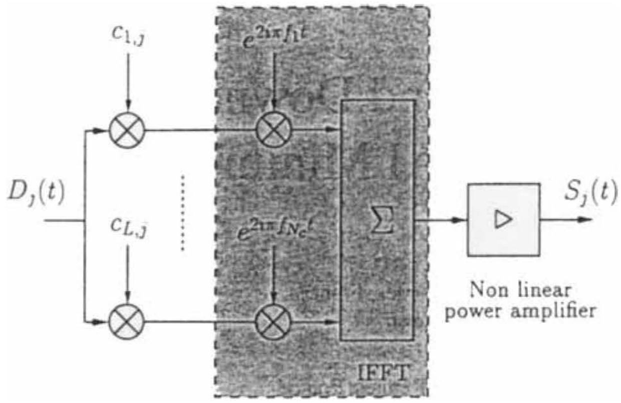

Figure 1: MC-CDMA transmitter for user j.

In this study, we focus on the realistic case of frequency correlated Rayleigh fading channels. We assume that inter symbol interference is avoided thanks to the insertion of a guard interval, which is longer than the delay spread of the channel. Moreover, frequency non-selective fading per subcarrier and time invariance during one OFDM symbol are supposed. Besides, as we consider single-user detection techniques, the complex channel response and the equalization coefficient for the subcarrier k of user j are respectively denoted $h_{k,j}$ and $g_{k,j}$.

Usually, for downlink transmissions, using orthogonal codes such as Walsh-Hadamard spreading sequences guarantees the absence of MAI in a Gaussian channel. However, in frequency selective fading channels, all the subcarriers of the MC-CDMA signal are received with different amplitude levels and different phase shifts, which generates MAI. To combat this interference, one may use various Single-user Detection (SD), linear or nonlinear Multi-user Detection (MD) techniques [3]. For downlink transmissions and for a given user terminal, the desired signal and the disturbing signals are affected by the same channel distortions. Then, it is easy, for example with the well-known Zero Forcing SD, to benefit from the orthogonality between the spreading codes by multiplying the received signals by coefficients equal to the inverse of the channel frequency response.

By contrast, for uplink transmissions, the N_u MC-CDMA signals received at the base station from the N_u active users suffer from different degradations introduced by the N_u independent channels. Consequently, using orthogonal codes for uplink transmissions is no longer mandatory and non-orthogonal codes may be considered.

3 SPREADING SEQUENCES AND SELECTION CRITERIA

Spreading sequences have to be selected in order to minimize on the one hand the Peak-to-Average Power Ratio (PAPR) or the Crest Factor (CF) of the transmitted multi-carrier signal envelope and on the other hand the MAI in the receiver.

3.1 SPREADING SEQUENCES

Taking into account the uplink and downlink specificities, two kinds of sequences, orthogonal or non-orthogonal, are investigated.

3.1.1 Orthogonal sequences

- *Walsh-Hadamard sequences*

An important set of orthogonal codes is the Walsh-Hadamard set. Walsh functions are generated using a Hadamard matrix, starting with $H_1 = [+1]$. The $(L \times L)$ Hadamard matrix is recursively built by:

$$H_L = \begin{bmatrix} H_{L/2} & H_{L/2} \\ H_{L/2} & -H_{L/2} \end{bmatrix} \quad (1)$$

Then, the Walsh-Hadamard sequences are given by the rows or the columns of the matrix H_L. These sequences are generally proposed for MC-CDMA synchronous systems due to their implementation facilities as depicted in [4].

- *Complementary Golay sequences*

Let $(A_i,\ 1 \leq i < p)$ be a set of finite sequences (± 1) of length L and let $\psi_{A_i A_i}(k)$ denote the k-th element of the autocorrelation function of the sequence A_i. A set of sequences is a complementary set if and only if [5]:

$$\sum_{i=1}^{p} \psi_{A_i A_i}(k) = 0, \qquad k \neq 0 \quad (2)$$

Golay sequences, both complementary and orthogonal, are recursively defined by the rows of the matrix CG_L starting with CG_2 [6]:

$$CG_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \end{bmatrix} \quad (3)$$

and more generally

$$CG_L = \begin{bmatrix} A_L & B_L \end{bmatrix} \quad (4)$$

with,

$$\begin{cases} A_L = \begin{bmatrix} A_{L/2} & B_{L/2} \\ A_{L/2} & B_{L/2} \end{bmatrix} \\ B_L = \begin{bmatrix} A_{L/2} & -B_{L/2} \\ -A_{L/2} & B_{L/2} \end{bmatrix} \end{cases} \quad (5)$$

where matrix A_L et B_L are of size $L \times L/2$.
For example, if $L = 4$:

$$CG_4 = \begin{bmatrix} +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 \end{bmatrix} \quad (6)$$

Moreover, Golay sequences are also complementary in two-two time ($i \neq j$), *i.e.*:

$$\psi_{A_i A_i}(k) + \psi_{A_j A_j}(k) = \begin{cases} 2L & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases} \quad (7)$$

• *Orthogonal Gold sequences*
The orthogonal Gold sequences [7][8] are developed from a set of original Gold sequences, which contain elements of the alphabet $\{1, -1\}$, by appending an additional "1" to the end of each sequence. The set $OG(.)$ of L sequences of length $L = 2^n$ (with n mod $4 \neq 0$) of orthogonal Gold codes is given by:

$$OG(A, B) = (U, V_0, V_1, \ldots, V_{L-2}) \quad (8)$$

with

$$U = (A, 1)$$

$$V_j = (A \oplus T^j B, 1)$$

and where

- $A = (a_0, \ldots, a_{L-2})$ and $B = (b_0, \ldots, b_{L-2})$ are a preferred pair of m-sequences of length $L - 1$,
- $T^j B$ is the sequence B after j-chip cyclic shift,
- and $\oplus$ is the modulo 2 addition operator.

3.1.2 *Non-orthogonal sequences*

• *Gold sequences*
This family of Gold codes $G(.)$ is constructed from a preferred pair of m-sequences of length $L = 2^n - 1$ (with n mod $4 \neq 0$) by adding modulo 2 these two m-sequences [9]:

$$G(A, B) = (A, B, V_0, V_1, \ldots, V_{L-1}) \quad (9)$$

with

$$V_j = (A \oplus T^j B)$$

and where $A = (a_0, \ldots, a_{L-1})$ and $B = (b_0, \ldots, b_{L-1})$ are a preferred pair of m-sequences of length L.

$L + 2$ Gold sequences of length L are available. Gold codes have correlation functions with three values $\{-1, -t(n), t(n) - 2\}$, where:

$$t(n) = \begin{cases} 2^{\frac{n+1}{2}} + 1 & \text{for } n \text{ odd} \\ 2^{\frac{n+2}{2}} + 1 & \text{for } n \text{ even} \end{cases} \quad (10)$$

• *Zadoff-Chu codes*
The Zadoff-Chu codes are the special case of the generalized Chirp-Like polyphase sequences having optimum correlation properties. Indeed, Zadoff-Chu sequences of length L offer on the one hand an ideal periodic autocorrelation, and on the other hand a constant magnitude ($\sqrt{L}$) periodic cross-correlation. They are defined by:

$$Z_{C_r}(k) = \begin{cases} e^{j\frac{2\pi r}{L}\left(\frac{k^2}{2} + qk\right)} & \text{for } L \text{ even} \\ e^{j\frac{2\pi r}{L}\left(\frac{k(k+1)}{2} + qk\right)} & \text{for } L \text{ odd} \end{cases} \quad (11)$$

where q is any integer, $k = 0, 1, \ldots, L - 1$ and r is the code index, prime with L [10]. Consequently, if L is a prime number, the set of Zadoff-Chu is composed of $L - 1$ sequences.

3.2 Peak-to-Average Power Ratio and Crest Factor

The MC-CDMA technique offers many advantages but presents also a significant drawback, which is due to the multicarrier feature. Indeed, the MC-CDMA signal consists of the sum of several subcarriers, which may result in a large dynamic transmitted signal. The envelope variation of a multicarrier signal can be estimated by the PAPR or the CF which are for a signal defined on the interval $[0, T[$ equal to [11]:

$$\mathrm{CF}\,(S_j(t)) = \sqrt{\mathrm{PAPR}\,(S_j(t))} = \sqrt{\frac{\max |S_j(t)|^2}{\frac{1}{T}\int_0^T |S_j(t)|^2\, dt}} \quad (12)$$

As a power amplifier has a limited peak output power, an increased PAPR or CF results in a reduced average radiated power in order to avoid nonlinear distortions. For the uplink mobile radio communication, each user's signal is transmitted by a different amplifier and the PAPR or CF of the spreading codes must be compared individually. By contrast, for the downlink, the different data multiplied by the orthogonal spreading codes of the N_u active users are added and transmitted synchronously by the same power amplifier at the base station. So, in that case, the quantity, which is of interest for the comparison between the different classes of sequences, is the global CF (GCF) of the global transmitted signal:

$$\mathrm{GCF}\left(\sum_{j=1}^{N_u} S_j(t)\right) = \sqrt{\frac{\max\left|\sum_{j=1}^{N_u} S_j(t)\right|^2}{\frac{1}{T}\int_0^T \left|\sum_{j=1}^{N_u} S_j(t)\right|^2 dt}} \quad (13)$$

3.3 Multiple access interference

A simple MAI limitation technique for downlink synchronous MC-CDMA transmission system, which consists in an optimized spreading sequence assignment, has been proposed in [12]. Considering SD techniques, the analytic expression of the MAI power associated to user j for the case of a synchronous MC-CDMA transmission is given by :

$$\sigma^2_{\mathrm{MAI},j} = \underbrace{(N_u - 1)R_j(0)L}_{\alpha} + \sum_{\substack{m=1 \\ m\neq j}}^{N_u} \left\{ \begin{array}{c} \underbrace{2R_j(1)\sum_{k=1}^{L-1} w_k^{(j,m)} w_{k+1}^{(j,m)}}_{\beta_{j,m}} + \\ \underbrace{2R_j(2)\sum_{k=1}^{L-2} w_k^{(j,m)} w_{k+2}^{(j,m)}}_{\gamma_{j,m}} + \ldots \\ 2R_j(L-1) w_1^{(j,m)} w_L^{(j,m)} \end{array} \right\} \quad (14)$$

where $R_j(i)$ is the autocorrelation defined as $R_j(p-q) = E[a_{p,j}.a_{q,j}]$, $a_{k,j} = h_{k,j}.g_{k,j}$ is the coefficient affecting the subcarrier k after equalization, $w_k^{(j,m)} = c_{k,j}.c_{k,m}$ defines the product between the chip element used by users j and m at the subcarrier k, and $N_u \leq L$ is the number of active users.

Whatever the frequency correlation of the transmission channel, the MAI minimization procedure detailed in [12] leads to retain a subgroup of N_u spreading sequences for which the minimum number of transitions $(+1/-1)$ among each possible product vector $W^{(j,m)} = (w_1^{(j,m)}, w_2^{(j,m)}, \ldots, w_L^{(j,m)})$ is maximum. Indeed, each product vector $W^{(j,m)}$ can have between 0 and $L-1$ transitions. So depending on the set of selected spreading sequences, the set of corresponding product vectors has a given minimum which can be different from the minimum of an other set. And we select the set of spreading sequences which offer the minimum corresponding product vectors which is maximal. In that case, the sum over m of negative terms $\beta_{j,m}$ of Equation (14) decreases, which reduces the MAI due to large positive value α. Here, $W^{(j,m)}$ must be understood as a measure of the ability to mitigate interference between users j and m. Thus, this first criterion aims at minimizing the largest degradation among two distinct users. Nevertheless, we may obtain several equivalent optimized subgroups. Then, the selection procedure can include a complementary criterion in order to further reduce the MAI.

For that purpose, as a complementary criterion, we compare the three following approaches :

- Complementary criterion: MEAN
  which consists in maximizing the average number of transitions among the different product vectors $W^{(j,m)}$, which ensures a minimization of the sum of terms $\beta_{j,m}$.

- Complementary criterion: STD
  aiming at minimizing the standard deviation of the number of transitions among the different product vectors $W^{(j,m)}$. The application of this complementary criterion further avoids privileging a given user.

- Complementary criterion: 2$^{\text{nd}}$ order
  which consists in maximizing the minimum number of transitions $(+1/-1)$ among each possible second order product vectors
  $W'^{(j,m)} = (w_1^{(j,m)}, w_3^{(j,m)}, \ldots, w_{L-3}^{(j,m)}, w_{L-1}^{(j,m)})$
  and
  $W''^{(j,m)} = (w_2^{(j,m)}, w_4^{(j,m)}, \ldots, w_{L-2}^{(j,m)}, w_L^{(j,m)})$.
  According to the first criterion, the minimization of the sum of negative terms $\beta_{j,m}$ results in a maximization of the sum of other negative terms $\gamma_{j,m}$ of equation (14). Hence, in order to mitigate this effect, this last approach aims at minimizing the sum over m of $\gamma_{j,m}$ which further reduces the MAI.

Criteria based on MAI are expected to be all the more efficient as the channel is frequency correlated [12].

4 Crest factor analytical results

4.1 Uplink context

In uplink context, the MC-CDMA signal, which is transmitted thanks to a high power amplifier for user j, is given by:

$$S_j(t) = \Re\left(\sum_{k=1}^{N_c} D_j(t) c_{k,j} e^{2i\pi f_k t}\right) \quad (15)$$

where $f_k = f_0 + k/T$, T is the "useful" duration of the MC symbol of the transmitted signal $S_j(t)$, N_c is the number of subcarriers and $|D_j(t)| = 1$, as we consider PSK modulations.

The maximum power of the signal $S_j(t)$ is defined by the maximum square absolute value of $S_j(t)$ equal to:

$$\max|S_j(t)|^2 = \max\left|\Re\left(\sum_{k=1}^{N_c} D_j(t)c_{k,j}e^{2i\pi kt/T}e^{2i\pi f_0 t}\right)\right|^2$$

$$\leq \max\left|D_j(t)\sum_{k=1}^{N_c} c_{k,j}e^{2i\pi kt/T}\right|^2 \left|e^{2i\pi f_0 t}\right|^2$$

$$\leq \max\left|\sum_{k=1}^{N_c} c_{k,j}e^{2i\pi kt/T}\right|^2$$

$$\leq \max|\mathcal{C}_j(t)|^2 \qquad (16)$$

where,

$$\mathcal{C}_j(t) = \sum_{k=1}^{N_c} c_{k,j}e^{2i\pi kt/T} \qquad (17)$$

is nothing else than the inverse Fourier transform of the sequence SC_j assigned to user j.

As the mean square value of the signal amplitude $S_j(t)$ equals to $N_c/2$, from Equation (12), we obtain the upper bound for the crest factor for an uplink MC-CDMA signal [11][13]:

$$\mathrm{CF}(S_j(t)) \leq \sqrt{\frac{\max|\mathcal{C}_j(t)|^2}{L/2}} \qquad (18)$$

• *Walsh-Hadamard sequences*

According to Equation (18), we need to evaluate the maximum square absolute value of the inverse Fourier transform of the Walsh-Hadamard sequence SC_j. Undoubtedly, this value is maximum when the Walsh-Hadamard sequences are only composed of elements +1. Consequently, $\max|\mathcal{C}_j(t)|^2 = L^2$ and the upper bound for the Walsh-Hadamard crest factor is given by:

$$\mathrm{CF}_{\mathrm{WH}}(S_j(t)) \leq \sqrt{2L} \qquad (19)$$

• *Golay sequences*

For each pair of complementary sequences assigned to users i and j ($i \neq j$), by calculating the inverse Fourier transform of Equation (7) and applying the well-known autocorrelation theorem [13], we obtain the following relation:

$$|\mathcal{C}_i(t)|^2 + |\mathcal{C}_j(t)|^2 = 2L \qquad (20)$$

From (20), it follows that:

$$|\mathcal{C}_x(t)|^2 \leq 2L \qquad (21)$$

where $\mathcal{C}_x(t)$ is the inverse Fourier transform of any complementary Golay sequence.

So, the upper bound for the Golay sequences crest factor is given by:

$$\mathrm{CF}_{\mathrm{Golay}}(S_j(t)) \leq 2 \qquad (22)$$

• *Gold sequences*

Gold codes have three-valued correlation properties. Thus, autocorrelation function of any Gold sequence can be overestimated by:

$$\psi_{G,G}(k) \leq \begin{cases} L & \text{for } k = 0 \\ t(n) - 2 & \text{for } k \neq 0 \end{cases} \qquad (23)$$

By applying the autocorrelation theorem, we obtain the inverse Fourier transform of any Gold sequence and then:

$$|\mathcal{C}_G(t)|^2 \leq \begin{cases} L[t(n) - 1] + 2 - t(n) & \text{for } t = 0 \\ L - t(n) + 2 & \text{for } t \neq 0 \end{cases} \qquad (24)$$

Hence,

$$\max|\mathcal{C}_G(t)|^2 \leq L[t(n) - 1] + 2 - t(n) \qquad (25)$$

It follows that the upper bound of Gold codes crest factor is given by:

$$\mathrm{CF}_{\mathrm{Gold}}(S_j(t)) \leq \sqrt{2\left[t(n) - 1 - \frac{t(n)}{L} + \frac{2}{L}\right]} \qquad (26)$$

• *Zadoff-Chu sequences*

The autocorrelation function of Zadoff-Chu codes is defined to be ideal, *i.e.*:

$$\psi_{Z_{C_r},Z_{C_r}}(k) = \begin{cases} L & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases} \qquad (27)$$

By applying the autocorrelation theorem, we can obtain the inverse Fourier transform of any Zadoff-Chu sequence and then:

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