Summary of UCRL Pyrotron (Mirror Machine) Program*

By R. F. Post*

Under the sponsorship of the Atomic Energy Commission, work has been going forward at the University of California Radiation Laboratory since 1952 to investigate the application of the so-called " magnetic mirror " effect to the creation and confinement of a high temperature plasma. By the middle of 1953 several specific ways by which this principle could be applied to the problem had been conceived and analyzed and were integrated into a proposed approach which has come to be dubbed "The Mirror Machine".[1] Study of the various aspects of the physics of the Mirror Machine has been the responsibility of a Laboratory experimental group (called the "Pyrotron" Group) under the scientific direction of the author. During the intervening years of effort, experiments have been performed which demonstrate the confinement properties of the Mirror Machine geometry and confirm several of its basic principles of operation. There remain, however, many basic quantitative and practical questions to be answered before the possibility of producing self-sustained fusion reactions in a Mirror Machine could be properly assessed. Nevertheless, the experimental and theoretical investigations to date have amply demonstrated the usefulness of the mirror principle in the experimental study of magnetically confined plasmas. This report presents some of the theory of operation of the Mirror Machine, and summarizes the experimental work which has been carried out.

PRINCIPLE OF THE MAGNETIC MIRROR

The modus operandi of the Mirror Machine is to create, heat and control a high temperature plasma by means of externally generated magnetic fields. The magnetic mirror principle is an essential element, not only in the confinement, but in the various manipulations which are performed in order to create and to heat the plasma.

Confinement

The magnetic mirror principle is an old one in the realm of charged particle dynamics. It is encountered for example, in the reflection of charged cosmic ray particles by the earth's magnetic field. As here to be understood, the magnetic mirror effect arises whenever a charged particle moves into a region of magnetic field where the strength of the field increases in a direction parallel to the local direction of the field lines, i.e., wherever the lines of force converge toward each other. Such regions of converging field lines tend to reflect charged particles, that is, they are " magnetic mirrors ". The basic confinement geometry of the Mirror Machine thus is formed by a cage of magnetic field lines lying between two mirrors, so that configuration of magnetic lines resembles a two-ended wine bottle, with the ends of the bottle defining the mirror regions. This is illustrated in Fig. 1, which also shows schematically the location of the external coils which produce the confining fields. The central, uniform field, region can in principle be of arbitrary length, as dictated by experimental convenience or other considerations. For various reasons, it has been found highly desirable to maintain axial symmetry in the fields, although the general principle of particle confinement by mirrors does not require this.

The confinement of a plasma between magnetic mirrors can be understood in terms of an individual particle picture. The conditions which determine the binding of individual particles between magnetic mirrors can be obtained through the use of certain adiabatic invariants applying generally to the motion of charged particles in a magnetic field.

The first of these invariants is the magnetic moment, μ, associated with the rotational component of motion of a charged particle as it carries out its helical motion in the magnetic field.[2] The assumption that μ is an absolute constant of the motion is not strictly valid, but, as later noted, it represents a very good approximation in nearly all cases of practical interest in the Mirror Machine.

The magnitude of μ is given by the expression

$$\mu = W_{\perp}/H = \tfrac{1}{2}mv_{\perp}{}^2/H \text{ ergs/gauss} \qquad (1)$$

which states that the ratio of rotational energy to magnetic field remains a constant at any point along the helical trajectory of the charged particle. In this case the magnetic field is to be evaluated at the line of force on which the guiding center of the particle is moving.

Now, at the point at which a particle moving toward a magnetic mirror is reflected, its entire energy of

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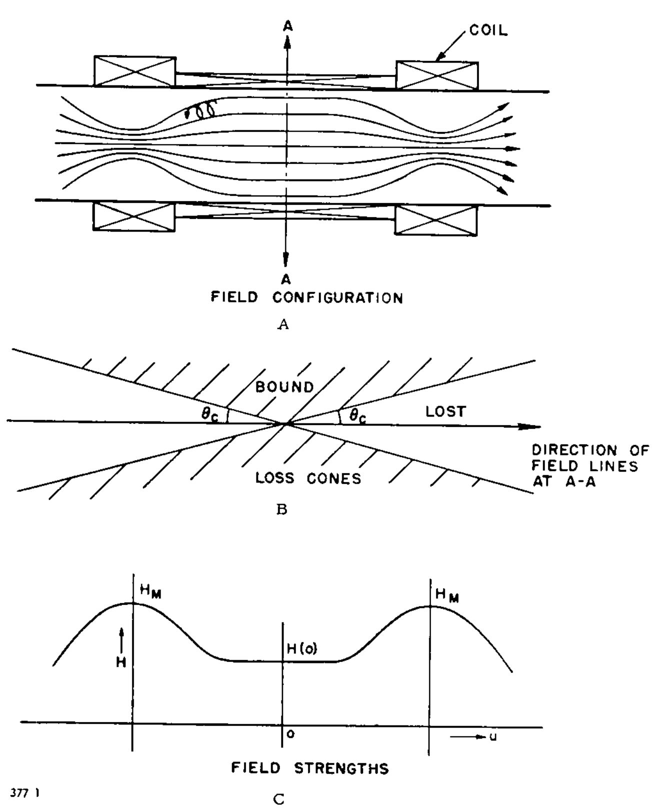

Figure 1. (*a*) Schematic of magnetic field lines and magnetic coils, (*b*) mirror loss cones, (*c*) axial magnetic field variation

motion, W, is in rotation, so that at this point $\mu H = W$. The condition for binding charged particles between two magnetic mirrors of equal strength is therefore simply that for bound particles

$$\mu H_M > W \tag{2}$$

where H_M is the strength of the magnetic field at the peak of the mirror. This condition may also be expressed in terms of field strength and energies through use of (1). If H_0 is the lowest value of the magnetic field between the mirrors, and H_M is the peak value at the mirror (both being evaluated on the same flux surface) then (2) becomes

$$W/W_{\perp}(0) < R_M \tag{3}$$

where $R_M = H_M/H_0$ (the "mirror ratio") and $W_{\perp}(0)$ is the value of W at the point where H takes on the value H_0.

It is to be noted that the condition that particles be bound does not depend on their mass, charge, absolute energy, or spatial position, nor on the absolute strength or detailed configuration of the magnetic field. Instead, it depends only on the ratios of energy components and magnetic field strengths. Such an insensitivity to the details of particle types or orbits is what is needed to permit the achievement of a confined plasma; i.e., a gas composed of charged particles of different types, charges and energies. The simplicity of the binding conditions reflects the insensitivity to detailed motion required to accomplish this.

Further insight into the nature of confinement of a plasma by mirrors can be obtained by manipulation of the reflection condition. By differentiation, holding the total particle energy constant, it can be shown that a particle moving along a magnetic line of force (or flux surface) into a magnetic mirror always experiences a retarding force which is parallel to the local direction of the magnetic field and is given by the expression[3]

$$F_u = \mu \frac{\partial H}{\partial u} \text{ dynes,} \tag{4}$$

where u is the axial distance from the plane of symmetry.

This force, which is the same as that expected on a classical magnetic dipole moving in a magnetic field gradient, represents the reflecting force of the mirror. Integrated up to the peak of the mirrors, (4) yields again the binding condition (3).

Since μ has been assumed constant, (4) may also be written in the form

$$F_u = -\nabla_u(\mu H) \tag{5}$$

This shows that the quantity μH acts as a potential so that the region between two magnetic mirrors of equal strength lies in a potential well between two potential maxima of height $\mu H_{\max}$.

Loss Cones

It is evident, either from (2), (3) or (5), that it is not possible to confine a plasma which is isotropic in its velocity conditions—one in which all instantaneous spatial directions of motion are allowed—between two magnetic mirrors: confinement of an isotropic plasma is only possible to contemplate in special cases where multiple mirrors might be employed. This limitation can be understood in terms of the concept of *loss cones*. The pitch angle, θ, of the helical motion of particles moving along a line of force is transformed in accordance with a well-known relationship.[4]

$$\sin\theta(u) = [R(u)]^{\frac{1}{2}} \sin\theta(0) \tag{6}$$

where the angle $\theta(u)$ is measured with respect to the local direction of the magnetic field. $R(u)$ is the mirror ratio, evaluated at u, $R(u) \equiv H(u)/H(0)$.

Expression (6) bears a resemblance to Snell's Law of classical optics, which relates refraction angles within optically dense media. Here $\sqrt{R}$ is analogous to the index of refraction of Snell's Law. As in the optical case, total internal reflection can occur for those angles larger than a critical angle, θ_c, found by setting $\sin\theta$ equal to its maximum value of 1. Thus

$$\sin\theta_c = R_M^{-\frac{1}{2}}. \tag{7}$$

This relation defines the loss cones for particles bound between magnetic mirrors, illustrated in Fig. 1. All particles with pitch angles lying outside the loss cones are bound, while all with pitch angles within the loss cone will be lost upon their first encounter with either mirror (for equal mirror strengths). It should be emphasized that the concept of the mirror loss cone pertains to the velocity space of the trapped particles and has nothing to do with the spatial dimensions of the confining fields.

Although the binding of particles between magnetic mirrors can be accomplished with fields which are not axially symmetric, there are substantial advantages to the adoption of axial symmetry. One of these advantages arises from the fact that all " magnetic bottles " involve magnetic fields with gradients or curvature of the magnetic field lines in the confinement zone, the existence of which gives rise to systematic drifts of the confined particles across the magnetic lines of force.[5] If the particle orbit diameters are small, compared to the dimensional scale of the field gradients or field curvature, these drifts will be slow compared to the particle velocities themselves. If directed across the field, however, they would still be too rapid to be tolerated and would effectively destroy the confinement. Furthermore, drifts of this type are oppositely directed for electrons and ions and may thus give rise to charge separation and electric fields within the plasma. These electric fields then may induce a general drift of the plasma across the field to the walls. However, if the magnetic field is axially symmetric, as for example in the mirror field configuration of Fig. 1, the particle drifts will also be axially symmetric, leading only to a rotational drift of the plasma particles around the axis of symmetry, positive ions and electrons drifting in opposite directions. However, this will not result in a tendency for charge separation to occur, since each flow closes on itself.

Another consequence of the use of axial symmetry is that even though the confined particles may be reflected back and forth between the mirrors a very large number of times, this fact will not lead to a progressive " walking " across the field. In fact, it is possible to show that all particles trapped between the mirrors are also bound to a high order of approximation to the flux surfaces on which they move and may not move outward or inward to another flux tube (apart from the normal slow diffusion effects arising from interparticle collisions).

Once bound, there is no tendency for particles to escape the confining fields, within the assumptions made to this point. However, in predicting the conditions for confinement of a *plasma* by means of conditions applying to the individual particles of the plasma, one makes the tacit assumption that co-operative effects will not act in such a way as to destroy the confinement. Such cooperative effects can modify the confinement through static or time-varying electric fields arising from charge separation, or through diamagnetic effects which change the local strength of the confining fields themselves. However, if the confining magnetic fields are axially symmetric, and if the presence of the plasma does not destroy this symmetry then, as has already been explained, charge separation effects cannot give rise to systematic drifts across the field. Similarly, in such a circumstance the diamagnetic effect of the plasma can only lead to an axially symmetric depression of the confining fields in the central regions of the confinement zone, but will leave the fields at the mirrors essentially unaltered (since the density falls nearly to zero at the peak of the mirrors, where particles are escaping). Symmetric diamagnetic effects therefore tend to increase the mirror ratios above the vacuum field value. Of course, there will exist a critical value of β, the ratio of plasma pressure to magnetic pressure, above which stable confinement is not possible. This critical relative pressure will be dependent on various parameters of the system, such as: the mirror ratio; the shape, symmetry and aspect ratio of the fields; and the plasma boundary conditions. Although the precise conditions for stability of plasmas confined by magnetic mirrors are not at this time sufficiently well understood theoretically to predict them with confidence, there is now agreement that it should be possible to confine plasmas with a substantial value of β between magnetic mirrors.[6] This conclusion is borne out by the experiments here reported, which indicate stable confinement. It should be emphasized, however, that, encouraging as this may be, neither the theoretical predictions nor the experimental results are yet sufficiently advanced to guarantee that plasmas of the size, temperature and pressure necessary to produce self-sustaining fusion reactions in a Mirror Machine would be stable. As in all other known " magnetic bottles " the ultimate role of plasma instabilities has not yet been determined in the Mirror Machine.

LOSS PROCESSES

Non-Adiabatic Effects

Although it has been shown that trapping conditions based on adiabatic invariants predict that a plasma might, in principle, be confined within a Mirror Machine for indefinitely long periods of time, it is clear that mechanisms will exist for the escape of particles, in spite of the trapping.

One might first of all question the validity of the assumption of constant magnetic moment, since trapping depends critically on this assumption. It is clear that this assumption cannot be exactly satisfied in actual magnetic fields, where particle orbits are not infinitesimal compared to the dimensions of the confining magnetic field.

It can be seen from first principles, as for example shown by Alfvén[2] that the magnetic moment will be very nearly a constant in situations where the magnetic field varies by only a small amount in the course of a single rotation period of the particle. It might also be suspected, in the light of the analogy between this problem and the general theory of non-adiabatic effects of classical mechanics, that the deviations from adiabaticity should rapidly diminish as the relative orbit size is reduced. Kruskal[7] has shown that the convergence is indeed rapid but his results are not readily applicable to the Mirror Machine. Using numerical methods, however, it has been shown[8]: (*a*) that the fluctuations in magnetic moment associated with non-adiabatic effects are of importance only for relatively large orbits, (*b*) that these fluctuations seem to be cyclic in nature, rather than cumulative, and (*c*) that they diminish approximately exponentially with the reciprocal of the particle orbit size, so that it should always be possible to scale an experiment in such a way as to make non-adiabatic orbit effects negligible. The results may be roughly summarized by noting that, for the typical orbits which were considered, the maximum amplitude of the fractional changes in the magnetic moment, which occurred after successive periods of back-and-forth motion of trapped particles, could be well represented by an expression of the form

$$|\delta\mu/\mu| = ae^{-b/\nu} \tag{8}$$

where $\nu = 2\pi\mathscr{R}/L$, i.e., the mean circumference of the particle orbit divided by the distance between the mirrors. The mirror fields were represented by the function $H_z = H_0\,[1+\alpha \cos uI(\rho)]$ plus the appropriate curl-free function for H_r. Here, in dimensionless form, $u = 2\pi z/L$ and $\rho = 2\pi r/L$. With $\alpha = 0.25$, typical values of the constants a and b were in the range $4 < a < 6$, $1.5 < b < 2$, for various radial positions of the orbits. From these values it can be seen that provided $\nu < 0.2$, the variations in μ are totally negligible, so that the adiabatic orbit approximation is well satisfied. For example, if $L = 100$ cm then all orbits with mean radii less than about 3 cm can be considered to behave adiabatically. This imposes restrictions on the minimum values of magnetic fields which can be used, or on the minimum size of the confinement zones, but it is clear that, as the scale of the apparatus is increased, the assumption of orbital adiabaticity becomes increasingly well satisfied.

Collision Losses

Even under conditions where the adiabatic invariants establish effective trapping of particles, there still remains a simple and direct mechanism for the loss of particles from a mirror machine. This is, of course, the mechanism of interparticle collisions, the mechanism which limits the confinement time of any stable, magnetically confined plasma. Collisions can induce changes in either the magnetic moment or energy of a trapped particle, and thus can cause its velocity vector to enter the escape cone. The dominant collision cross section in a totally ionized plasma is the Coulomb cross section, which varies inversely with the square of the relative energy of the colliding particles. Thus the rate of losses (if dominated by collisions) can be reduced by increasing the kinetic temperature of the plasma. The basic rate of these loss processes may be estimated by consideration of the "relaxation" time of energetic particles in a plasma, as calculated by Chandrasekhar or by Spitzer. In this case, the relevant quantity is the mean rate of dispersion in pitch angle of a particle as a result of collisions, since, if a given trapped particle is scattered through a large angle in velocity space, the probability is large that it will have been scattered into the escape cone and thus lost. As is usual in a plasma, distant collisions (within a Debye sphere) play a greater role than single scattering events. The time for the growth of the angular dispersion in this (multiple) scattering process for a particle of given energy is governed by the relation

$$\bar{\theta}^2 = t/t_{\mathrm{D}} \tag{9}$$

where

$$t_{\mathrm{D}} = \frac{M^2v^3}{\pi ne^4\,\mathscr{H}(x)\log\Lambda}, \tag{10}$$

M and v are the mass and velocity of the scattered particle, $x^2 = W/kT = \frac{1}{2}Mv^2/kT$ and $\log\Lambda$ has the usual value[5] of about 20. $\mathscr{H}(x)$ is a slowly varying function of x and is approximately equal to 0.5 for typical values.

In terms of deuteron energies in kev,

$$t_{\mathrm{D}} = 2.6\times 10^{10}W^{\frac{3}{2}}/n \text{ seconds.} \tag{11}$$

For $\bar{\theta}^2 = 1$, the scattering time becomes equal to t. Thus t_{D} represents a rough estimate of the confinement time of ions in a Mirror Machine. It is clear that the confinement time will also depend on the mirror ratio R, but detailed calculations show that for large values of this ratio, the confinement varies only slowly with R.

Some numerical values are of interest in this connection. Suppose $W = 0.01$ kev, and $n = 10^{14}$, a mean particle energy and density which might be achieved in a simple discharge plasma. In this case $t_{\mathrm{D}} = 0.26$ μsec. This means that the confinement of low temperature plasmas by simple mirrors will be of very short duration, unless means for rapid heating are provided, which would extend the confinement time. On the other hand, for $W = 150$ kev, the same particle density would give $t_{\mathrm{D}} = 0.5$ sec, which is long enough to provide adequate confinement for experimental studies and is within a factor of about 20 of the mean reaction time of a tritium-deuteron plasma at a corresponding temperature. Since the energy released in a single nuclear reaction would be about 100 times the mean energy of the plasma particles, it can be seen that the possibility exists for producing an energetically self-sustaining reaction, with a modest margin of energy profit, in this case about 4 to 1.

The problem of end losses may be more precisely formulated by noting that these losses arise from binary collision processes, so that it should be possible at all times to write the loss rate in the form

$$\dot{n} = -n^2\langle\sigma v\rangle_{\mathrm{s}}f(R) \tag{12}$$

where $\langle\sigma v\rangle_{\mathrm{s}}$ represents a scattering rate parameter and $f(R)$ measures the effective fractional escape cone of the mirrors for diffusing particles. If $\langle\sigma v\rangle_{\mathrm{s}}$ remains approximately constant during the decay, then integration of the equation shows that a given initial density will decay with time as

$$n = n_0\tau/(t+\tau) \tag{13}$$

where

$$\tau = [n_0\langle\sigma v\rangle_{\mathrm{s}}f(R)]^{-1}. \tag{14}$$

The relaxation time approximation consists of setting $f(R) = 1$—i.e., ignoring the dependence on mirror ratio, thereby overestimating the loss rate—and, at the same time, approximating the true value of $n_0\langle\sigma v\rangle_s$ by $1/t_D$, which tends to underestimate the loss rate. Thus, in this approximation, the transient decay of the plasma in a Mirror Machine is given by

$$n = n_0 t_D/(t+t_D). \qquad (15)$$

In this approximation, t_D represents the time for one-half of the original plasma to escape through the mirrors. Now, the instantaneous rate of nuclear reactions which could occur in the plasma is proportional to n^2. Integrating n^2 from (15) over all time, it is found that the total number of reactions which will occur is the same as that calculated by assuming that the density has the constant value n_0 for time t_D and then immediately drops to zero; i.e., number of reactions is proportional to $n_0{}^2 t_D$.

Actually, although they correctly portray the basic physical processes involved in end losses, estimates of confinement time based on simple relaxation considerations are likely to be in error by factors of two or more. To obtain accurate values of the confinement time more sophisticated methods are required. Judd, McDonald and Rosenbluth,[9] and others,[8] have applied the Fokker-Planck equation to this problem and have derived results which should be much closer to the true situation, even though it was necessary to introduce simplifying approximations in their calculations. They find that calculations based on the simple relaxation considerations tend to overestimate the confinement times. They are also able to calculate explicitly the otherwise intuitive result that the mean energy of the escaping group of particles is always substantially less than the mean energy of the remaining particles, a circumstance which arises because low energy particles are more rapidly scattered than high energy ones.

The most detailed results in these end loss calculations have been obtained by numerical integration. However, before these results were obtained, D. Judd, W. McDonald, and M. Rosenbluth derived an approximate analytic solution which, in many cases, differs only slightly from the more accurate numerical calculations. Their results can be expressed by two equations which describe the dominant mode of decay of an eigenvalue equation for the diffusion of particles in the velocity space of the Mirror Machine.

The first equation, that for the density, is:

$$\dot{n} = -n^2[\tfrac{4}{3}\pi(e^4/m^2)\langle v^{-1}\rangle\langle v^{-2}\rangle \log\Lambda]\lambda(R), \qquad (16)$$

where the angle brackets denote averages over the particle distribution and $\log\Lambda$ is the screening factor.[5] The eigenvalue $\lambda(R)$ is closely approximated by the expression $\lambda(R) = 1/\log_{10}(R)$, showing that the confinement time varies linearly with the mirror ratio, at small mirror ratios, but that it varies more slowly at large values of R. We see immediately from the form of the equation that one can define a scattering time as in Eq. (14):

$$\tau_\lambda = \{[\tfrac{4}{3}\pi(n_0e^4/m^2v_0{}^3)\log\Lambda][v_0{}^3\langle v^{-1}\rangle\langle v^{-2}\rangle]\lambda(R)\}^{-1}. \qquad (17)$$

The term in the first bracket is almost identical with that for the relaxation time of a particle with the mean velocity v_0. The velocity averages give rise to small departures from the simple relaxation values but, as noted, the corrections are usually not large.

In a similar way, an equation can be written for the rate of energy transport through the mirror by particle escape. The basic rate for this is, of course, simply given by the value of $\dot{n}\bar{W}_s$; but $\bar{W}_s$, the mean energy of escaping particles, as noted, is not the same as the mean energy of the remaining particles. They find, for the rate of energy loss:

$$\dot{n}\bar{W}_s = -n^2[\tfrac{2}{3}\pi(e^4/m)\langle v^{-1}\rangle \log\Lambda]\lambda(R). \qquad (18)$$

Since the value of $\langle v^{-1}\rangle$ is not particularly sensitive to the velocity distribution, one may calculate this for a Maxwellian distribution without committing excessive error.

The mean energy of the escaping ions, $\bar{W}_s$, may be evaluated from the equations. Dividing (18) by (16), $\bar{W}_s$ is seen to be $\frac{1}{2}m\langle v^{-2}\rangle^{-1}$. Approximating the actual distribution by a Maxwellian, $\bar{W}_s$ turns out to be $\frac{1}{2}kT$, or $\frac{1}{3}$ of the Maxwellian mean energy. This is roughly the same value as was obtained by the more accurate numerical calculations.

As far as eventual practical applications are concerned, the over-all result of the detailed end loss calculations is to show that although a power balance can in principle be obtained, both with the DT and DD reactions, by operation at sufficiently high temperatures, the margin is less favorable than the rough calculations indicated. Thus, any contemplated application of the Mirror Machine to the generation of power would doubtless require care in minimizing or effectively recovering the energy carried from the confinement zone by escaping particles. Some possible ways by which this might be accomplished will be discussed in sections to follow. Other methods to reduce end losses through magnetic mirrors have been suggested and are under study. One of these, involving a rotation of the plasma to "enhance" the mirror effect, is under study at this Laboratory and at the Los Alamos Scientific Laboratory.

In the present Mirror Machine program, scattering losses impose a condition on the experimental use of simple mirror systems for the heating and confinement of plasmas. This condition is that operations such as injection and heating must be carried out in times shorter than the scattering times. These requirements can be readily met, however, in most circumstances so that end losses do not present an appreciable barrier to the studies.

Ambipolar Effects

Since the scattering rate for electrons is more rapid than for ions of the same energy, the intrinsic end loss rate for electrons and ions will be markedly different. In an isolated plasma this situation will not persist,

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