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Eigenstructure Assignment for Design of Multimode Flight Control Systems

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ABSTRACT: Advanced aircraft such as control configured vehicles (CCV) provide the capability for implementing multimode control laws, which allow the aircraft performance to be tailored to match the characteristics of a specific task or mission. This is accomplished by generating decoupled aircraft motions, which can be used to improve aircraft effectiveness. In this article, we describe a task-tailored multimode flight control system, which was designed by using eigenstructure assignment.

Introduction

Advanced aircraft such as control configured vehicles (CCV) provide the capability to control the aircraft in unconventional ways. One such approach is to generate decoupled motions, which can be used to improve tracking and accuracy. The decoupled motions are obtained by utilizing a task-tailored multimode flight control system, which implements feedback gains not only as a function of flight condition but also as a function of the mode selected. The aircraft performance can then be tailored to match the desired characteristics of a specific task or mission.

For the longitudinal dynamics of a control configured vehicle, the flaperons and elevator form a set of redundant control surfaces capable of decoupling normal control forces and pitching moments. The decoupled motions include pitch pointing, vertical translation, and direct lift control. Pitch pointing is characterized by pitch attitude command without a change in flight path angle. Vertical translation is characterized by flight path command without a change in pitch attitude. Direct lift control is characterized by normal acceleration command without a change in the angle of attack.

For the lateral dynamics of a control configured vehicle, the vertical canard and rudder form a set of redundant surfaces that is capable of producing lateral forces and yawing moments independently. The decoupled motions include yaw pointing, lateral translation, and direct sideforce. Yaw pointing is characterized by heading command without a change in lateral directional flight path angle. Lateral translation is characterized by lateral directional flight path command without a change in heading. Direct sideforce is characterized by lateral acceleration command without a change in sideslip angle. All three lateral modes also require that there be no change in bank angle.

The application of eigenstructure assignment to conventional flight control design has been described by Shapiro et al. in Ref. [1]. A design methodology that uses eigenstructure assignment to obtain decoupled aircraft motions has been described by Sobel et al. in Refs. [2]–[5]. In this article, we use eigenstructure assignment to design a task-tailored multimode flight control system. The longitudinal design is illustrated by using the unstable dynamics of an advanced fighter aircraft, and the lateral design is illustrated by using the dynamics of the flight propulsion control coupling (FPCC) vehicle.

Eigenstructure Assignment Basics

Consider an aircraft modeled by the linear time-invariant matrix differential equation given by

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where x is the state vector ($n \times 1$), u is the control vector ($m \times 1$), and y is the output vector ($r \times 1$). Without loss of generality, we assume that the m inputs are independent and the r outputs are independent. Also, as is usually the case in aircraft problems, we assume that m, the number of inputs, is less than r, the number of outputs. If there are no pilot commands, the control vector u equals a matrix times the output vector y.

$$u = -Fy$$

The feedback problem can be stated as follows [1]: Given a set of desired eigenvalues, (λ_i^d), $i = 1, 2, \ldots, r$ and a corresponding set of desired eigenvectors, (v_i^d), $i = 1, 2, \ldots, r$, find a real $m \times r$ matrix F such that the eigenvalues of $A - BFC$ contain (λ_i^d) as a subset, and the corresponding eigenvectors of $A - BFC$ are close to the respective members of the set (v_i^d).

The feedback gain matrix F will exactly assign r eigenvalues. It will also assign the corresponding eigenvectors, provided that they were chosen to be in the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$. This subspace is of dimension m, which is the number of independent control variables. In general, a chosen or desired eigenvector v_i^d will not reside in the prescribed subspace and, hence, cannot be achieved. Instead, a "best possible" choice for an achievable eigenvector is made. This "best possible" eigenvector is the projection of v_i^d onto the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$.

We summarize with the following:

- The matrix F will exactly assign r eigenvalues. It will also exactly assign each of the corresponding r eigenvectors to m-dimensional subspaces, which are constrained by λ_i^d, A, and B.
- If more than m elements are specified for a particular eigenvector, then an achievable eigenvector is computed by projecting the desired eigenvector onto the allowable subspace. This is the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$.
- If control over a larger number of eigenvalues is required, then additional independent sensors must be added.
- If improved eigenvector assignability is required, then additional independent control surfaces must be added.

Now suppose that in addition to transient shaping, we desire the controlled (or tracked) aircraft variables y_t to follow the command vector u_c with zero steady-state error where

$$y_t = Hx \tag{3}$$

The complete control law is derived by Broussard [6] and Davison [7]. If the command inputs u_c are constant, and if the tracking objective is to have the aircraft variables y_t approach the command inputs in the limit, then the control input vector is given by

$$u = \underbrace{(\Omega_{22} + FC\Omega_{12})u_c}_{\text{feedforward}} \quad \underbrace{- Fy}_{\text{feedback}} \tag{4}$$

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where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ H & O \end{bmatrix}^{-1}$$

Further details of the eigenstructure assignment algorithm may be found in [1].

Longitudinal Multimode Flight Control Design

The model of the advanced fighter aircraft [8] will be described by the short period approximation equations augmented by control actuator dynamics (elevator and flaperons). The equations of motion are described by Eqs. (1) and (2) where the state x has five components and the control u has two components.

$$x = \begin{bmatrix} \theta \\ q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix} \begin{matrix} \text{—pitch attitude} \\ \text{—pitch rate} \\ \text{—angle of attack} \\ \text{—elevator deflection} \\ \text{—flaperon deflection} \end{matrix}$$

$$u = \begin{bmatrix} \delta_{ec} \\ \delta_{fc} \end{bmatrix} \begin{matrix} \text{—elevator deflection command} \\ \text{—flaperon deflection command} \end{matrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -0.8693 & 43.223 & -17.251 & -1.5766 \\ 0 & 0.9933 & -1.341 & -0.1689 & -0.2518 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix}$$

The eigenvalues of the open-loop system from matrix A are given by

$$\left.\begin{matrix} \lambda_1 = -7.662 \\ \lambda_2 = 5.452 \end{matrix}\right\} \text{ unstable short period mode}$$

$\lambda_3 = 0.0$ pitch attitude mode

$\lambda_4 = -20$ elevator actuator mode

$\lambda_5 = -20$ flaperon actuator mode

The normal acceleration at the pilot's station n_{zp} is used as a controlled aircraft variable for pitch pointing.

$$n_{zp} = [-0.268, 47.76, -4.56, 4.45] \begin{bmatrix} q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix} \quad (5)$$

where n_{zp} is in g's and q, α, δ_e, and δ_f are implement modes 1 and 2, using the same gain matrix.

Pitch Pointing (Mode 1) and Vertical Translation (Mode 2)

The objective in pitch pointing control is to command the pitch attitude while maintaining zero perturbation in the flight path angle. The measurements are chosen to be pitch rate, normal acceleration, altitude rate, and control surface deflections. The altitude rate is obtained from the air data computer, and it is used to obtain the flight path angle via the relationship

$$\gamma \approx \dot{h}/\text{TAS} \quad (6)$$

where TAS is true airspeed. The surface deflections are measured by using linear variable differential transformers (LVDT).

We include γ as a state because this is the variable whose perturbation we require to remain zero. Thus, we replace θ by $\gamma + \alpha$ in the state equations and obtain an equation for γ. The resulting state-space model is given by Eqs. (1) and (2) with

$$x = [\gamma, q, \alpha, \delta_e, \delta_f]^T \quad (7)$$

$$u = [\delta_{ec}, \delta_{fc}]^T \quad (8)$$

$$y = [q, n_{zp}, \gamma, \delta_e, \delta_f]^T \quad (9)$$

Our first step in the design is to compute the feedback matrix F. The desired short period frequency and damping are chosen to be $\zeta = 0.8$ and $\omega_n = 7$ rad/sec. These values were chosen to meet MIL-F-8785C specifications for category A, level 1 flight. Category A includes nonterminal flight phases that require rapid maneuvering, precision tracking, or precise flight path control. Level 1 flying qualities are those that are clearly adequate for the mission objectives.

We can arbitrarily place all five eigenvalues because we have five measurements. We can also arbitrarily assign two entries in each eigenvector because we have two inputs. Alternatively, we can specify more than two entries in a particular eigenvector, and then the algorithm will compute a corresponding achievable eigenvector by taking the projection of the desired eigenvector onto the allowable subspace.

We choose the desired eigenvectors to decouple pitch rate and flight path angle. Such a choice should prevent an attitude command from causing significant flight path change. The desired eigenvectors and achievable eigenvectors are shown in Table 1, from which we observe that we have achieved an exact decoupling between pitch rate and flight path angle. The "X" elements in the desired eigenvectors represent elements that are not specified because they are not directly related to the decoupling objective.

We now compute the feedforward gains by using Eq. (4). For the pitch pointing problem

$$y_t = Hx = [\theta, \gamma]^T \quad (10a)$$

$$u_c = [\theta_c, \gamma_c]^T \quad (10b)$$

where

θ_c = pilot's pitch attitude command

γ_c = pilot's flight path angle command

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The feedforward gain matrix consists of four gains, which couple the commands θ_c and γ_c to the actuator inputs. The control law is described by

$$u = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \theta_c \\ \gamma_c \end{bmatrix} - Fy \quad (11)$$

When $\gamma_c = 0$, we can command pitch attitude without a change in flight path angle (pitch pointing). Alternatively, when $\theta_c = 0$, we can command flight path angle without a change in pitch attitude (vertical translation). The feedback and feedforward gains are shown in Table 2. The pitch pointing and vertical translation responses are shown in Fig. 1. We observe that both responses exhibit excellent decoupling between pitch attitude and flight path angle. An additional feature of the design is that the aircraft is stable with good handling qualities in the event of a flaperon failure. Of course, decoupled mode control would no longer be possible.

Table 1
Eigenvectors for Pitch Pointing/Vertical Translation

Desired Eigenvectors						Achievable Eigenvectors				
0	0	1	X	X	γ	0	0	1	−0.0057	−0.0137
1	X	0	X	X	q	1	−9.5	0	1.07	0.0601
X	1	X	X	X	α	−0.9286	1	−1	−0.0508	0.0106
X	X	X	1	X	δ_e	−5.13	0.1286	−2.80	1	0
X	X	X	X	1	δ_f	8.36	−5.16	3.23	0	1

Table 2
Pitch Pointing/Vertical Translation Control Law

Desired Eigenvalues	Feedforward Gains	Feedback Gains
$\lambda_{1,2}^d = -5.6 \pm j4.2$ $\lambda_3^d = -1.0$ $\lambda_4^d = -19.0$ $\lambda_5^d = -19.5$	$\begin{bmatrix} -2.88 & -0.367 \\ 2.02 & 4.08 \end{bmatrix}$	$\begin{array}{ccccc} q & n_z & \gamma & \delta_e & \delta_f \end{array}$ $\begin{bmatrix} -0.931 & -0.149 & -3.25 & -0.153 & 0.747 \\ 0.954 & 0.210 & 6.10 & 0.537 & -1.04 \end{bmatrix}$

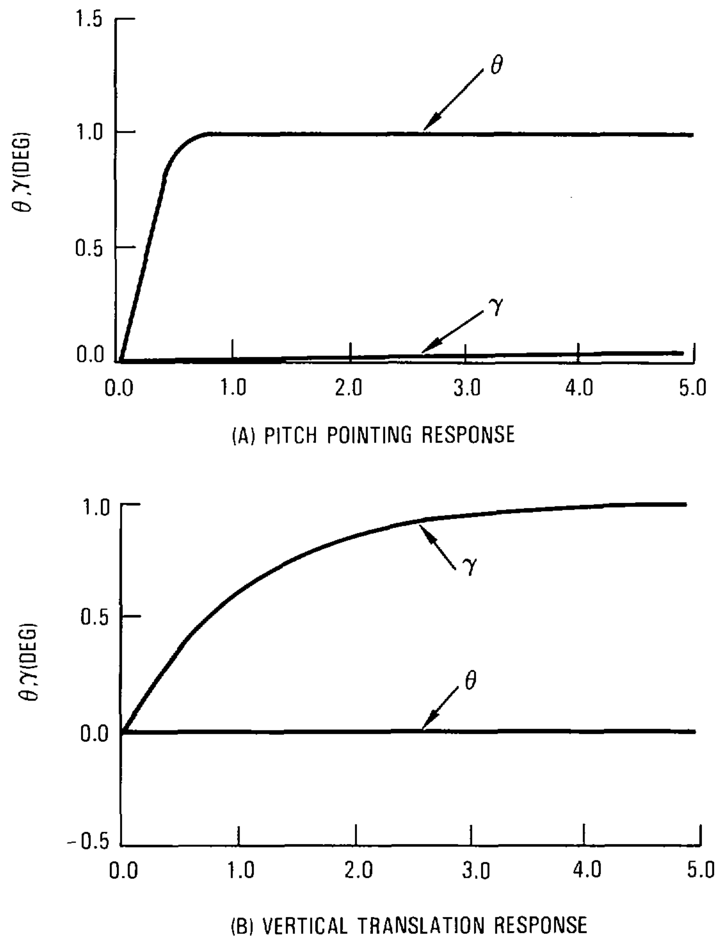

Fig. 1. Longitudinal decoupled responses.

Mode 3: Direct Lift Control

The objective in direct lift control (DLC) is to command normal acceleration (or equivalently flight path angle rate) without a change in angle of attack. To achieve acceleration command following, we include integrated normal acceleration in the state vector. Thus, for the DLC problem, we choose the state vector to be

$$x = [q, \alpha, \delta_e, \delta_f, n_{zI}]^T$$

where

$$n_{zI} = \begin{cases} \text{integral of normal acceleration} \\ \text{at the pilot's station} \end{cases}$$

The measurement vector is chosen to be

$$y = [q, \alpha, n_{zI}, \delta_e, \delta_f]^T$$

The desired eigenvalues and desired eigenvectors are shown in Table 3. Observe that the zeros in the desired eigenvectors are chosen to decouple the short period motion from the normal acceleration. The feedback gain matrix is also shown in Table 3.

To obtain normal acceleration command following, we feedback the integral of the error between measured n_{zp} and commanded n_{zp}. The control law is described by

$$u = \begin{bmatrix} -f_{11} & -f_{12} & -f_{14} & -f_{15} \\ -f_{21} & -f_{22} & -f_{24} & -f_{25} \end{bmatrix} \begin{bmatrix} q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix} + \begin{bmatrix} -f_{13} \\ -f_{23} \end{bmatrix} \int e(t)\, dt \qquad (12)$$

where

$$e(t) = n_{zp} - (n_{zp})_{\text{command}}$$

The DLC responses to a $1g$ normal acceleration command are shown in Fig. 2. Observe that we achieve a large change in flight path angle with an insignificant deviation in angle of attack. Thus, the aircraft is climbing with almost no change in angle of attack.

Table 3
Direct Lift Control Summary

Desired Eigenvalues	Desired Eigenvectors	Feedback Gains
$\lambda_{1,2} = -5.6 \pm j4.2$ $\lambda_3 = -5.6$ $\lambda_4 = -19.0$ $\lambda_5 = -19.5$	mode decoupling $\begin{bmatrix} 1 \\ X \\ X \\ X \\ 0 \end{bmatrix} \begin{bmatrix} X \\ 1 \\ X \\ X \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ X \\ X \\ 1 \end{bmatrix} \begin{bmatrix} X \\ X \\ 1 \\ X \\ X \end{bmatrix} \begin{bmatrix} X \\ X \\ X \\ 1 \\ X \end{bmatrix} \begin{matrix} q \\ \alpha \\ \delta_e \\ \delta_f \\ n_{zI} \end{matrix}$ $q \quad \alpha \quad n_{zI} \quad \delta_e \quad \delta_f$ short period	$\begin{bmatrix} -0.722 & -6.70 & -0.220 & 0.468 & 0.0587 \\ -0.301 & 5.51 & 0.994 & 0.223 & 0.187 \end{bmatrix}$

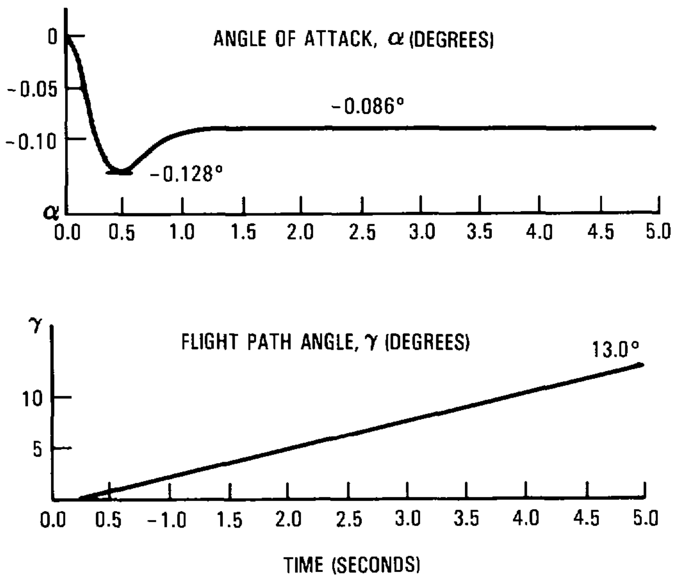

Fig. 2. DLC responses.

Lateral Multimode Control Law Design

The model of the flight propulsion control coupling (FPCC) lateral dynamics is described by eight state variables x, three control variables u, and five measurement variables y. The eight state variables are sideslip angle (β), bank angle (ϕ), roll rate (p), yaw rate (r), lateral directional flight path ($\gamma = \psi + \beta$), rudder deflection (δ_r), aileron deflection (δ_a), and canard deflection (δ_c).

The three control variables are rudder command (δ_{rc}), aileron command (δ_{ac}), and canard command (δ_{cc}). The five measurement variables are r, β, p, ϕ, γ. Because of space limitations, the detailed numerical results are not presented, but the general approach is outlined. Further details may be found in [9]. In what follows, we implement modes 4 and 5 using the same gain matrix.

Yaw Pointing (Mode 4) and Lateral Translation (Mode 5)

We desire to decouple the lateral directional flight path response from the bank angle, roll rate, and yaw rate responses. Thus, the desired eigenvectors are chosen such that the flight path mode will not affect the bank angle, roll rate, or yaw rate responses and so that the flight path response will consist only of the flight path mode. For design #1, which corresponds to the full feedback gain matrix, the achievable eigenvectors are very close to those that were desired. The control law gives achievable eigenvalues almost exactly those that were desired.

We compute the feedforward gains by using Eq. (4). The tracked variables are given by

$$y_t = [\psi, \gamma, \phi]^T$$

and the pilot commands are given by

$$u_c = [\psi_c, \gamma_c, \phi_c]^T$$

where

ψ_c = commanded heading

γ_c = commanded lateral directional flight path

$\phi_c = 0$

Since bank angle is commanded to be zero, we need not implement the gains that multiply ϕ_c. It is included in the numerical computations only to avoid the need for a pseudo-inversion.

The time histories for design #1 are not shown; however, the yaw pointing responses to a unit step heading command are such that $|\gamma| \leqq 0.0004$ degree and $|\phi| \leqq 0.0031$ degree. The lateral translation responses to a unit step lateral flight path command are such that $|\psi| \leqq 0.008$ degree and $|\phi| \leqq 0.004$ degree.

Design #2 is characterized by an additional specification that seven of the feedback gains be constrained to be zero. The zero elements are chosen based upon the physical insight that the roll autopilot should be able to operate somewhat independently of the lateral directional control system. Of course, some degradation will result, but the responses will still be acceptable from a practical point of view. Furthermore, by reducing the number of gains, we have increased the reliability of the control system.

The couplings that we wanted to be zero are now greater than they were for design #1. Also, the eigenvalues are not quite where we had specified that they should be. The responses for design #2 are shown in Figs. 3 and 4. Figure 3 shows the lateral pointing response to a unit step heading command. The change in flight path angle is less than 0.01 degree, and the change in bank angle is less than 0.25 degree. Figure 4 shows the lateral translation response to a unit step flight path command. The change in heading is less than 0.012 degree, and the change in bank angle is less than 0.14 degree. Both designs are considered to achieve acceptable performance.

Mode 6: Direct Sideforce Control

The objective in direct sideforce control (DSC) is to command lateral acceleration (or equivalently lateral directional flight path) without a change in sideslip angle. To achieve acceleration command following, we include integrated lateral acceleration in the state vector. Thus, for the DSC problem, we choose the state vector to be

$$x = [\beta, \phi, p, r, \gamma, \delta_r, \delta_a, \delta_c, n_{ypi}]^T$$

where

$$n_{ypi} = \begin{cases} \text{integral of lateral acceleration} \\ \text{at pilot's station} \end{cases}$$

The measurement vector is chosen to be

$$y = [r, \beta, p, \phi, n_{ypi}]^T$$

The desired eigenvectors were chosen such that the lateral acceleration mode would be decoupled from both the dutch roll mode and the roll mode. This choice yields the following:

$$\begin{bmatrix} X \\ 0 \\ 0 \\ 1 \\ X \\ X \\ X \\ X \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ X \\ X \\ X \\ X \\ X \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ X \\ 1 \\ 0 \\ X \\ X \\ X \\ X \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ X \\ 0 \\ X \\ X \\ X \\ X \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ X \\ X \\ X \\ X \\ 1 \end{bmatrix} \quad \begin{matrix} \beta \\ \phi \\ p \\ r \\ \psi + \beta \\ \delta_r \\ \delta_a \\ \delta_c \\ n_{ypi} \end{matrix}$$

dutch roll   roll mode   acc mode

mode decoupling

To obtain lateral acceleration command following, we feed back the integral of the error between measured n_{yp} and commanded n_{yp}. This approach is similar to that used for direct lift control.

Several designs are investigated. Design #1 is characterized by an output feedback

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