PGS v WESTERNGECO (IPR2014-00689)
WESTERNGECO Exhibit 2051
Part 2

Full derivation of streamer positioning equations

Equations of motion:

$$m \partial_t^2 x = \partial_s (T \cos\theta) + F_x$$
$$m \partial_t^2 y = \partial_s (T \sin\theta) + F_y$$

$C = \cos\theta$, $S = \sin\theta$.

$$x = x_0 + \int_0^s C \, ds'$$
$$y = y_0 + \int_0^s S \, ds'$$

$$\partial_t x = \dot{x}_0 - \int_0^s S \, \partial_t \theta \, ds'$$
$$\partial_t y = \dot{y}_0 + \int_0^s C \, \partial_t \theta \, ds'$$

$$\partial_t^2 x = \ddot{x}_0 - \int_0^s C \omega^2 + S \, \partial_t \omega \, ds'$$
$$\partial_t^2 y = \ddot{y}_0 - \int_0^s S \omega^2 - C \, \partial_t \omega \, ds'$$

$$m \left\{ \ddot{x}_0 - \int_0^s C \omega^2 + S \partial_t \omega \, ds' \right\} = F_x + \partial_s (TC)$$

$$m \left\{ \ddot{y}_0 - \int_0^s S \omega^2 - C \partial_t \omega \, ds' \right\} = F_y + \partial_s (TS)$$

Differentiating w.r.t. s.

$$-m (C\omega^2 + S \, \partial_t \omega) = \partial_s F_x + \partial_s^2 (TC)$$
$$-m (S\omega^2 - C \, \partial_t \omega) = \partial_s F_y + \partial_s^2 (TS)$$

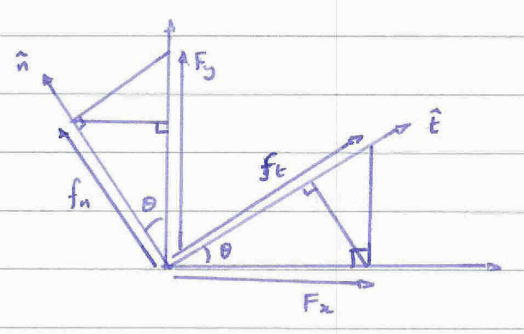
$$-m\omega^2 = c\,\partial_s F_x + s\,\partial_s F_y + c\,\partial_s^2(Tc) + s\,\partial_s^2(Ts)$$

$$\Rightarrow \quad c\,\partial_s^2(Tc) + s\,\partial_s^2(Ts) = -c\,\partial_s F_x - s\,\partial_s F_y - m\omega^2. \qquad \underline{1}$$

$$-m\,\partial_t \omega = s\,\partial_s F_x - c\,\partial_s F_y + s\,\partial_s^2(cT) - c\,\partial_s^2(sT).$$

$$m\,\partial_t \omega = c\,\partial_s F_y - s\,\partial_s F_x + c\,\partial_s^2(sT) - s\,\partial_s^2(cT). \qquad \underline{2}$$

$$\partial_t \theta = \omega$$



$$F_x = f_t c - f_n s$$
$$F_y = f_t s + f_n c.$$

$$RHS_1 = -c\,\partial_s(f_t c - f_n s) - s\,\partial_s(f_t s + f_n c) - m\omega^2$$
$$= -(c^2+s^2)\partial_s f_t + (cs\,\partial_s\theta f_t - sc\,\partial_s\theta f_t)$$
$$+ (cs\,\cancel{\partial_s f_n} - sc\,\cancel{\partial_s f_n}) + (c^2\partial_s\theta f_n + s^2\partial_s\theta f_n) - m\omega^2$$
$$= -\partial_s f_t + \partial_s\theta f_n - m\omega^2$$

$$RHS_2 = c\,\partial_s(f_t s + f_n c) - s\,\partial_s(f_t c - f_n s) + \ldots$$
$$= (cs\,\cancel{\partial_s f_t} - sc\,\cancel{\partial_s f_t}) + (c^2\partial_s\theta f_t + s^2\partial_s\theta f_t)$$
$$(c^2\partial_s f_n + s^2\partial_s f_n) + (-cs\,\cancel{\partial_s\theta f_n} + sc\,\cancel{\partial_s\theta f_n})$$
$$= \partial_s\theta f_t + \partial_s f_n + \ldots$$

Governing equations become:

$$\underline{\underline{C\,\partial_s^2(TC) + S\,\partial_s^2(TS) = -\partial_s f_t + \partial_s\theta\, f_n - m\omega^2}} \qquad \underline{3}$$

$$\underline{\underline{m\,\partial_t \omega = \partial_s\theta\, f_t + \partial_s f_n + C\,\partial_s^2(ST) - S\,\partial_s^2(CT)}} \qquad \underline{4}.$$

Expansions:

$$LHS_1 = C\partial_s\{ C\,\partial_s T - S\,\partial_s\theta\, T\} + S\partial_s\{ S\,\partial_s T + C\,\partial_s\theta\, T\}$$

$$= (C^2+S^2)\,\partial_s^2 T + (-CS\cancel{\partial_s\theta}\,\partial_s T + SC\cancel{\partial_s\theta}\,\partial_s T)$$

$$(-\cancel{S}C + \cancel{S}C)\,\partial_s\theta\,\partial_s T + (-SC\cancel{\partial_s^2}\theta\, T + SC\cancel{\partial_s^2}\theta\, T)$$

$$+ (-C^2(\partial_s\theta)^2 T - S^2(\partial_s\theta)^2 T)$$

$$= \partial_s^2 T - (\partial_s\theta)^2\, T$$

$$RHS_2\,(\text{last terms}) = C\,\partial_s\{S\,\partial_s T + C\,\partial_s\theta\, T\} - S\,\partial_s\{C\,\partial_s T - S\,\partial_s\theta\, T\}$$

$$= (C\cancel{S} - \cancel{S}C)\,\partial_s^2 T + (C^2\,\partial_s\theta\,\partial_s T + S^2\,\partial_s\theta\,\partial_s T)$$

$$+ (C^2+S^2)\,\partial_s\theta\,\partial_s T + (C^2+S^2)\,\partial_s^2\theta\, T$$

$$+ (-CS\cancel{(\partial_s\theta)^2}\, T + SC\cancel{(\partial_s\theta)^2}\, T)$$

$$= 2\,\partial_s\theta\,\partial_s T + \partial_s^2\theta\, T$$

Governing equations become:

$$\underline{\underline{\partial_s^2 T - (\partial_s\theta)^2\, T = -\partial_s f_t + \partial_s\theta\, f_n - m\,\omega^2}} \qquad \underline{5}.$$

$$\underline{\underline{m\,\partial_t\,\omega = \partial_s\theta\, f_t + \partial_s f_n + 2\,\partial_s\theta\,\partial_s T + \partial_s^2\theta\, T}} \qquad \underline{6}$$

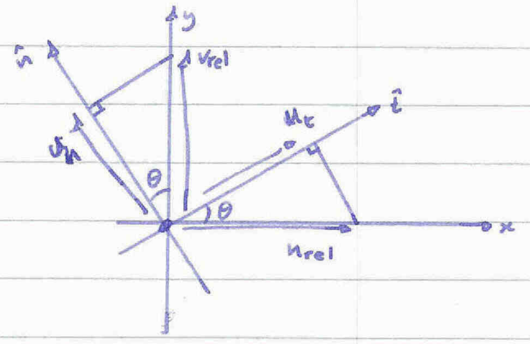
Forces on streamer:

$$f_t = 2\rho \pi a C_t \; u_t |u_t|$$

$$f_n = 2\rho a C_n \; u_n |u_n|.$$

$$u_t = (U - \partial_t x) c + (V - \partial_t y) s$$

$$u_n = -(U - \partial_t x) s + (V - \partial_t y) c$$



Boundary conditions $\quad T_{s=\ell} = 0 \qquad \partial_s \theta |_{s=\ell} = 0$

$$m \ddot{x}_0 = F_x|_{s=0} + \partial_s (Tc)|_{s=0}$$

$$m \ddot{y}_0 = F_y|_{s=0} + \partial_s (Ts)|_{s=0}.$$

Non-dimensionalisation.

$$\tilde{x} = x/\ell \qquad \tilde{y} = y/\ell \qquad \tilde{s} = s/\ell$$

$$\tilde{u}_t = u_t/u_b \qquad \tilde{u}_n = u_n/u_b \qquad \tilde{U} = U/u_b \qquad \tilde{V} = V/u_b$$

$$\tilde{t} = \frac{t u_b}{\ell}. \qquad \tilde{\omega} = \frac{\omega \ell}{u_b}.$$

(alternative non-dimensionalisation)

Forces:

$$\tilde{f}_t = \frac{f_t}{2a\rho u_b^2 \pi C_t} = \frac{\cancel{2}\cancel{\rho}\cancel{\pi}\cancel{a}\cancel{C_t}\cancel{u_b^2}\ \tilde{u}_t|\tilde{u}_t|}{\cancel{2}\cancel{a}\cancel{\rho}\cancel{u_b^2}\cancel{\pi}\cancel{C_t}} = \tilde{u}_t|\tilde{u}_t|.$$

$$\tilde{f}_n = \frac{f_n}{2a\rho u_b^2 C_n} = \frac{\cancel{2}\cancel{\rho}\cancel{a}\cancel{C_n}\cancel{u_b^2}\ \tilde{u}_n|\tilde{u}_n|}{\cancel{2}\cancel{a}\cancel{\rho}\cancel{u_b^2}\cancel{C_n}} = \tilde{u}_n|\tilde{u}_n|.$$

$$\tilde{T} = \frac{T}{2\pi a\rho u_b^2 \ell C_t}$$

Governing equations:

$$\frac{C_t}{\ell^{\cancel{2}}} 2\pi a\rho u_b^2 \cancel{\ell} \left\{ \partial_\xi^2 \tilde{T} - (\partial_\xi\theta)^2 \tilde{T} \right\} = -\frac{1}{\ell}.\, 2a\rho u_b^2 \pi C_t\, \partial_\xi \tilde{f}_t + \frac{1}{\ell}.\, 2a\rho u_b^2 C_n\, \partial_\xi\theta\, \tilde{f}_n$$

$$- \pi a^2\rho \frac{u_b^2}{\ell^2} \tilde{\omega}^2$$

$$\partial_\xi^2 \tilde{T} - (\partial_\xi\theta)^2 \tilde{T} = -\partial_\xi \tilde{f}_t + \frac{C_n}{\pi C_t} \partial_\xi\theta\, \tilde{f}_n - \frac{a}{2C_t \ell} \omega^2$$

Let $\varepsilon = \dfrac{a}{2C_t\ell}$ and $\lambda = \dfrac{C_n}{\pi C_t}$

$$\underline{\underline{\partial_\xi^2 \tilde{T} - (\partial_\xi\theta)^2 \tilde{T} = -\partial_\xi \tilde{f}_t + \lambda\,\partial_\xi\theta\, \tilde{f}_n - \varepsilon\,\omega^2}}$$

$$\pi a^2\rho \frac{u_b^2}{\ell^2} \partial_\eta \tilde{\omega} = \frac{1}{\ell}.\, 2a\rho u_b^2 \pi C_t\, \partial_\xi\theta\, \tilde{f}_t + \frac{1}{\ell}.\, 2a\rho u_b^2 C_n\, \partial_\xi \tilde{f}_n +$$

$$\frac{1}{\ell^{\cancel{2}}} 2\pi a\rho u_b^2 \cancel{\ell} C_t\ 2.\partial_\xi\theta\,\partial_\xi \tilde{T} + \frac{1}{\ell^{\cancel{2}}} 2\pi a\rho u_b^2 \cancel{\ell} C_t\ \partial_\xi^2\theta\, \tilde{T}$$

$$\frac{\pi a^{\cancel{2}}}{\ell^{\cancel{2}}}.\frac{\cancel{\ell}}{2\pi \cancel{a} C_t}\,\partial_t \tilde{\omega} = \partial_\xi\theta\,\tilde{f}_t + \frac{C_n}{\pi C_t}\partial_\xi \tilde{f}_n + 2\,\partial_\xi\theta\,\partial_\xi\tilde{T} + \partial_\xi^2\theta\,\tilde{T}$$

$$\epsilon\,\partial_{\tilde{t}}\tilde{\omega} = \partial_{\tilde{s}}\theta\,\tilde{f}_t + \lambda\,\partial_{\tilde{s}}\tilde{f}_n + 2\,\partial_{\tilde{s}}\theta\,\partial_{\tilde{s}}\tilde{T} + \partial^2_{\tilde{s}}\theta\,\tilde{T}.$$

$$\partial_{\tilde{t}}\theta = \tilde{\omega}$$

Non-dimensionalisation of boundary conditions.

$$\tilde{T}|_{s=1} = 0 \qquad \partial_{\tilde{s}}\theta\,|_{s=1} = 0.$$

$$\pi a^2 \rho \frac{\cancel{U_0^2}}{\ell}\,\ddot{\tilde{x}}_0 = 2a\rho \cancel{U_0^2}\pi C_t\,\tilde{f}_t\,c - 2a\rho \cancel{U_0^2} C_n\,\tilde{f}_n\,s + \frac{1}{\cancel{\ell}}\,2\pi a\rho \cancel{U_0^2}\,\cancel{\ell} C_t\,\partial_{\tilde{s}}(\tilde{T}c)$$

$$\epsilon\,\ddot{\tilde{x}}_0 = \tilde{f}_t\,c - \lambda\,\tilde{f}_n\,s + \partial_{\tilde{s}}(\tilde{T}c)$$

$$\pi a^2 \rho \frac{U_0^2}{\ell}\,\ddot{\tilde{y}}_0 = 2a\rho U_0^2\pi C_t\,\tilde{f}_t\,s + 2a\rho U_0^2 C_n\,\tilde{f}_n\,c + \frac{2\pi a\rho U_0^2\,\cancel{\ell} C_t}{\cancel{\ell}}\,\partial_{\tilde{s}}(\tilde{T}s)$$

$$\epsilon\,\ddot{\tilde{y}}_0 = \tilde{f}_t\,s + \lambda\tilde{f}_n\,c + \partial_{\tilde{s}}(\tilde{T}s)$$

Combining:

$$\epsilon a_t = \epsilon(\ddot{x}_0 c + \ddot{y}_0 s) = \tilde{f}_t + \partial_{\tilde{s}}\tilde{T}$$

$$\partial_{\tilde{s}}\tilde{T} = \epsilon a_t - \tilde{f}_t$$

$$\epsilon a_n = \epsilon(-s\ddot{x}_0 + c\ddot{y}_0) = \lambda\tilde{f}_n + \tilde{T}\,\partial_{\tilde{s}}\theta.$$

$$\tilde{T}\,\partial_{\tilde{s}}\theta = \epsilon a_n - \lambda\tilde{f}_n$$

Numerical solution : dropping ~'s

$$\partial_s^2 T - (\partial_s \theta)^2 T = p$$

$$\frac{T_{i+1}^{k+1} - 2T_i^{k+1} + T_{i-1}^{k+1}}{\Delta s^2} - (\partial_s \theta)^2\Big|_i^k T_i^{k+1} = p_i^k$$

$$T_{i+1}^{k+1} - \left(2 + \Delta s^2 (\partial_s \theta)^2\right)\Big|_i^k T_i^{k+1} + T_{i-1}^{k+1} = \Delta s^2 p_i^k$$

Boundary Conditions.

$$T_n^{k+1} = 0$$

$$T_1^{k+1} - T_0^{k+1} = \Delta s\, (\epsilon a_t - f_t)\Big|_0^k$$

Let $g = 2\partial_s T + f_t$. $\qquad Q = \lambda \partial_s f_n$

$$\epsilon \frac{\omega^{k+1} - \omega^k}{\Delta t} - \frac{1}{2} T_i^k \frac{\theta_{i+1}^{k+1} - 2\theta_i^{k+1} + \theta_{i-1}^{k+1}}{\Delta s^2} - \frac{1}{2} g_i^k \frac{\theta_{i+1}^{k+1} - \theta_{i-1}^{k+1}}{2\Delta s} =$$

$$Q_i^k + \frac{1}{2} T_i^k (\partial_s^2 \theta)_i^k + \frac{1}{2} g_i^k (\partial_s \theta)_i^k$$

$$\epsilon\omega^{k+1} - \theta_{i+1}^{k+1}\Delta t\left(\frac{\frac{1}{2}T_i^k}{\Delta s^2} + \frac{g_i^k}{4\Delta s}\right) + \frac{\Delta t\, \theta_i^{k+1} T_i^k}{\Delta s^2} - \theta_{i-1}^{k+1}\Delta t\left(\frac{T_i^k}{2\Delta s^2} - \frac{g_i^k}{4\Delta s}\right)$$

$$= \epsilon\omega^k + \Delta t\left\{ Q + \tfrac{1}{2} T_i^k (\partial_s^2 \theta)_i^k + \tfrac{1}{2} g_i^k (\partial_s \theta)_i \right\}$$

Let $A_i = -\frac{\Delta t}{2\Delta s^2}\left(T_i^k + \frac{g_i^k \Delta s}{2}\right)$ $\qquad B_i = \frac{\Delta t\, T_i^k}{\Delta s^2}$

$$C_i = -\frac{\Delta t}{2\Delta s^2}\left(T_i^k - \frac{g_i^k \Delta s}{2}\right)$$

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Let

$$R_i = \epsilon \omega_i^k + \Delta t \left\{ Q + \frac{1}{2} T_i^k (\partial_s^2 \theta)_i^k + \frac{1}{2} g_i^k (\partial_s \theta)_i \right\}$$

Then

$$\epsilon \omega^{k+1} + A_i \theta_{i+1}^{k+1} + B_i \theta_i^{k+1} + C_i \theta_{i-1}^{k+1} = R_i$$

And for the θ equation

$$\frac{\theta_i^{k+1} - \theta_i^k}{\Delta t} = \frac{1}{2}(\omega_i^{k+1} + \omega_i^k)$$

$$\theta_i^{k+1} = \frac{\Delta t}{2} \omega_i^{k+1} + \frac{\Delta t}{2} \omega_i^k + \theta_i^k$$

$$\theta_i^{k+1} = \frac{\Delta t}{2} \omega_i^{k+1} + h_i^k$$

where $h_i^k = \frac{\Delta t}{2} \omega_i^k + \theta_i^k$.

Substituting into governing:

$$\epsilon \omega_i^{k+1} + A_i \left(\frac{\Delta t}{2} \omega_{i+1}^{k+1} + h_{i+1}^k\right) + B_i \left(\frac{\Delta t}{2} \omega_i^{k+1} + h_i^k\right) + C_i \left(\frac{\Delta t}{2} \omega_{i-1}^{k+1} + h_{i-1}^k\right) = R_i$$

$$A_i \frac{\Delta t}{2} \omega_{i+1}^{k+1} + \left(B_i \frac{\Delta t}{2} + \epsilon\right) \omega_i^{k+1} + C_i \frac{\Delta t}{2} \omega_{i-1}^{k+1} = R_i - A_i h_{i+1}^k - B_i h_i^k - C_i h_{i-1}^k$$

$$A_i \frac{\Delta t}{2} \omega_{i+1}^{k+1} + \left(\frac{B_i \Delta t}{2} + \epsilon\right) \omega_i^{k+1} + \frac{C_i \Delta t}{2} \omega_{i-1}^{k+1} = R_i - A_i h_{i+1}^k - B_i h_i^k - C_i h_{i-1}^k$$

Boundary conditions - second equation.

$$\partial_s \theta|_{s=1} = 0.$$

$$\theta_n^{k+1} - \theta_{n-1}^{k+1} = 0.$$

$$\frac{\Delta t}{2} w_n^{k+1} + h_n^k - \frac{\Delta t}{2} w_{n-1}^{k+1} - h_{n-1}^k = 0.$$

$$w_n^{k+1} - w_{n-1}^{k+1} = -\frac{2}{\Delta t}(h_n^k - h_{n-1}^k).$$

$$T \partial_s \theta|_{s=0} = (\varepsilon a_n - \lambda f_n)_{s=0}.$$

$$T_0^k \frac{\theta_1^{k+1} - \theta_0^{k+1}}{\Delta s} = (\varepsilon a_n - \lambda f_n)|_{s=0}.$$

$$\theta_1^{k+1} - \theta_0^{k+1} = \frac{\Delta s}{T_0^k}(\varepsilon a_n - \lambda f_n)_{s=0}.$$

$$\frac{\Delta t}{2}(w_1^{k+1} - w_0^{k+1}) = -(h_1^k - h_0^k) + \frac{\Delta s}{T_0^k}(\varepsilon a_n - \lambda f_n)|_{s=0}.$$

$$w_1^{k+1} - w_0^{k+1} = -\frac{2}{\Delta t}(h_1 - h_0) + \frac{2\Delta s}{\Delta t}\left.\frac{(\varepsilon a_n - \lambda f_n)}{T}\right|_{s=0}$$

(A)

Block tri-diagonal solution of linearised equations

Governing equations :- (non-dimensional)

$$\partial_s T + u|u| = 0$$

$$T\partial_s \theta + \lambda v|v| = 0$$

$$\partial_s u - v\,\partial_s \theta = 0$$

$$\partial_s v + u\,\partial_s \theta = \partial_t \theta$$

Re-order for consistency with T, θ, u, v row

$$\partial_s T + u|u| = 0 \qquad T$$

$$\partial_s v + u\,\partial_s \theta = \partial_t \theta \qquad \theta$$

$$\partial_s u - v\,\partial_s \theta = 0 \qquad u$$

$$T\partial_s \theta + \lambda v|v| = 0 \qquad v$$

Linearisation:

$$T = T_0 + T_1 \qquad \theta = \theta_0 + \theta_1 \qquad u = u_0 + u_1 \qquad v = v_0 + v_1$$

$|u_0| >> |u_1| \Rightarrow \operatorname{sgn}(u_0 + u_1) = \operatorname{sgn} u_0$

$$\partial_s T_0 + \partial_s T_1 + u_0^2 \operatorname{sgn}(u_0) + 2u_0 u_1 \operatorname{sgn}(u_0) = 0$$

$$\underline{\underline{\partial_s T_1 + 2|u_0| u_1 = -\partial_s T_0 - u_0|u_0| = r_1}}$$

Implicit time stepping

$$\partial_s v + u\,\partial_s \theta = \frac{\theta - \theta^0}{\Delta t}$$

$$\partial_s v_0 + \partial_s v_1 + u_0 \partial_s \theta_0 + u_0 \partial_s \theta_1 + u_1 \partial_s \theta_0 = \frac{\theta_0 + \theta_1 - \theta^0}{\Delta t}$$

$$\partial_s v_1 + u_0 \partial_s \theta_1 + \partial_s \theta_0 \, u_1 - \frac{\theta_1}{\Delta t} = \frac{\theta_0 - \theta^0}{\Delta t} - \partial_s v_0 - u_0 \partial_s \theta_0 = r_2$$

$$\partial_s u_0 + \partial_s u_1 - v_0 \partial_s \theta_0 - v_1 \partial_s \theta_0 - \theta_0 \partial_s \theta_1 = 0$$

$$\partial_s u_1 - \partial_s \theta_0 \, v_1 - \theta_0 \partial_s \theta_1 = -\partial_s u_0 + v_0 \partial_s \theta_0 = r_3$$

$$T_0 \partial_s \theta_0 + T_1 \partial_s \theta_0 + T_0 \partial_s \theta_1 + \lambda v_0 |v_0| + 2|v_0| v_1 \lambda = 0$$

$$\partial_s \theta_0 \, T_1 + T_0 \partial_s \theta_1 + 2\lambda |v_0| v_1 = -T_0 \partial_s \theta_0 - \lambda v_0 |v_0| = r_4$$

Block tridiagonal vector $(T, \theta, u, v)^T$

Interlaced on f.d. grid $\underline{w} = (T_1, \theta_1, u_1, v_1, \ldots T_n, \theta_n, u_n, v_n)^T$

b.c.'s $T_n = T_{tail}$ $u_1 = u_0 c + v_0 s$ (or similarly) $v_1 = \ldots$ similarly.

(c)

Given T_0, θ_0, u_0 and v_0 Calculate updates.
also need u_b, v_b, λ, θ^0, Δt, Δs, n.
Grid is $i=1..n$ $\Delta s = 1/(n-1)$

Calculate residuals
Calculate body block - tri
Calc. end pts
Impose bc's
Solve
update values

Calculate residuals Given T_0, θ_0, u_0, v_0, λ, θ^0, Δt, Δs, n
resid [1..n][1..m] $m=4$
allocate dTds dθds du0ds dv0ds
deriv(T, ds, n, dTds); deriv(θ, ds, n, dθds), deriv(u0, ds, n, du0ds);
deriv(v0, ds, n, dv0ds)
for (i=1; i<=n; i++){
resid[i][1] = $-dTds_i - u_{0_i}|u_{0_i}|$
resid[i][2] = $\frac{1}{\Delta t}(\theta_{0_i} - \theta^0_i) - dv_0ds_i - u_{0_i}\, d\theta_0 ds_i$
resid[i][3] = $-du_0ds_i + v_{0_i}\, d\theta_0 ds_i$
resid[i][4] = $-T_{0_i}\, d\theta_0 ds_i - \lambda v_{0_i}|v_{0_i}|$
}
deallocate dv0ds, du0ds, dθds, dTds
}

```
    bp[4][1] = dΘ₀ds;
    cp[4][2] = T₀i /(2ds)
    ap[4][2] = −T₀i /(2ds)
    bp[4][4] = 2λ|v₀|;
}
```

and forward + backward at the ends.

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Matrix form.

$$\begin{pmatrix} \partial_s & 0 & 2|u_0| & 0 \\ 0 & u_0\partial_s - \frac{1}{\Delta t} & \partial_s\theta_0 & \partial_s \\ 0 & -\theta_0\partial_s & 0 & -\partial_s\theta_0 \\ \partial_s\theta_0 & T_0\partial_s & 0 & 2\lambda|v_0| \end{pmatrix} \begin{pmatrix} T_1 \\ \theta_1 \\ u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix}$$

Calculate block body T_0, θ_0, u_0, v_0, λ, ds, dt

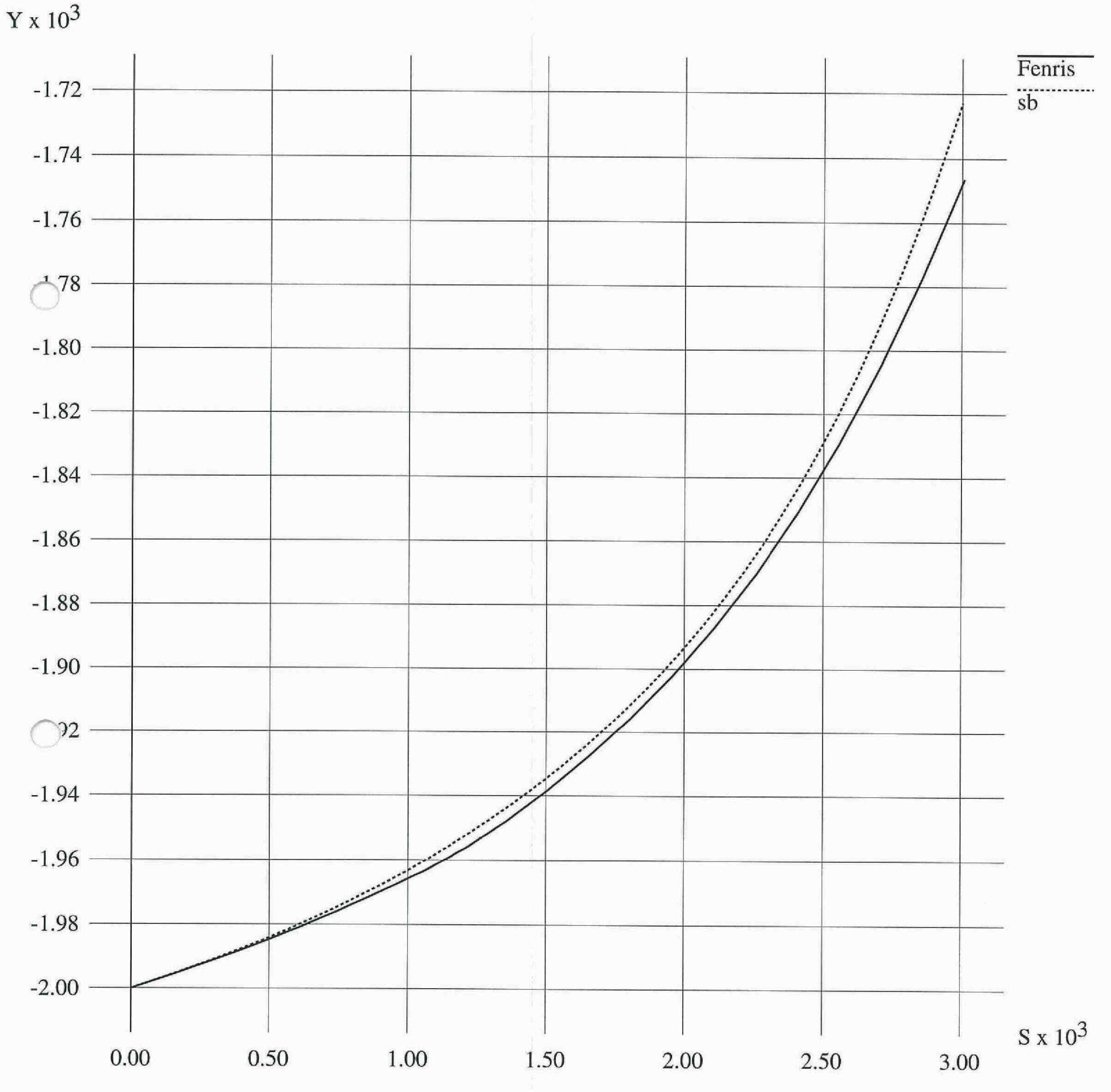
a[1..n][1..4][1..4]   b[1..n][1..4][1..4]

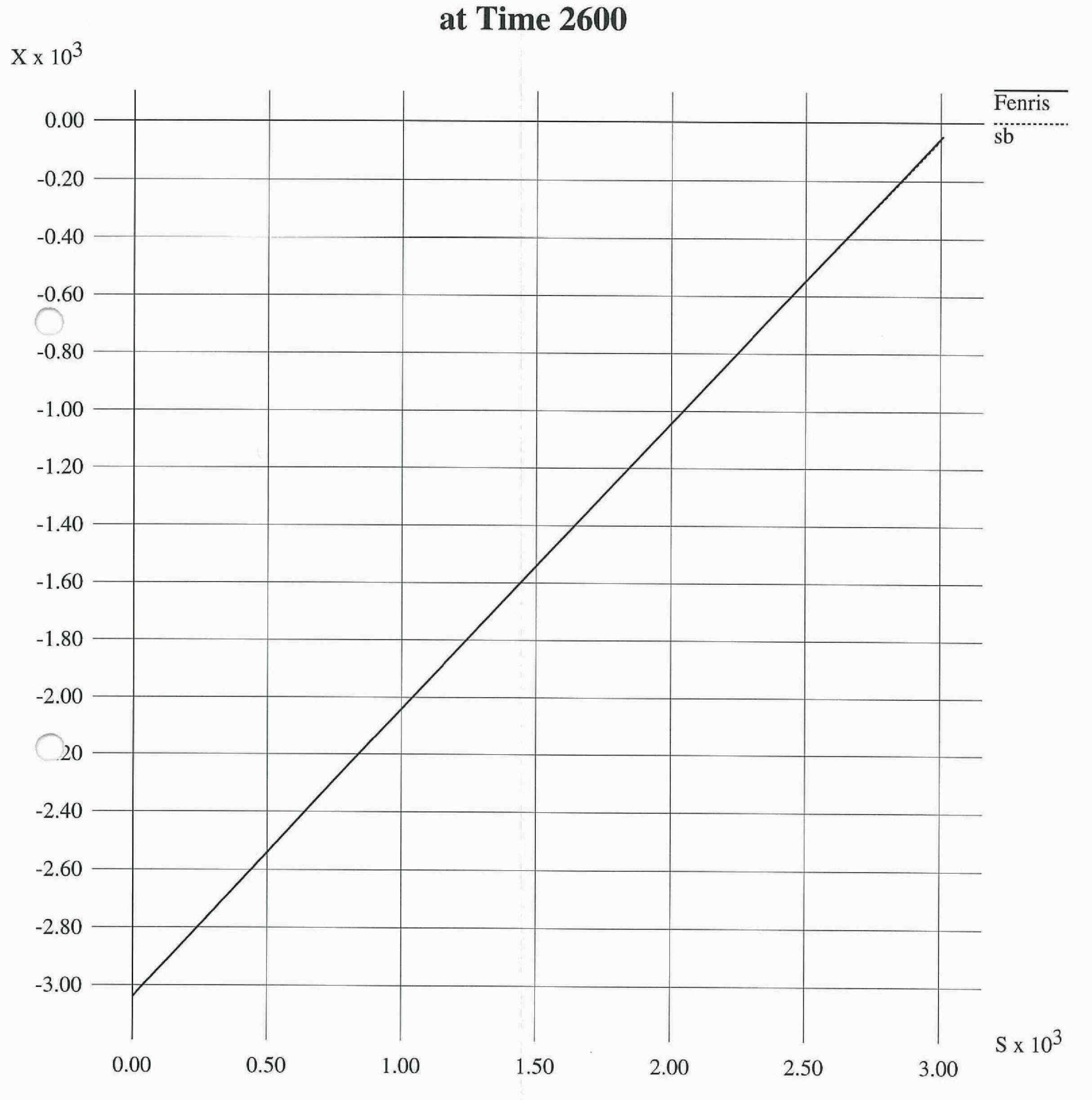
c[1..n][1..4][1..4]

for ($i \in 1..n$, $j \in 1..m$, $k \in 1..m$) $a_{ijk} = b_{ijk} = c_{ijk} = 0$.

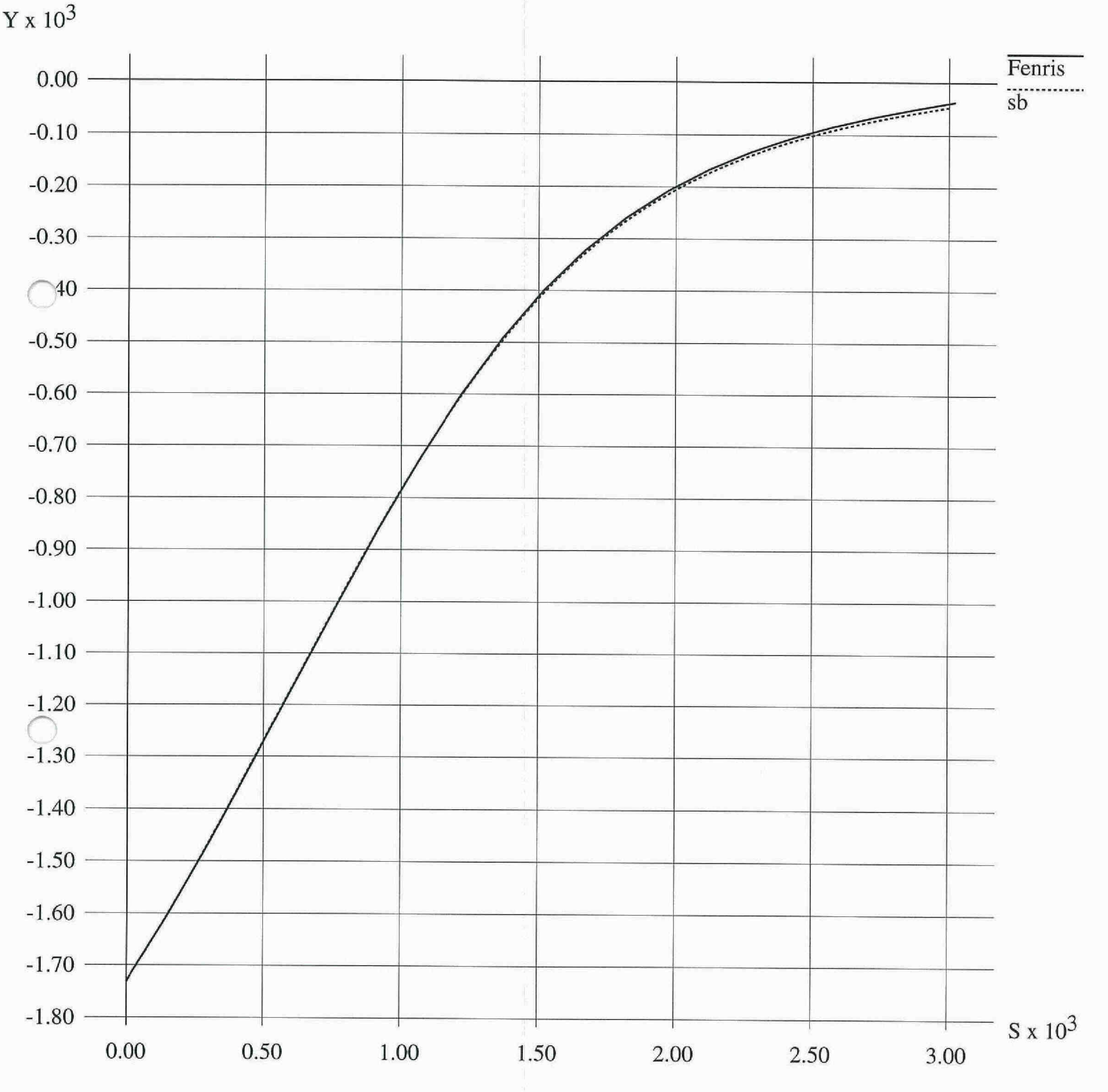
```
deriv(θ, ds, n, dθ0 ds)
for (i=2 ; i<n ; i+1) {
    ap = a[i] ; bp = b[i] ; cp = c[i];
    cp[1][1] = 1/(ds*2)
    ap[1][1] = -1/(ds*2)
    bp[1][3] = 2|u0|i
    cp[2][2] = u0i /(ds*2)
    bp[2][2] = -1/Δt
    ap[2][2] = -u0i /(ds*2)
    bp[2][3] = dθ0 dsi
    cp[2][4] = 1/(2ds)
    ap[2][4] = 1/(2ds)
    cp[3][2] = - θ0i /(2ds)
    ap[3][2] = +θ0i /(2ds)
    bp[3][4] = - dθ0 ds[i]
```

at Time 2600


Y x 10^3
-1.72
-1.74
-1.76
-1.78
-1.80
-1.82
-1.84
-1.86
-1.88
-1.90
-1.92
-1.94
-1.96
-1.98
-2.00
Fenris
sb
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3

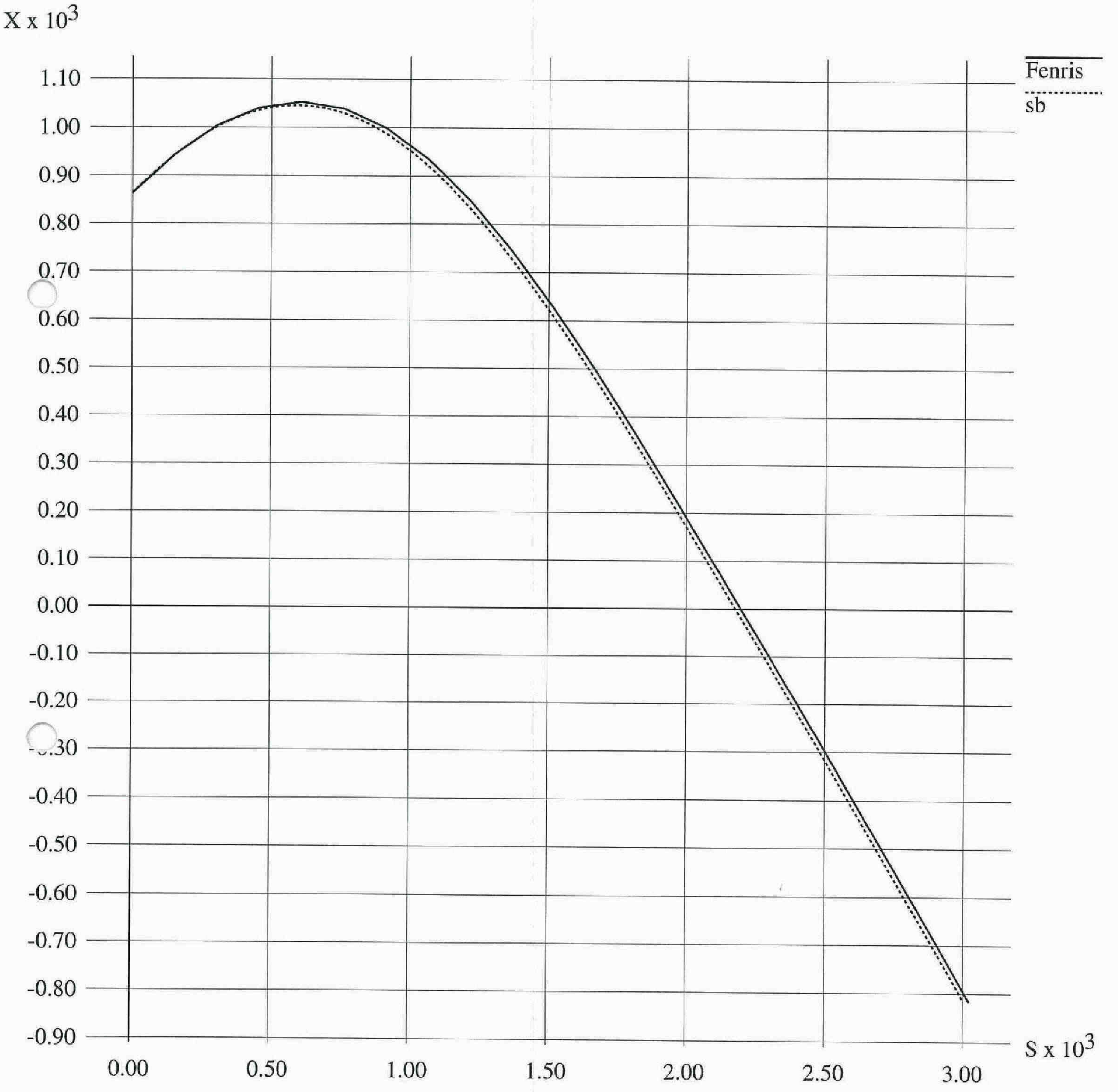

at Time 2600
X x 10^3
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
-1.20
-1.40
-1.60
-1.80
-2.00
-2.20
-2.40
-2.60
-2.80
-3.00
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3
Fenris
sb

at Time 1000



Y x 10^3
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
-1.10
-1.20
-1.30
-1.40
-1.50
-1.60
-1.70
-1.80
Fenris
sb
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3

at Time 1000


X x 10^3
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3
Fenris
sb

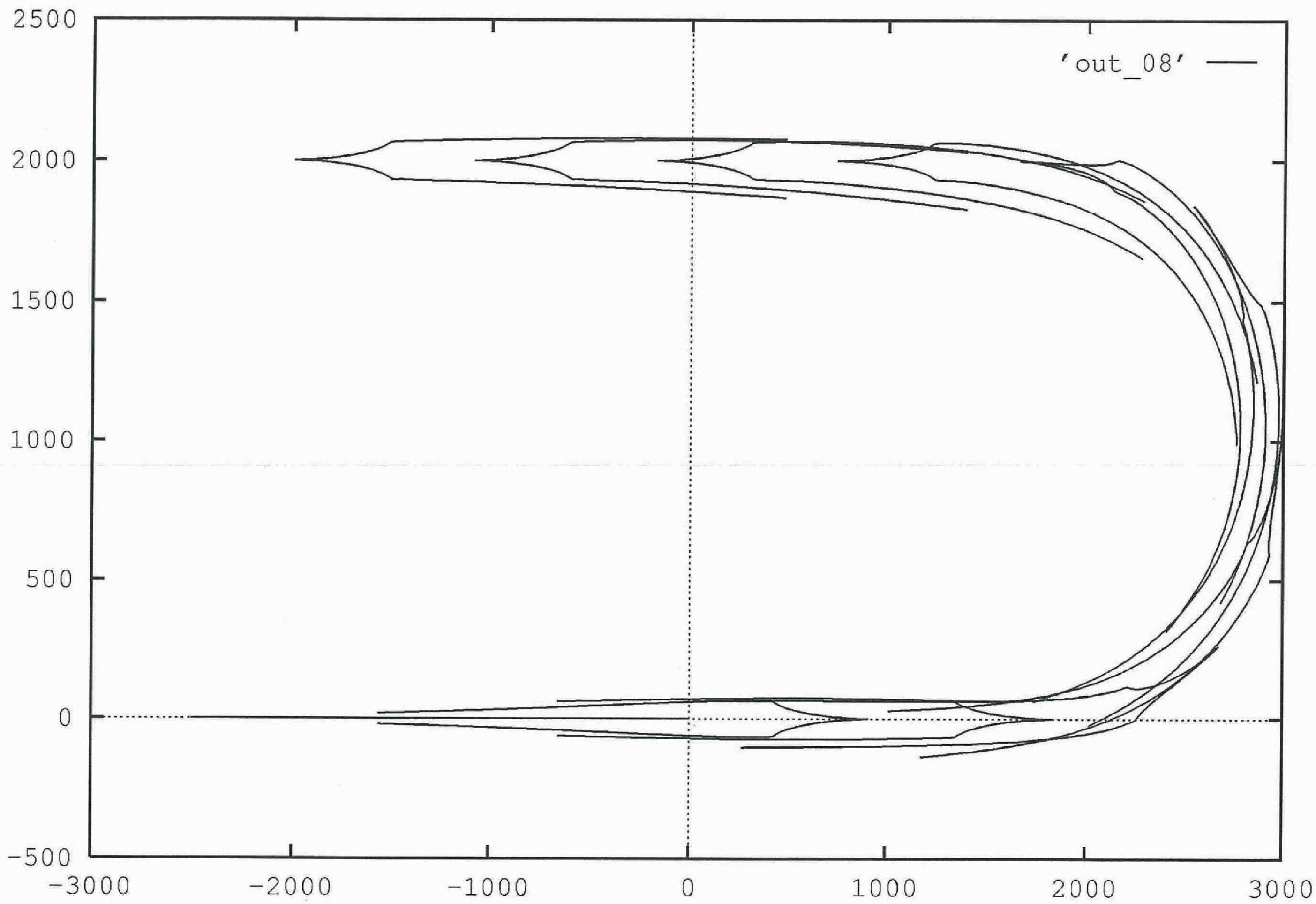
ANNEX B : HMPE YARN CREEP DATA @ 20°C

Yarn Type	At 15% BL: % Strain Increment: to 1 min	At 15% BL: % Strain Increment: 1 min to 1 day	At 15% BL: % Strain Increment: 1 day to 10 days	At 15% BL: % Strain Increment: 10 to 100 days	days to break	At 30% BL: % Strain Increment: to 1 min	At 30% BL: % Strain Increment: 1 min to 1 day	At 30% BL: % Strain Increment: 1 day to 10 days	At 30% BL: % Strain Increment: 10 to 100 days	days to break
Spectra 900	0.8	1.1	1.6	12.9	180	1.5	3.3	break		4
Spectra 1000	0.5	0.6	1.0	6.2	330	1.0	1.4	8.0		30
Dyneema SK60	0.8	0.4	0.1	0.4	>350	1.4	0.7	0.9		120

SK75

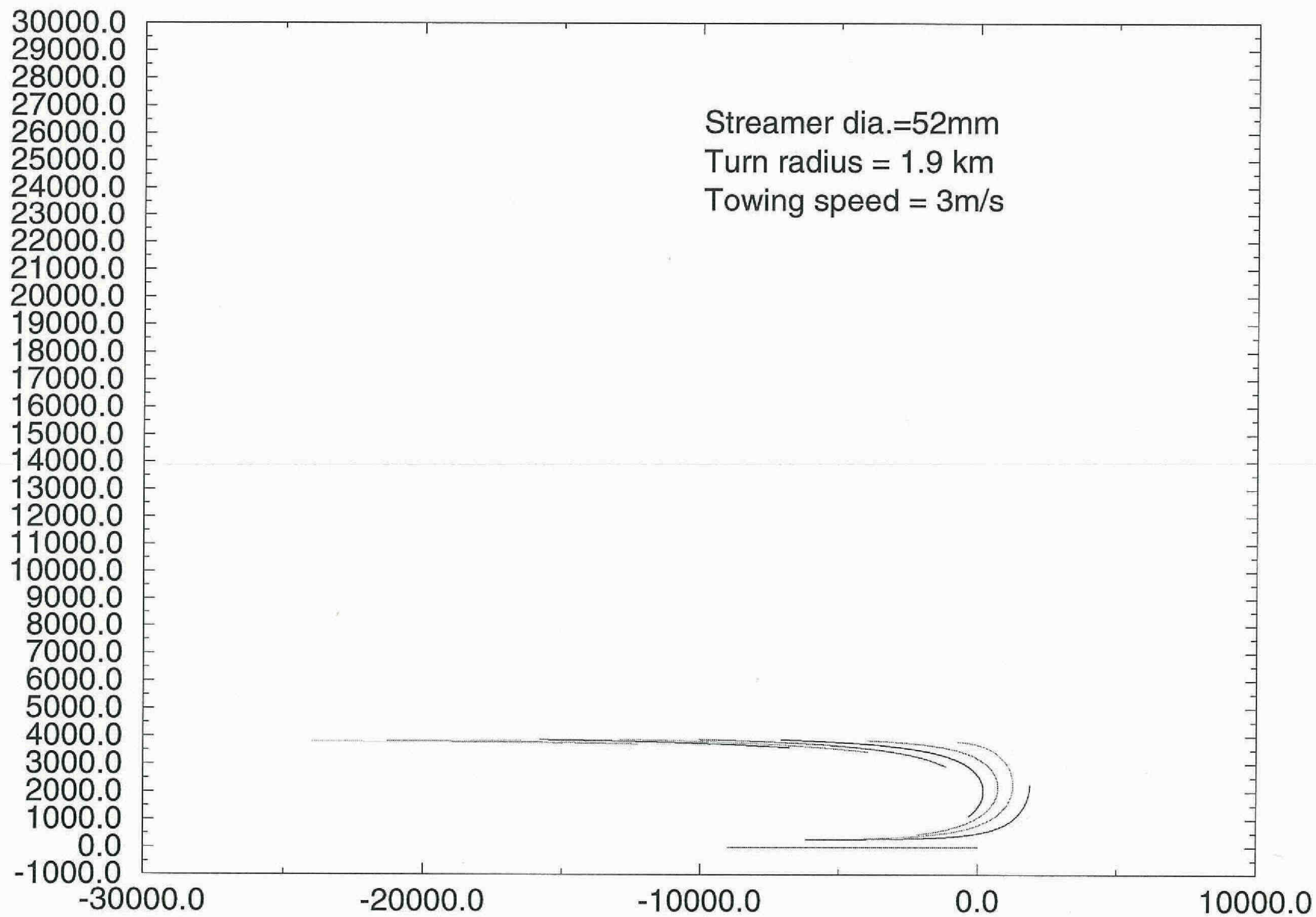
polyethylene.

SK60
65
SK.75



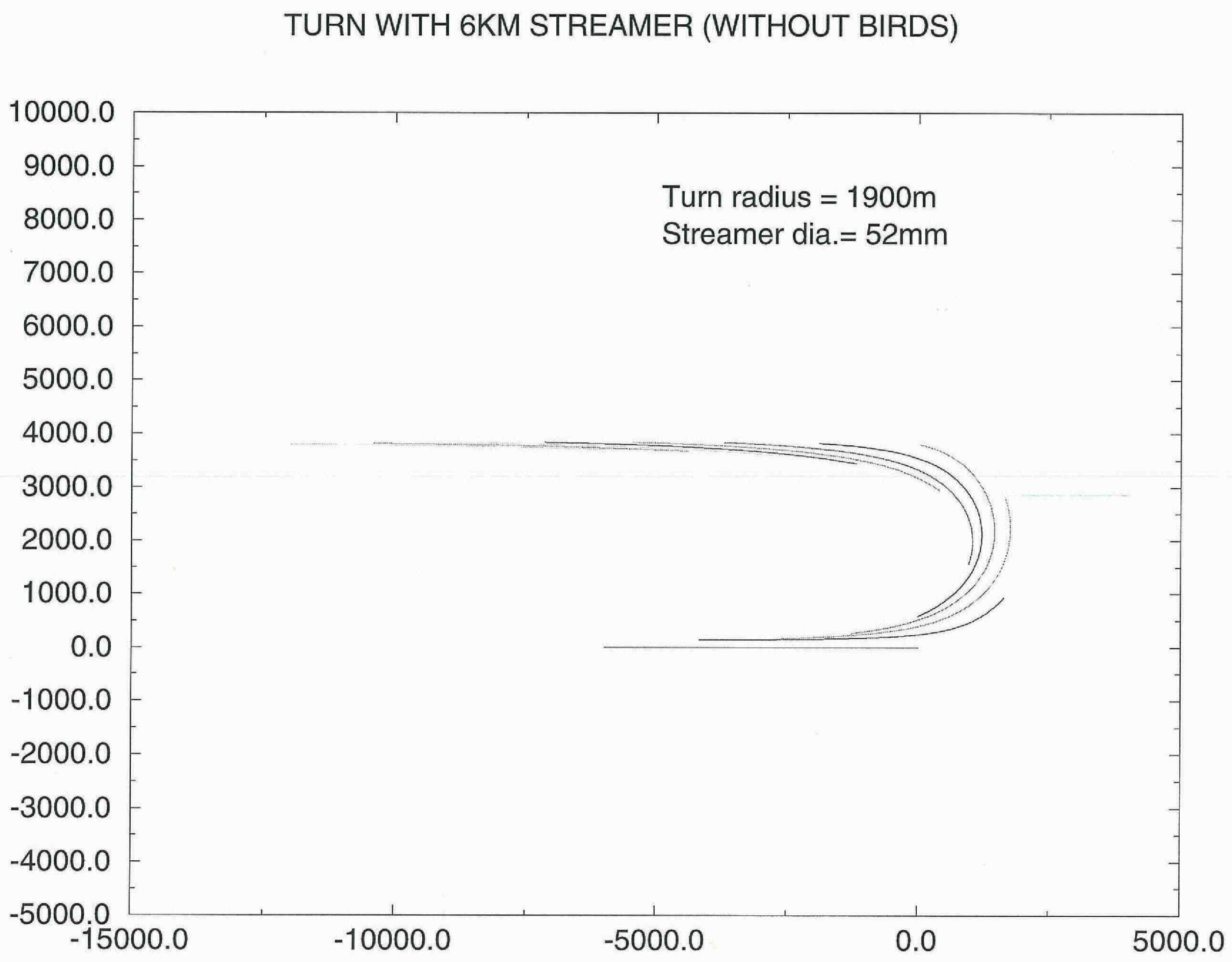
2500
2000
1500
1000
500
0
-500
-3000
-2000
-1000
0
1000
2000
3000
'out_08'

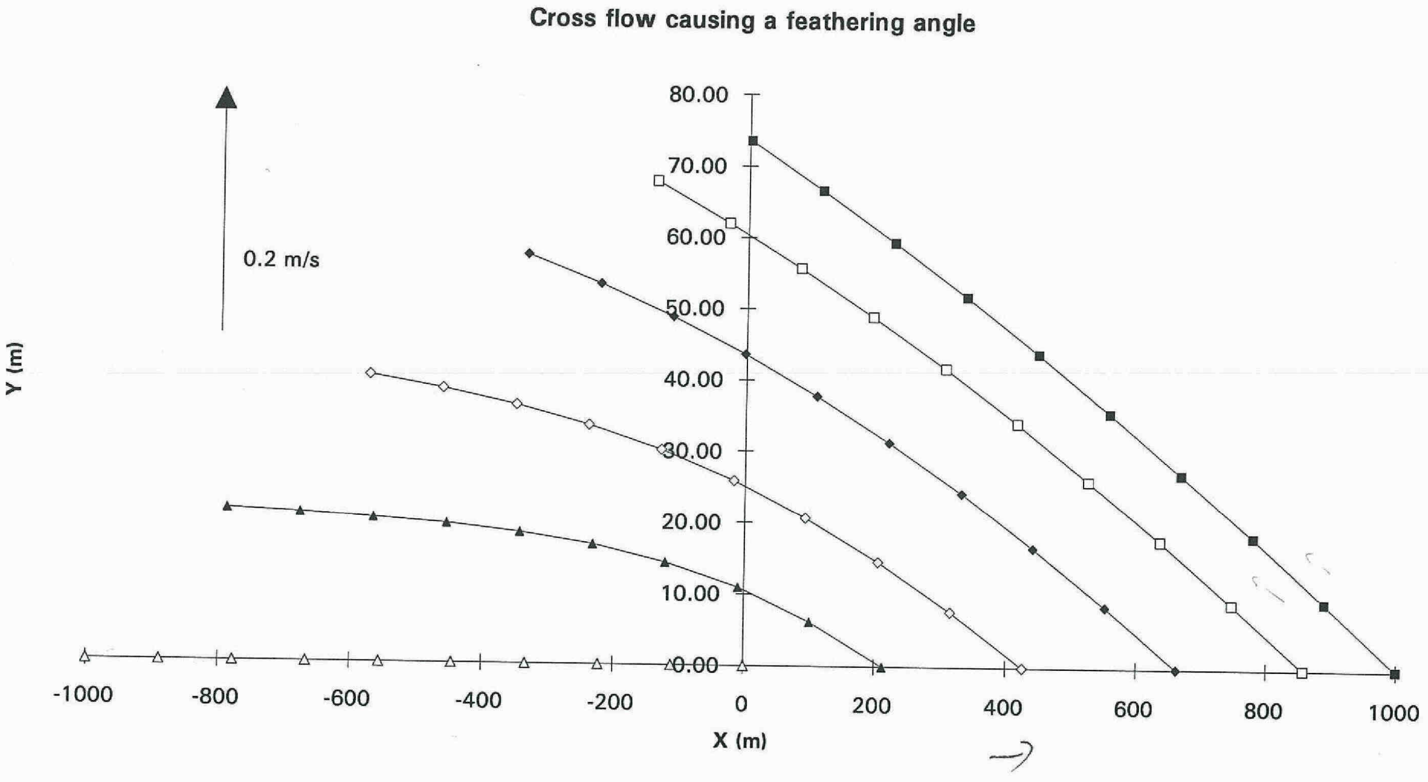
TURN WITH 9KM STREAMER (WITHOUT BIRDS)



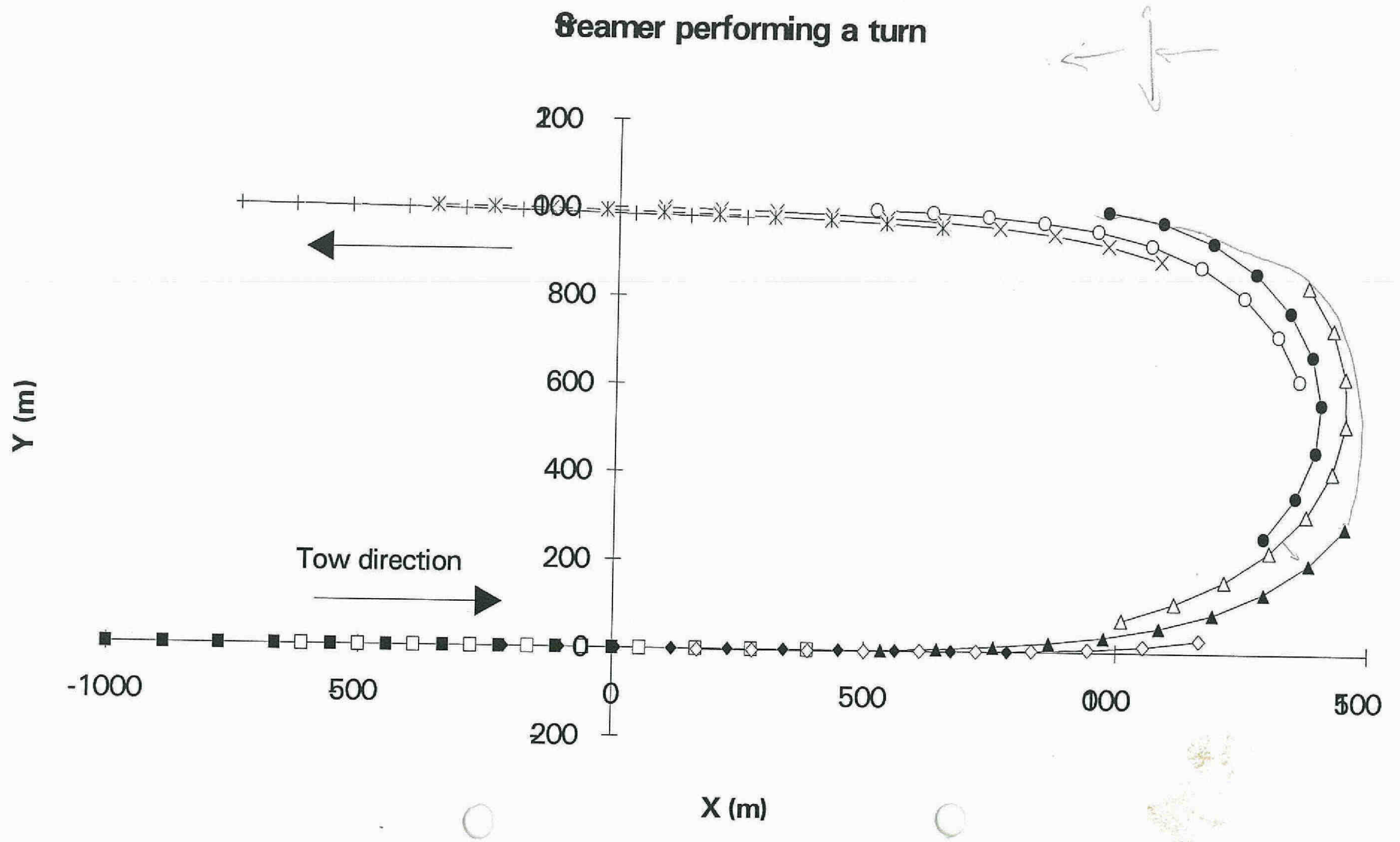
24th 2 wks GR
25th 3 days ØH
18th 5 days. KØ.

7th aug. 2 – 3 wks.


TURN WITH 6KM STREAMER (WITHOUT BIRDS)
Turn radius = 1900m
Streamer dia.= 52mm
10000.0
9000.0
8000.0
7000.0
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
-1000.0
-2000.0
-3000.0
-4000.0
-5000.0
-15000.0
-10000.0
-5000.0
0.0
5000.0


Cross flow causing a feathering angle
0.2 m/s
Y (m)
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
-1000
-800
-600
-400
-200
0
200
400
600
800
1000
X (m)

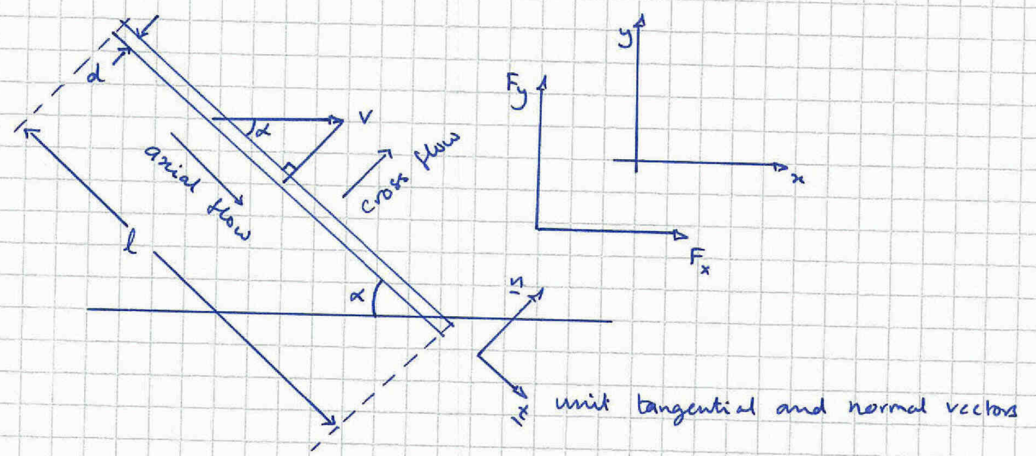
Streamer performing a turn



CONFIDENTIAL INFORMATION -- SUBJECT TO PROTECTIVE ORDER

WG00014262

Notes on streamer balancing program.



Force due to cross flow $v \sin\alpha$

$$\underline{F_c} = C_D \tfrac{1}{2} \rho \, v^2 \sin^2\alpha \, dL \; \underline{n}$$

Contribution in cartesians

$$\underline{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} C_D \frac{1}{2}\rho v^2 \sin^2\alpha \, dL \, . \, \sin\alpha \\ C_D \frac{1}{2}\rho v^2 \sin^2\alpha \, dL \, . \, \cos\alpha \end{pmatrix}$$

Friction force along streamer. due to axial flow $v\cos\alpha$

$$\underline{F_a} = \pi d L \, C_f \tfrac{1}{2}\rho \, v^2 \cos^2\alpha$$

Defn of $C_f = \dfrac{\tau_\omega}{\frac{1}{2}\rho v^2}$

Force $= \tau_\omega \times$ surface area

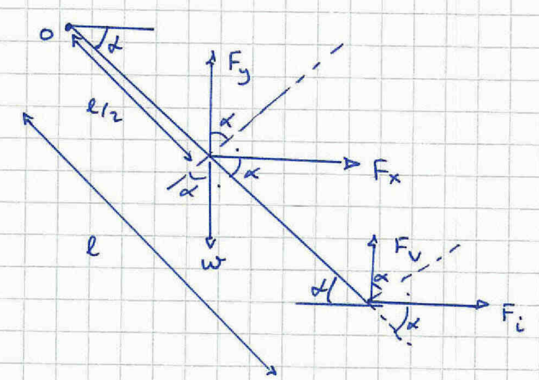
Contribution in cartesians

$$\underline{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} \pi d L C_f \frac{1}{2}\rho v^2 \cos^2\alpha \, . \, \cos\alpha \\ \pi d L C_f \frac{1}{2}\rho v^2 \cos^2\alpha \, . \, \sin\alpha \end{pmatrix}$$

$$\therefore \quad \underline{F} = \begin{pmatrix} \frac{1}{2}\rho v^2 dL \, (C_D \sin^3\alpha + \pi C_f \cos^3\alpha) \\ \frac{1}{2}\rho v^2 dL \, (C_D \sin^2\alpha \cos\alpha + \pi C_f \cos^2\alpha \sin\alpha) \end{pmatrix}$$

$$\underline{F} = \tfrac{1}{2}\rho v^2 dL \begin{pmatrix} C_D \sin^3\alpha + \pi C_f \cos^3\alpha \\ C_D \sin^2\alpha \cos\alpha + \pi C_f \cos^2\alpha \sin\alpha \end{pmatrix}$$

Moments balance about o

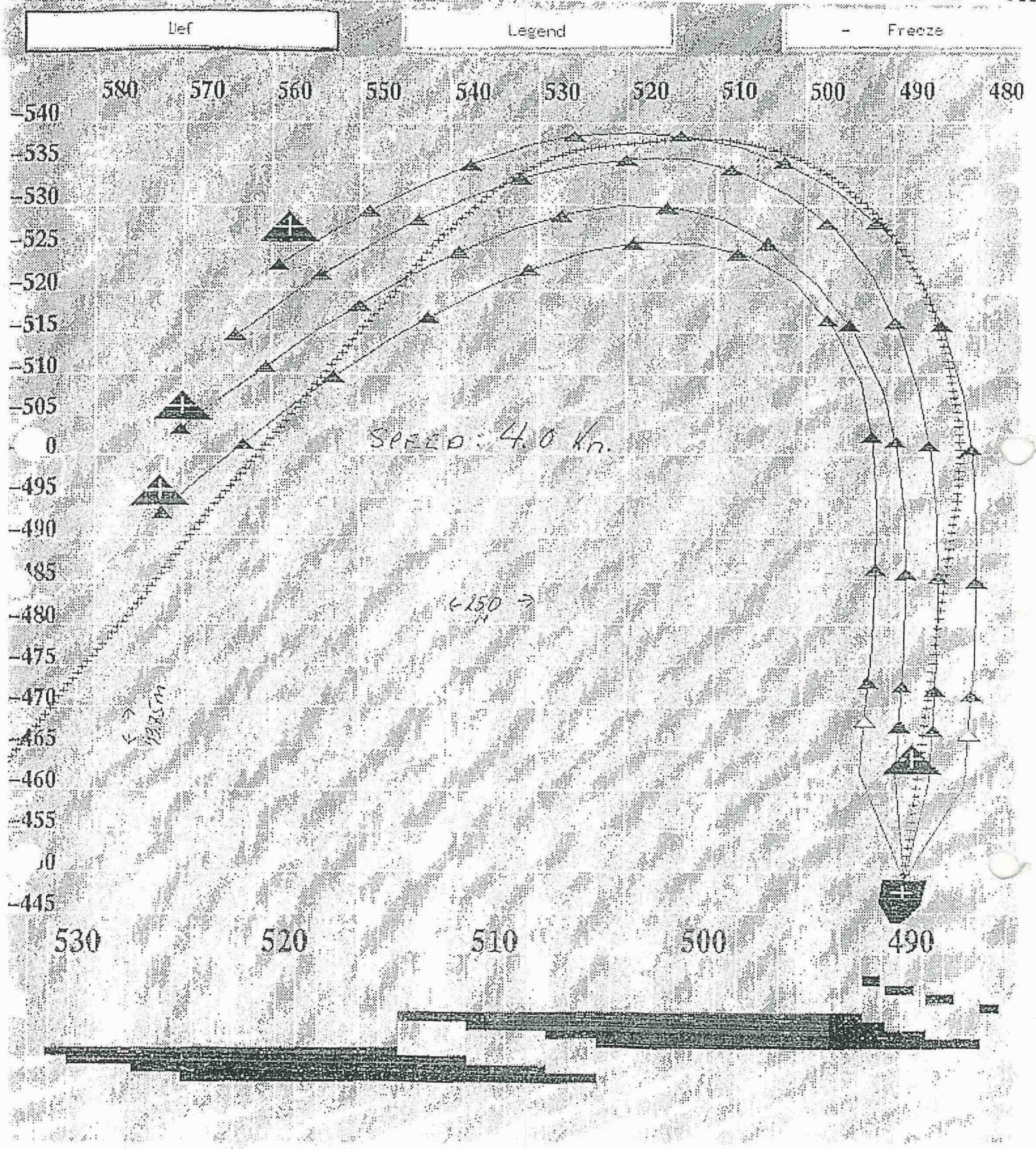


$$F_y \cos\alpha \cdot \frac{l}{2} + F_x \sin\alpha \cdot \frac{l}{2} - w \cos\alpha \, \frac{l}{2} + F_v \cos\alpha \cdot l + F_i \sin\alpha \cdot l = 0$$

$$\left(\frac{F_x}{2} + F_i \right) \sin\alpha = \left(\frac{F_y}{2} - \frac{w}{2} + F_v \right) \cos\alpha$$

$$\tan\alpha = \frac{\frac{F_y}{2} - \frac{w}{2} + F_v}{\frac{F_x}{2} + F_i}$$

25/07/93 15:04 GECO DIAMOND → 47 2 475537 NO.796 P03



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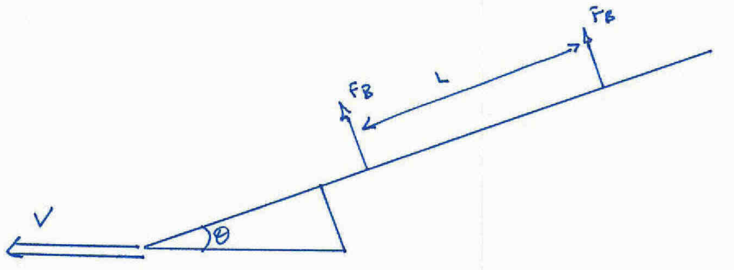
```
Input file for dynamic.C dynamic streamer program - bird deflection
-------------------------------------------------------------------

Basic conditions
----------------
water_density = 1020.0      // rho (Kg/m3)
diameter      = 60.0e-3     // diameter (m)
length        = 3000.0      // length of streamer (m)

theat0        = 3.14159     // Initial angle to horizontal
ucross        = 0.0         // Cross flow velocity (m/s)
tail_tension  = 10.0        // Tail Tension (N)

Bird control
------------
nbird         = 2           // number of birds (evenly distributed)

Bird_functions
1500.0  300.0 0.0 0.0 0.0
2000.0 -300.0 0.0 0.0 0.0

Boat control
------------
time        x_boat       yboat
Boat_positions
0.0         0.0          0.0
10.0        25.0         0.0
100.0       250.0        0.0

Numerical constants
-------------------
nsave         = 8           // number of points at which to output streamer po
number_nodes  = 101         // number of points along streamer
number_tsteps = 200         // number time steps (increase this if calcs crash
itmax         = 50          // maximum iterations for the non-linear iterator
epsnlit       = 1.0e-6      // epsilon for the non-linear solver

Fluid mechanics constants
-------------------------
Ct            = 0.007       // Ct Tangential drag coefficient
Cn            = 1.1         // Cn Normal drag coefficient
```

SIMPLE ESTIMATE FOR HORIZONTAL STEERING



Boat Speed = V

Cross flow = $V \sin\theta$

Bird Force = F_B

Distance between birds = L

θ = Feathering Angle

ρ = water density

d = Streamer diameter

C_n = Drag coeff normal to streamer.

Assume forces normal to streamer balance. F_B distributed evenly over L. $\therefore$

$$\tfrac{1}{2} \rho d C_n V^2 \sin^2\theta \, L = F_B$$

$\therefore$

$$\sin\theta = \left(\frac{2 F_B}{\rho d C_n V^2 L} \right)^{1/2}$$

Typical $F_B = 500$ N, $\rho = 1000$ Kg / m^2, $d = 60 \times 10^{-3}$, $C_n = 1$

$V = 3$ ms^{-1}, $L = 300$ m

θ = 4.5 degrees.

```
// FILE:  blocksolve.cpp

// block tri-diagonal solver
//
//    ( b[1] c[1]                        ) ( y[1] )       ( d[1] )
//    ( a[2] b[2] c[2]                   ) ( y[2] )       ( d[2] )
//    (      a[3] b[3] c[3]              ) ( y[3] )       ( d[3] )
//    ( ................................ ) ( .... )   =   ( .... )
//    (                       b[n] c[n]  ) ( y[n] )       ( d[n] )
//
// where a[i],b[i],c[i] are m x m matricies
// y, d at m vectors

#include <stdio.h>
#include <math.h>
#define  NRANSI
#include "nrutil.h"

void blocksolve(float ***a, float ***b, float ***c, float **y,
                float **d, int m, int n)
// a[1..n][1..m][1..m] , similarly b and c
// y[1..n][1..m]
// d[1..n][1..m]
{
    int    i;
    float  ***e, **f, **minv, **mtemp, *vtemp;

    void matinv(float **ainv, float **a, int n);
    void matmul(float **ab, float **a, float **b, int n);
    void mscal(float **a, float s, int m);
    void matvec(float *v1, float **a, float *v2, int m);
    void vscal(float *v1, float s, int m);
    void matadd(float **a, float **b, int m);
    void veccop(float *a, float *b, int m);
    void vecadd(float *a, float *b, int m);

    e     = f3tensor(1, n, 1, m, 1, m);
    f     = matrix(1, n, 1, m);
    minv  = matrix(1, m, 1, m);
    mtemp = matrix(1, m, 1, m);
    vtemp = vector(1, m);

    matinv(minv, b[1], m);
    matmul(e[1], minv, c[1], m);
    mscal(e[1], -1.0, m);
    matvec(f[1], minv, d[1], m);

    for (i=2; i<=n; i++) {

        matmul(mtemp, a[i], e[i-1], m);
        matadd(mtemp, b[i], m);
        matinv(minv, mtemp, m);
        matmul(e[i], minv, c[i], m);
        mscal(e[i], -1.0, m);

        matvec(vtemp, a[i], f[i-1], m);
        vscal(vtemp, -1.0, m);
        vecadd(vtemp, d[i], m);
        matvec(f[i], minv, vtemp, m);

    }

    veccop(y[n], f[n], m);
    for (i=n-1; i>=1; i--) {
```

```
        matvec(y[i], e[i], y[i+1], m);
        vecadd(y[i], f[i], m);

    }

    free_vector(vtemp, 1, m);
    free_matrix(mtemp, 1, m, 1, m);
    free_matrix(minv, 1, m, 1, m);
    free_matrix(f, 1, n, 1, m);
    free_f3tensor(e, 1, n, 1, m, 1, m);
}
```