PGS v WESTERNGECO (IPR2014-00688)
WESTERNGECO Exhibit 2051
Part 2

Full derivation of streamer positioning equations

Equations of motion:

$$m \partial_t^2 x = \partial_s (T \cos\theta) + F_x$$
$$m \partial_t^2 y = \partial_s (T \sin\theta) + F_y$$

$C = \cos\theta, \quad S = \sin\theta.$

$$x = x_0 + \int_0^s C \, ds'$$
$$y = y_0 + \int_0^s S \, ds'$$

$$\partial_t x = \dot{x}_0 - \int_0^s S \, \partial_t \theta \, ds'$$
$$\partial_t y = \dot{y}_0 + \int_0^s C \, \partial_t \theta \, ds'$$

$$\partial_t^2 x = \ddot{x}_0 - \int_0^s C w^2 + S \partial_t w \, ds'$$
$$\partial_t^2 y = \ddot{y}_0 - \int_0^s S w^2 - C \partial_t w \, ds'$$

$$m \left\{ \ddot{x}_0 - \int_0^s C w^2 + S \partial_t w \, ds' \right\} = F_x + \partial_s (TC)$$

$$m \left\{ \ddot{y}_0 - \int_0^s S w^2 - C \partial_t w \, ds' \right\} = F_y + \partial_s (TS)$$

Differentiating w.r.t. s.

$$- m (C w^2 + S \partial_t w) = \partial_s F_x + \partial_s^2 (TC)$$
$$- m (S w^2 - C \partial_t w) = \partial_s F_y + \partial_s^2 (TS)$$

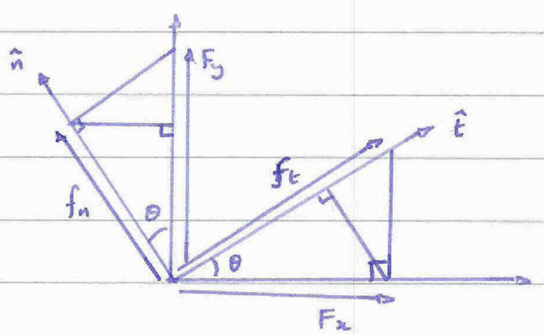
$$-m\omega^2 = C\,\partial_s F_x + S\,\partial_s F_y + C\,\partial_s^2(TC) + S\,\partial_s^2(TS)$$

$$\Rightarrow \quad \underline{\underline{C\,\partial_s^2(TC) + S\,\partial_s^2(TS) = -C\,\partial_s F_x - S\,\partial_s F_y - m\omega^2.}} \qquad \underline{1}$$

$$-m\,\partial_t \omega = S\,\partial_s F_x - C\,\partial_s F_y + S\,\partial_s^2(CT) - C\,\partial_s^2(ST).$$

$$\underline{\underline{m\,\partial_t \omega = C\,\partial_s F_y - S\,\partial_s F_x + C\,\partial_s^2(ST) - S\,\partial_s^2(CT).}} \qquad \underline{2}$$

$$\underline{\underline{\partial_t \theta = \omega}}$$



$$F_x = f_t C - f_n S$$

$$F_y = f_t S + f_n C.$$

$$RHS_1 = -C\,\partial_s(f_t C - f_n S) - S\,\partial_s(f_t S + f_n C) - m\omega^2$$

$$= -(C^2+S^2)\,\partial_s f_t + (CS\,\cancel{\partial_s\theta} f_t - SC\,\cancel{\partial_s\theta} f_t)$$

$$+ (CS\,\cancel{\partial_s} f_n - SC\,\cancel{\partial_s} f_n) + (C^2\,\partial_s\theta\, f_n + S^2\,\partial_s\theta\, f_n) - m\omega^2$$

$$= -\partial_s f_t + \partial_s\theta\, f_n - m\omega^2$$

$$RHS_2 = C\,\partial_s(f_t S + f_n C) - S\,\partial_s(f_t C - f_n S) + \ldots$$

$$= (CS\,\cancel{\partial_s f_t} - SC\,\cancel{\partial_s f_t}) + (C^2\,\partial_s\theta\, f_t + S^2\,\partial_s\theta\, f_t)$$

$$(C^2\,\partial_s f_n + S^2\,\partial_s f_n) + (-CS\,\cancel{\partial_s\theta} f_n + SC\,\cancel{\partial_s\theta} f_n)$$

$$= \partial_s\theta\, f_t + \partial_s f_n + \ldots$$

Governing equations become:

$$C\,\partial_s^2(TC) + S\,\partial_s^2(TS) = -\partial_s f_t + \partial_s\theta\, f_n - m\omega^2 \qquad \underline{3}$$

$$m\,\partial_t\omega = \partial_s\theta\, f_t + \partial_s f_n + C\,\partial_s^2(ST) - S\,\partial_s^2(CT) \qquad \underline{4}.$$

Expansions:

$$LHS_1 = C\partial_s\{C\,\partial_s T - S\,\partial_s\theta\, T\} + S\partial_s\{S\,\partial_s T + C\,\partial_s\theta\, T\}$$

$$= (C^2+S^2)\partial_s^2 T + (-CS\cancel{\partial_s\theta}\,\partial_s T + SC\cancel{\partial_s\theta}\,\partial_s T)$$

$$(-\cancel{S}C + \cancel{S}C)\,\partial_s\theta\,\partial_s T + (-SC\,\cancel{\partial_s^2}\theta\, T + SC\,\cancel{\partial_s^2}\theta\, T)$$

$$+ (-C^2(\partial_s\theta)^2 T - S^2(\partial_s\theta)^2 T)$$

$$= \partial_s^2 T - (\partial_s\theta)^2 T$$

$$RHS_2\,(\text{last terms}) = C\,\partial_s\{S\,\partial_s T + C\,\partial_s\theta\, T\} - S\,\partial_s\{C\partial_s T - S\,\partial_s\theta\, T\}$$

$$= (C\cancel{S} - \cancel{S}C)\,\partial_s^2 T + (C^2\partial_s\theta\,\partial_s T + S^2\partial_s\theta\,\partial_s T)$$

$$+ (C^2+S^2)\partial_s\theta\,\partial_s T + (C^2+S^2)\,\partial_s^2\theta\, T$$

$$+ (-CS\cancel{(\partial_s\theta)^2}\, T + SC\cancel{(\partial_s\theta)^2}\, T)$$

$$= 2\,\partial_s\theta\,\partial_s T + \partial_s^2\theta\, T$$

Governing equations become:

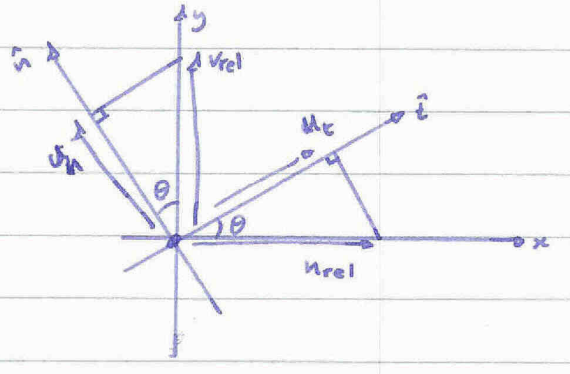
$$\partial_s^2 T - (\partial_s\theta)^2 T = -\partial_s f_t + \partial_s\theta\, f_n - m\,\omega^2 \qquad \underline{5}.$$

$$m\,\partial_t\omega = \partial_s\theta\, f_t + \partial_s f_n + 2\,\partial_s\theta\,\partial_s T + \partial_s^2\theta\, T \qquad \underline{6}$$

Forces on streamer:

$$f_t = 2\rho \pi a C_t \; u_t |u_t|$$
$$f_n = 2\rho a C_n \; u_n |u_n|.$$

$$u_t = (U - \partial_t x)c + (V - \partial_t y)s$$
$$u_n = -(U - \partial_t x)s + (V - \partial_t y)c$$



Boundary conditions $\quad T_{s=\ell} = 0 \qquad \partial_s \theta |_{s=\ell} = 0$

$$m\ddot{x}_0 = F_x|_{s=0} + \partial_s (Tc)|_{s=0}$$

$$m\ddot{y}_0 = F_y|_{s=0} + \partial_s (Ts)|_{s=0}.$$

Non-dimensionalisation.

$$\tilde{x} = x/\ell \qquad \tilde{y} = y/\ell \qquad \tilde{s} = s/\ell$$

$$\tilde{u}_t = u_t / u_b \qquad \tilde{u}_n = u_n / u_b \qquad \tilde{U} = U/u_b \qquad \tilde{V} = V/u_b$$

$$\tilde{t} = \frac{t u_b}{\ell}. \qquad \tilde{\omega} = \frac{\omega \ell}{u_b}.$$

(alternative non-dimensionalisation)

Forces:

$$\tilde f_t = \frac{f_t}{2a\rho u_b^2 \pi C_t} = \frac{2\rho\pi a C_t u_b^2\, \tilde u_t |\tilde u_t|}{2 a\rho u_b^2 \pi C_t} = \tilde u_t |\tilde u_t|.$$

$$\tilde f_n = \frac{f_n}{2a\rho u_b^2 C_n} = \frac{2\rho a C_n u_b^2\, \tilde u_n |\tilde u_n|}{2 a\rho u_b^2 C_n} = \tilde u_n |\tilde u_n|.$$

$$\tilde T = \frac{T}{2\pi a \rho u_b^2 l C_t}$$

Governing equations:

$$\frac{C_t}{l^2} 2\pi a\rho u_b^2 l \left\{ \partial_\xi^2 \tilde T - (\partial_\xi \theta)^2 \tilde T \right\} = -\frac{1}{l}\cdot 2a\rho u_b^2 \pi C_t\, \partial_\xi \tilde f_t + \frac{1}{l}\cdot 2a\rho u_b^2 C_n\, \partial_\xi \theta \tilde f_n - \pi a^2 \rho \frac{u_b^2}{l^2} \tilde\omega^2$$

$$\partial_\xi^2 \tilde T - (\partial_\xi \theta)^2 \tilde T = -\partial_\xi \tilde f_t + \frac{C_n}{\pi C_t} \partial_\xi \theta \tilde f_n - \frac{a}{2C_t l} \omega^2$$

let $\varepsilon = \frac{a}{2C_t l}$ and $\lambda = \frac{C_n}{\pi C_t}$

$$\underline{\underline{\partial_\xi^2 \tilde T - (\partial_\xi \theta)^2 \tilde T = -\partial_\xi \tilde f_t + \lambda \partial_\xi \theta \tilde f_n - \varepsilon \omega^2}}$$

$$\pi a^2 \rho \frac{u_b^2}{l^2} \partial_t \tilde\omega = \frac{1}{l}\cdot 2a\rho u_b^2 \pi C_t\, \partial_\xi \theta \tilde f_t + \frac{1}{l}\cdot 2a\rho u_b^2 C_n\, \partial_\xi \tilde f_n + \frac{1}{l^2} 2\pi a\rho u_b^2 l C_t\, 2\cdot \partial_\xi \theta \partial_\xi \tilde T + \frac{1}{l^2} 2\pi a \rho u_b^2 l C_t\, \partial_\xi^2 \theta \tilde T$$

$$\frac{\pi a^2}{l^2}\cdot\frac{l}{2\pi a C_t} \partial_t \tilde\omega = \partial_\xi \theta \tilde f_t + \frac{C_n}{\pi C_t} \partial_\xi \tilde f_n + 2\partial_\xi \theta \partial_\xi \tilde T + \partial_\xi^2 \theta \tilde T$$

$$\epsilon\, \partial_{\tilde t}\, \tilde\omega = \partial_{\tilde s}\theta\, \tilde f_t + \lambda\, \partial_{\tilde s} \tilde f_n + 2\, \partial_{\tilde s}\theta\, \partial_{\tilde s}\tilde T + \partial^2_{\tilde s}\theta\, \tilde T.$$

$$\partial_{\tilde t}\theta = \tilde\omega$$

Non-dimensionalisation of boundary conditions.

$$\tilde T|_{s=1} = 0 \qquad \partial_{\tilde s}\theta\,|_{s=1} = 0.$$

$$\pi a^2 \rho \frac{v_0^2}{l}\, \ddot{\tilde x}_0 = 2a\rho v_0^2 \pi C_t\, \tilde f_t\, c - 2a\rho v_0^2 C_n\, \tilde f_n\, s + \frac{1}{\cancel{l}} \cdot 2\pi \cancel{a\rho} v_0^2 \cancel{l} C_t\, \partial_{\tilde s}(\tilde T c)$$

$$\epsilon\, \ddot{\tilde x}_0 = \tilde f_t\, c - \lambda\, \tilde f_n\, s + \partial_{\tilde s}(\tilde T c)$$

$$\pi a^2 \rho \frac{v_0^2}{l}\, \ddot{\tilde y}_0 = 2a\rho v_0^2 \pi C_t\, \tilde f_t\, s + 2a\rho v_0^2 C_n\, \tilde f_n\, c + \frac{2\pi \cancel{a\rho} v_0^2 \cancel{l} C_t}{\cancel{l}}\, \partial_{\tilde s}(\tilde T s)$$

$$\epsilon\, \ddot{\tilde y}_0 = \tilde f_t\, s + \lambda \tilde f_n\, c + \partial_{\tilde s}(\tilde T s)$$

Combining:

$$\epsilon a_t = \epsilon(\ddot x_0 c + \ddot y_0 s) = \tilde f_t + \partial_{\tilde s}\tilde T$$

$$\partial_{\tilde s}\tilde T = \epsilon a_t - \tilde f_t$$

$$\epsilon a_n = \epsilon(-s\ddot x_0 + c\ddot y_0) = \lambda \tilde f_n + \tilde T\, \partial_{\tilde s}\theta.$$

$$\tilde T\, \partial_{\tilde s}\theta = \epsilon a_n - \lambda \tilde f_n$$

Numerical solution : dropping ~'s

$$\partial_s^2 T - (\partial_s \theta)^2 T = P$$

$$\frac{T_{i+1}^{k+1} - 2T_i^{k+1} + T_{i-1}^{k+1}}{\Delta s^2} - (\partial_s \theta)^2\Big|_i^k T_i^{k+1} = P_i^k$$

$$T_{i+1}^{k+1} - \left(2 + \Delta s^2 (\partial_s \theta)^2\right)\Big|_i^k T_i^{k+1} + T_{i-1}^{k+1} = \Delta s^2 P_i^k$$

Boundary Conditions.

$$T_n^{k+1} = 0$$

$$T_1^{k+1} - T_0^{k+1} = \Delta s \,(\epsilon a_t - f_t)\Big|_0^k$$

Let $g = 2\partial_s T + f_t$. $\quad Q = \lambda \partial_s f_n$

$$\epsilon \frac{\omega^{k+1} - \omega^k}{\Delta t} - \frac{1}{2} T_i^k \frac{\theta_{i+1}^{k+1} - 2\theta_i^{k+1} + \theta_{i-1}^{k+1}}{\Delta s^2} - \frac{1}{2} g_i^k \frac{\theta_{i+1}^{k+1} - \theta_{i-1}^{k+1}}{2\Delta s} =$$

$$Q_i^k + \frac{1}{2} T_i^k (\partial_s^2 \theta)_i^k + \frac{1}{2} g_i^k (\partial_s \theta)_i^k$$

$$\epsilon \omega^{k+1} - \theta_{i+1}^{k+1} \Delta t \left(\frac{1}{2}\frac{T_i^k}{\Delta s^2} + \frac{g_i^k}{4\Delta s}\right) + \frac{\Delta t \,\theta_i^{k+1} T_i^k}{\Delta s^2} - \theta_{i-1}^{k+1} \Delta t \left(\frac{T_i^k}{2\Delta s^2} - \frac{g_i^k}{4\Delta s}\right)$$

$$= \epsilon \omega^k + \Delta t \left\{ Q + \frac{1}{2} T_i^k (\partial_s^2 \theta)_i^k + \frac{1}{2} g_i^k (\partial_s \theta)_i \right\}$$

Let $A_i = -\dfrac{\Delta t}{2\Delta s^2}\left(T_i^k + \dfrac{g_i^k \Delta s}{2}\right)$ $\qquad B_i = \dfrac{\Delta t\, T_i^k}{\Delta s^2}$

$$C_i = -\frac{\Delta t}{2\Delta s^2}\left(T_i^k - \frac{g_i^k}{2}\Delta s\right)$$

let

$$R_i = \epsilon \omega_i^k + \Delta t \left\{ Q + \tfrac{1}{2} T_i^k (\partial_s^2 \theta)_i^k + \tfrac{1}{2} g_i^k (\partial_s \theta)_i \right\}$$

Then

$$\epsilon \omega^{k+1} + A_i \theta_{i+1}^{k+1} + B_i \theta_i^{k+1} + C_i \theta_{i-1}^{k+1} = R_i$$

And for the θ equation

$$\frac{\theta_i^{k+1} - \theta_i^k}{\Delta t} = \frac{1}{2}(\omega_i^{k+1} + \omega_i^k)$$

$$\theta_i^{k+1} = \frac{\Delta t}{2} \omega_i^{k+1} + \frac{\Delta t}{2} \omega_i^k + \theta_i^k$$

$$\theta_i^{k+1} = \frac{\Delta t}{2} \omega_i^{k+1} + h_i^k$$

where $h_i^k = \frac{\Delta t}{2} \omega_i^k + \theta_i^k$.

Substituting into governing:

$$\epsilon \omega_i^{k+1} + A_i \left(\frac{\Delta t}{2} \omega_{i+1}^{k+1} + h_{i+1}^k \right) + B_i \left(\frac{\Delta t}{2} \omega_i^{k+1} + h_i^k \right) + C_i \left(\frac{\Delta t}{2} \omega_{i-1}^{k+1} + h_{i-1}^k \right) = R_i$$

$$A_i \frac{\Delta t}{2} \omega_{i+1}^{k+1} + \left(B_i \frac{\Delta t}{2} + \epsilon \right) \omega_i^{k+1} + C_i \frac{\Delta t}{2} \omega_{i-1}^{k+1} = R_i - A_i h_{i+1}^k - B_i h_i^k - C_i h_{i-1}^k$$

$$A_i \frac{\Delta t}{2} \omega_{i+1}^{k+1} + \left(B_i \frac{\Delta t}{2} + \epsilon \right) \omega_i^{k+1} + C_i \frac{\Delta t}{2} \omega_{i-1}^{k+1} = R_i - A_i h_{i+1}^k - B_i h_i^k - C_i h_{i-1}^k$$

Boundary conditions - second equation.

$$\partial_s \theta \big|_{s=1} = 0.$$

$$\theta_n^{k+1} - \theta_{n-1}^{k+1} = 0.$$

$$\frac{\Delta t}{2} w_n^{k+1} + h_n^k - \frac{\Delta t}{2} w_{n-1}^{k+1} - h_{n-1}^k = 0.$$

$$w_n^{k+1} - w_{n-1}^{k+1} = -\frac{2}{\Delta t}\left(h_n^k - h_{n-1}^k\right).$$

$$T \partial_s \theta \big|_{s=0} = \left(\varepsilon a_n - \lambda f_n\right)_{s=0}.$$

$$T_0^k \frac{\theta_1^{k+1} - \theta_0^{k+1}}{\Delta s} = \left(\varepsilon a_n - \lambda f_n\right)\big|_{s=0}.$$

$$\theta_1^{k+1} - \theta_0^{k+1} = \frac{\Delta s}{T_0^k}\left(\varepsilon a_n - \lambda f_n\right)_{s=0}.$$

$$\frac{\Delta t}{2}\left(w_1^{k+1} - w_0^{k+1}\right) = -\left(h_1^k - h_0^k\right) + \frac{\Delta s}{T_0^k}\left(\varepsilon a_n - \lambda f_n\right)\big|_{s=0}.$$

$$w_1^{k+1} - w_0^{k+1} = -\frac{2}{\Delta t}\left(h_1 - h_0\right) + \frac{2\Delta s}{\Delta t}\left.\frac{\left(\varepsilon a_n - \lambda f_n\right)}{T}\right|_{s=0}$$

(A)

Block tri-diagonal solution of linearised equations

Governing equations :- (non-dimensional)

$$\partial_s T + u|u| = 0$$

$$T\partial_s \theta + \lambda v|v| = 0$$

$$\partial_s u - v\,\partial_s \theta = 0$$

$$\partial_s v + u\,\partial_s \theta = \partial_t \theta$$

Re-order for consistency with T, θ, u, v unk

$$\partial_s T + u|u| = 0 \qquad T$$

$$\partial_s v + u\,\partial_s \theta = \partial_t \theta \qquad \theta$$

$$\partial_s u - v\,\partial_s \theta = 0 \qquad u$$

$$T\,\partial_s \theta + \lambda v|v| = 0 \qquad v$$

Linearisation:

$$T = T_0 + T_1 \qquad \theta = \theta_0 + \theta_1 \qquad u = u_0 + u_1 \qquad v = v_0 + v_1$$

$|u_0| >> |u_1| \Rightarrow \operatorname{sgn}(u_0 + u_1) = \operatorname{sgn} u_0$

$$\partial_s T_0 + \partial_s T_1 + u_0^2 \operatorname{sgn}(u_0) + 2u_0 u_1 \operatorname{sgn}(u_0) = 0$$

$$\underline{\underline{\partial_s T_1 + 2|u_0|\,u_1 = -\partial_s T_0 - u_0|u_0| = r_1}}$$

Implicit time stepping

$$\partial_s v + u\,\partial_s \theta = \frac{\theta - \theta^0}{\Delta t}.$$

$$\partial_s v_0 + \partial_s v_1 + u_0\,\partial_s \theta_0 + u_0\,\partial_s \theta_1 + u_1\,\partial_s \theta_0 = \frac{\theta_0 + \theta_1 - \theta^0}{\Delta t}$$

$$\partial_s \vartheta_1 + u_0 \partial_s \theta_1 + \partial_s \theta_0 u_1 - \frac{\theta_1}{\Delta t} = \frac{\theta_0 - \theta^0}{\Delta t} - \partial_s v_0 - u_0 \partial_s \theta_0 = r_2$$

$$\partial_s u_0 + \partial_s u_1 - v_0 \partial_s \theta_0 - v_1 \partial_s \theta_0 - \theta_0 \partial_s \theta_1 = 0.$$

$$\partial_s u_1 - \partial_s \theta_0 v_1 - \theta_0 \partial_s \theta_1 = -\partial_s u_0 + v_0 \partial_s \theta_0 = r_3$$

$$T_0 \partial_s \theta_0 + T_1 \partial_s \theta_0 + T_0 \partial_s \theta_1 + \lambda v_0 |v_0| + 2|v_0| v_1 \lambda = 0.$$

$$\partial_s \theta_0 T_1 + T_0 \partial_s \theta_1 + 2\lambda |v_0| v_1 = -T_0 \partial_s \theta_0 - \lambda v_0 |v_0| = r_4$$

Block tridiagonal vector $(T, \theta, u, \vartheta)^T$

Interlaced on f.d. grid $\underline{\omega} = (T_1, \theta_1, u_1, v_1, \ldots T_n, \theta_n, u_n, v_n)^T$

b.c.'s $T_n = T_{tail}$ $u_1 = u_0 c + v_0 s$ (or something) $\vartheta_1 = \ldots$ similarly.

(c)

Given T_o, θ_o, u_o and v_o Calculate updates.
also need u_b, v_b, λ, θ^o, Δt, Δs, n.
Grid is $i = 1..n$ $\Delta s = 1/(n-1)$

Calculate residuals
Calculate bdy block - bi
Calc. end pts.
Impose bc's
Solve
update values

Calculate residuals Given T_o, θ_o, u_o, v_o, λ, θ^o, Δt, Δs, n

```
resid [1..n][1..m]      m = 4
allocate  dTds  dθds  duods  dvods
deriv(T, ds, n, dTds); deriv(θ, ds, n, dθds), deriv(uo, ds, n, duods);
deriv(vo, ds, n, dvods)
for (i=1; i<=n; i++){
    resid[i][1] =      - dTds_i - uo_i |uo_i|
    resid[i][2] = 1/Δt (θo_i - θ°_i) - dvods_i - uo_i dθods_i
    resid[i][3] =      - duods_i + vo_i dθods_i
    resid[i][4] =      - To_i dθods_i - λ vo_i |vo_i|
}
deallocate dvods, duods, dθds, dTds
}
```

```
    bp[4][1] =  dθ₀ds;
    cp[4][2] =  T₀i /(2ds)
    ap[4][2] = −T₀i /(2ds)
    bp[4][4] =    2λ|v₀|;
}
```

and forward + backward at the ends.

Matrix form.

$$\begin{pmatrix} \partial_s & 0 & 2|u_0| & 0 \\ 0 & u_0\partial_s - \frac{1}{\Delta t} & \partial_s\theta_0 & \partial_s \\ 0 & -\theta_0\partial_s & 0 & -\partial_s\theta_0 \\ \partial_s\theta_0 & T_0\partial_s & 0 & 2\lambda|v_0| \end{pmatrix} \begin{pmatrix} T_1 \\ \theta_1 \\ u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix}$$

Calculate block body T_0, θ_0, u_0, v_0, λ, ds, dt

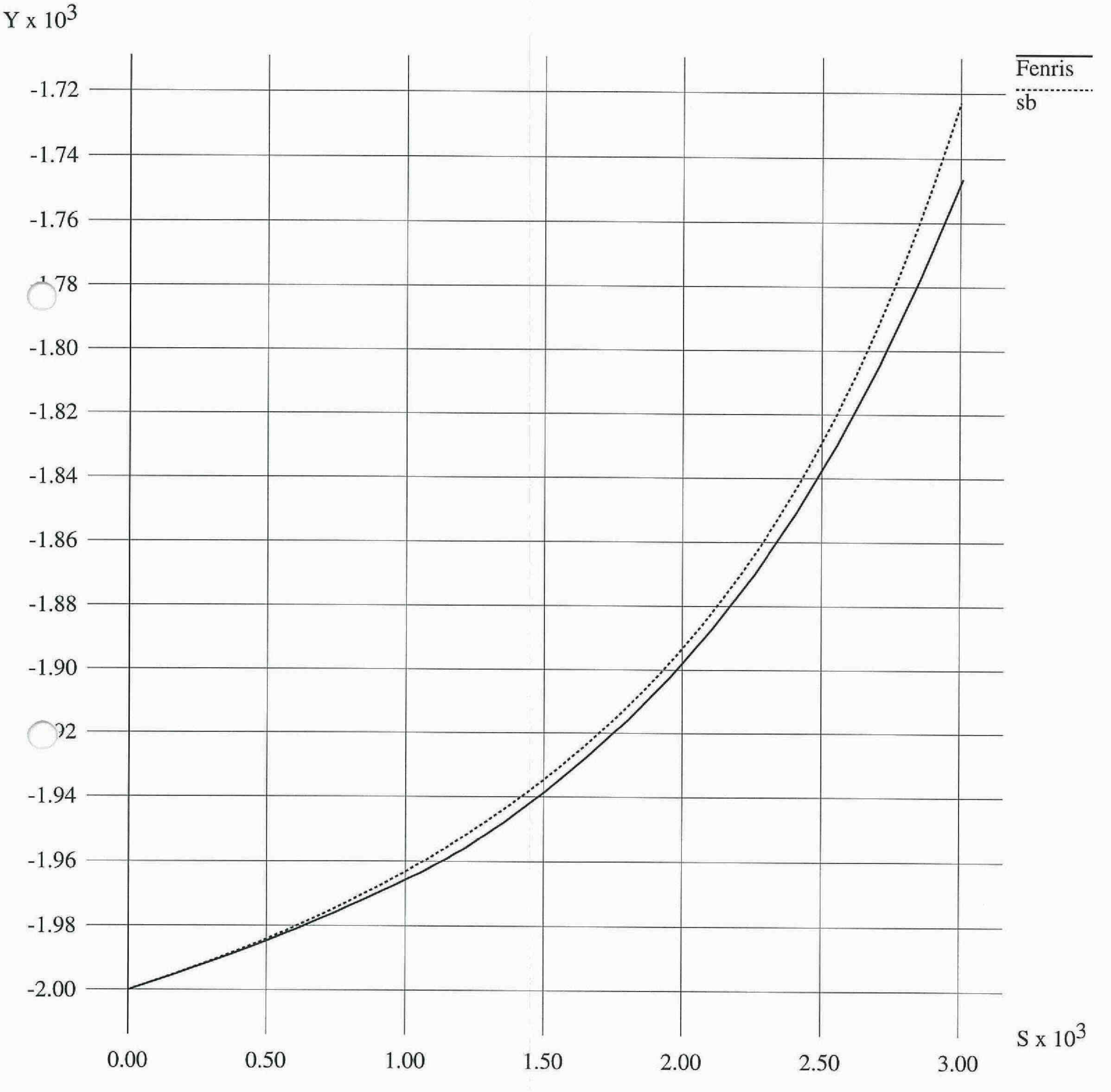
a[1..n][1..4][1..4] b[1..n][1..4][1..4]

c[1..n][1..4][1..4]

for ($i \in 1..n$, $j \in 1..m$, $k \in 1..m$) $a_{ijk} = b_{ijk} = c_{ijk} = 0$.

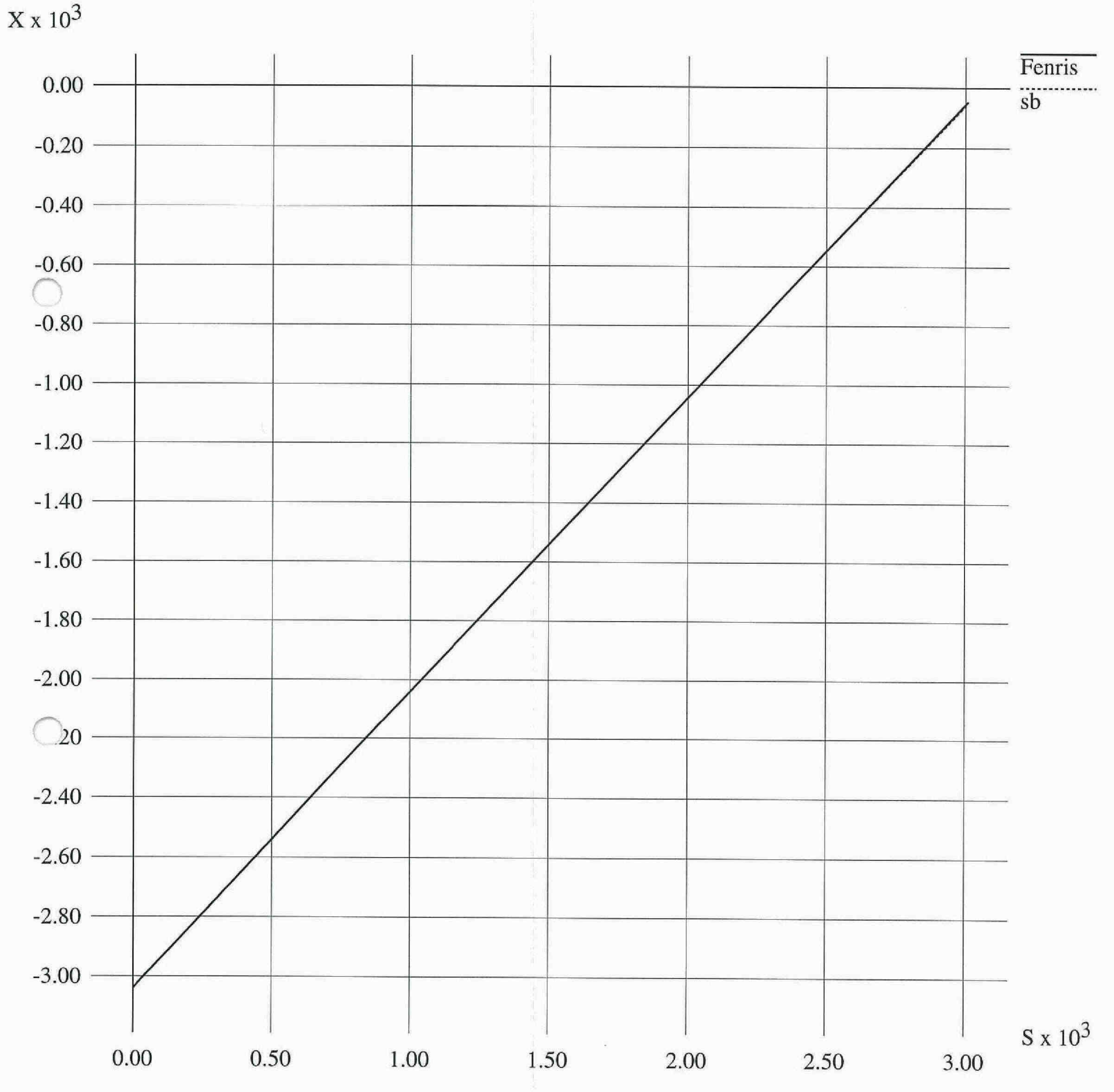
```
deriv(θ, ds, n, dθ0 ds)
for (i=2, i<n; i++) {
    ap = a[i]; bp = b[i]; cp = c[i];
    cp[1][1] = 1/(ds*2)
    ap[1][1] = -1/(ds*2)
    bp[1][3] = 2|u0|i
    cp[2][2] = u0i /(ds*2)
    bp[2][2] = -1/Δt
    ap[2][2] = -u0i /(ds*2)
    bp[2][3] = dθ0 dsi
    cp[2][4] = 1/(2ds)
    ap[2][4] = 1/(2ds)
    cp[3][2] = -θ0i /(2ds)
    ap[3][2] = +θ0i /(2ds)
    bp[3][4] = -dθ0 ds[i]
```

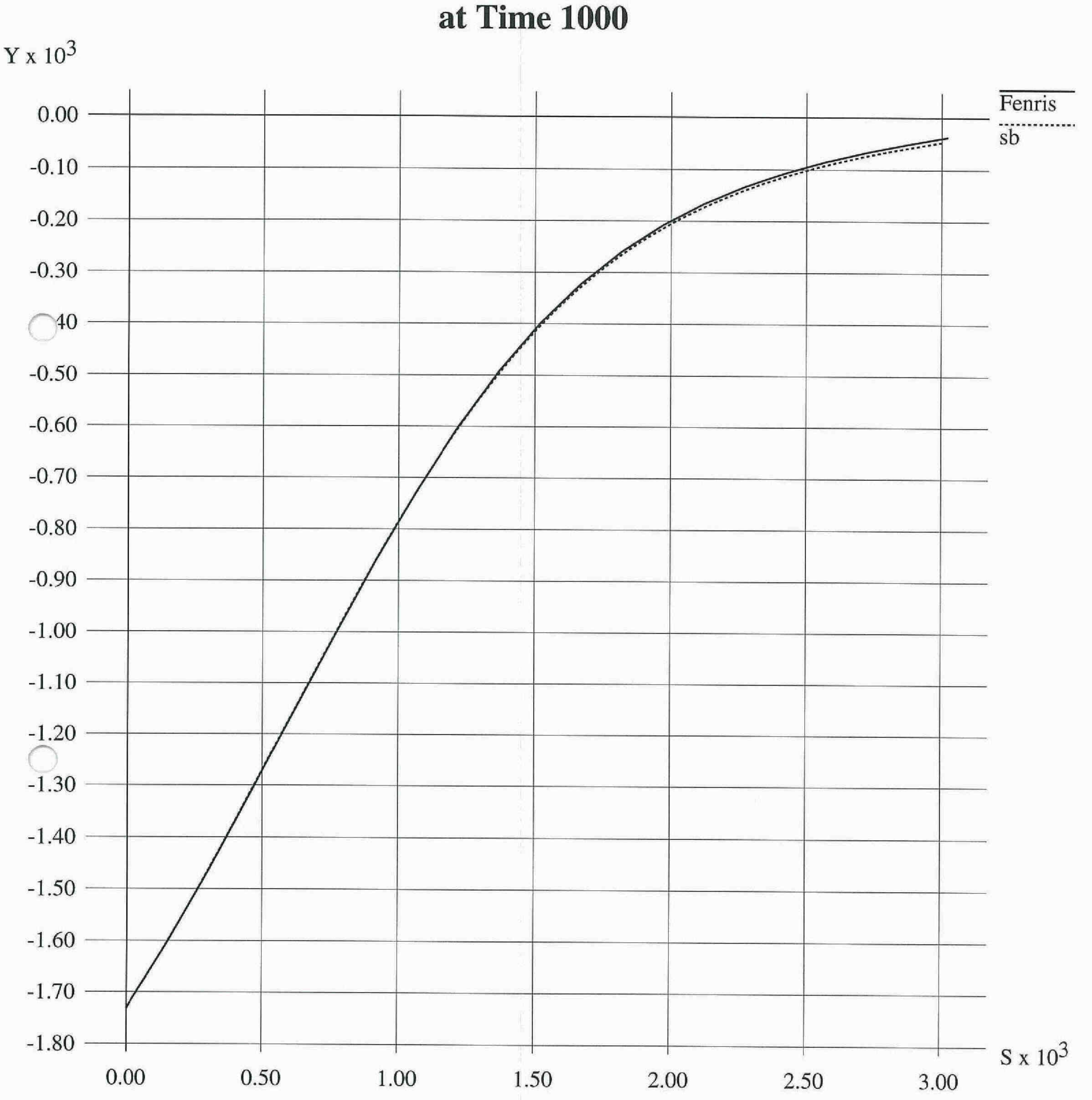
at Time 2600

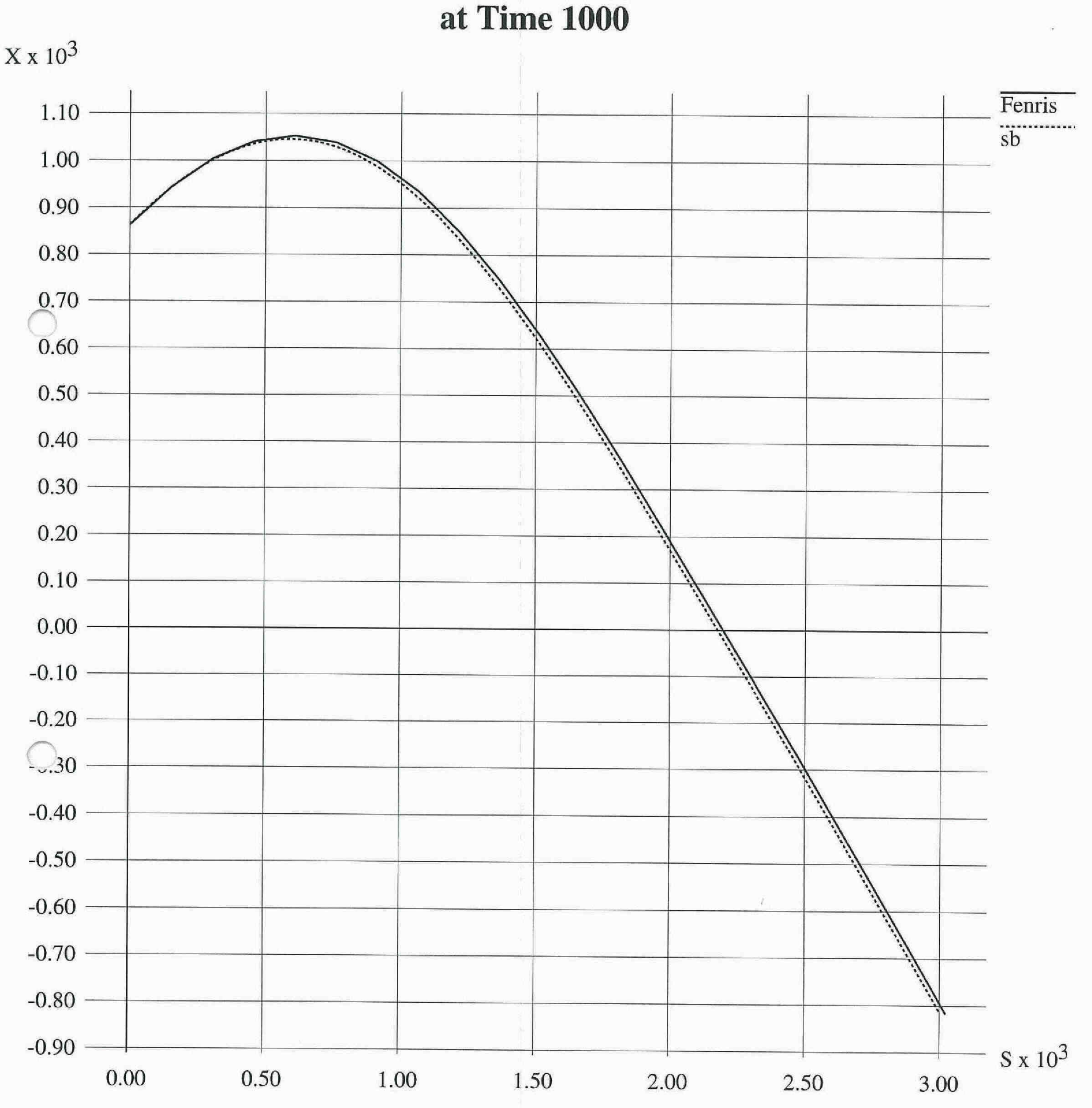


Y x 10^3
-1.72
-1.74
-1.76
-1.78
-1.80
-1.82
-1.84
-1.86
-1.88
-1.90
-1.92
-1.94
-1.96
-1.98
-2.00
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3
Fenris
sb

at Time 2600


X x 10^3
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
-1.20
-1.40
-1.60
-1.80
-2.00
.20
-2.40
-2.60
-2.80
-3.00
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3
Fenris
sb


at Time 1000
Y x 10^3
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
-1.10
-1.20
-1.30
-1.40
-1.50
-1.60
-1.70
-1.80
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3
Fenris
sb


at Time 1000
X x 10^3
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
0.00
0.50
1.00
1.50
2.00
2.50
3.00
S x 10^3
Fenris
sb

CONFIDENTIAL

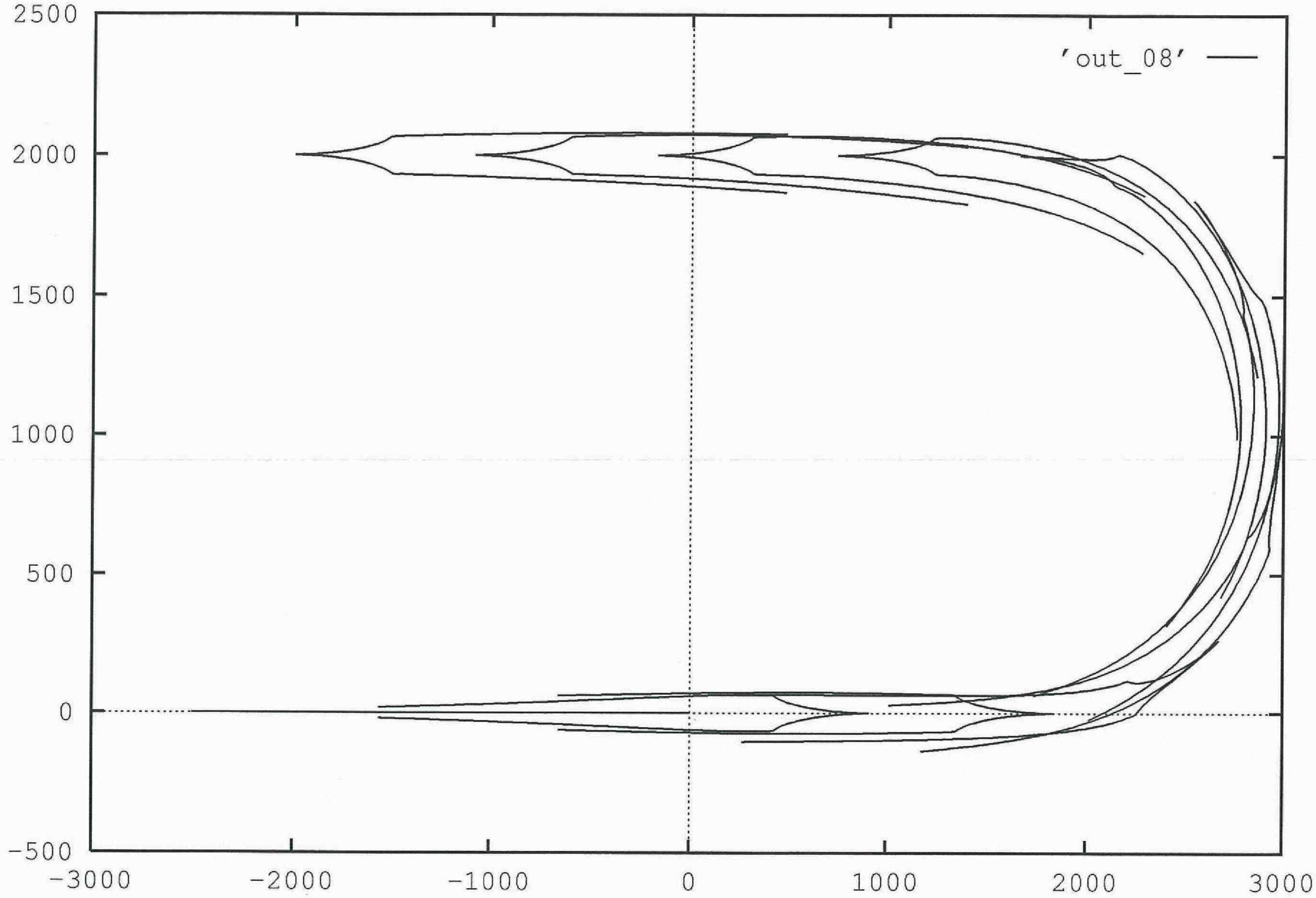
ANNEX B : HMPE YARN CREEP DATA @ 20ºC

Yarn Type	At 15% BL — % Strain Increment: to 1 min	1 min to 1 day	1 day to 10 days	10 to 100 days	days to break	At 30% BL — % Strain Increment: to 1 min	1 min to 1 day	1 day to 10 days	10 to 100 days	days to break
Spectra 900	0.8	1.1	1.6	12.9	180	1.5	3.3	break		4
Spectra 1000	0.5	0.6	1.0	6.2	330	1.0	1.4	8.0		30
Dyneema SK60	0.8	0.4	0.1	0.4	>350	1.4	0.7	0.9		120

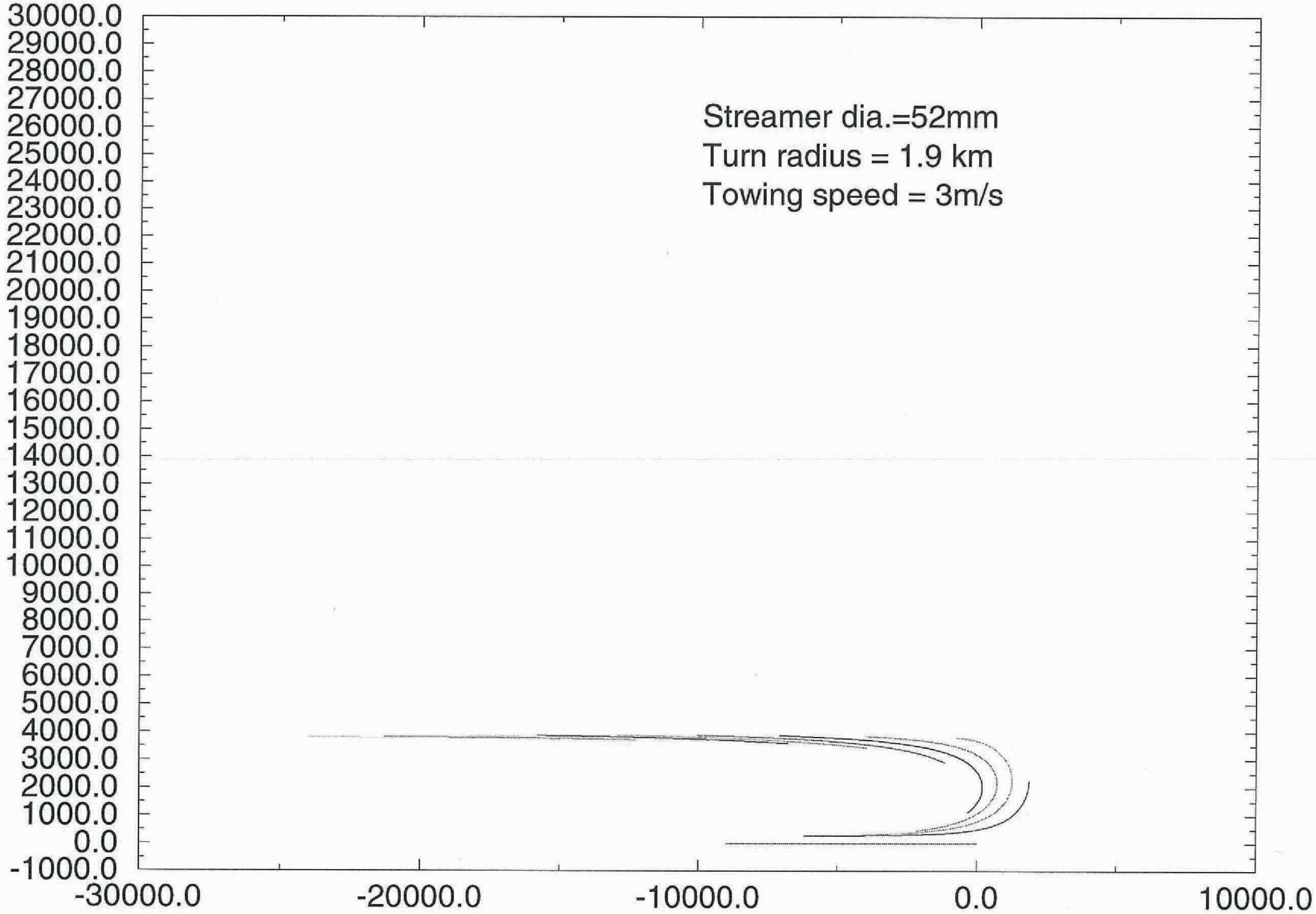
SK75

polyethylene.

SK60
65
SK.75


'out_08'
2500
2000
1500
1000
500
0
-500
-3000
-2000
-1000
0
1000
2000
3000

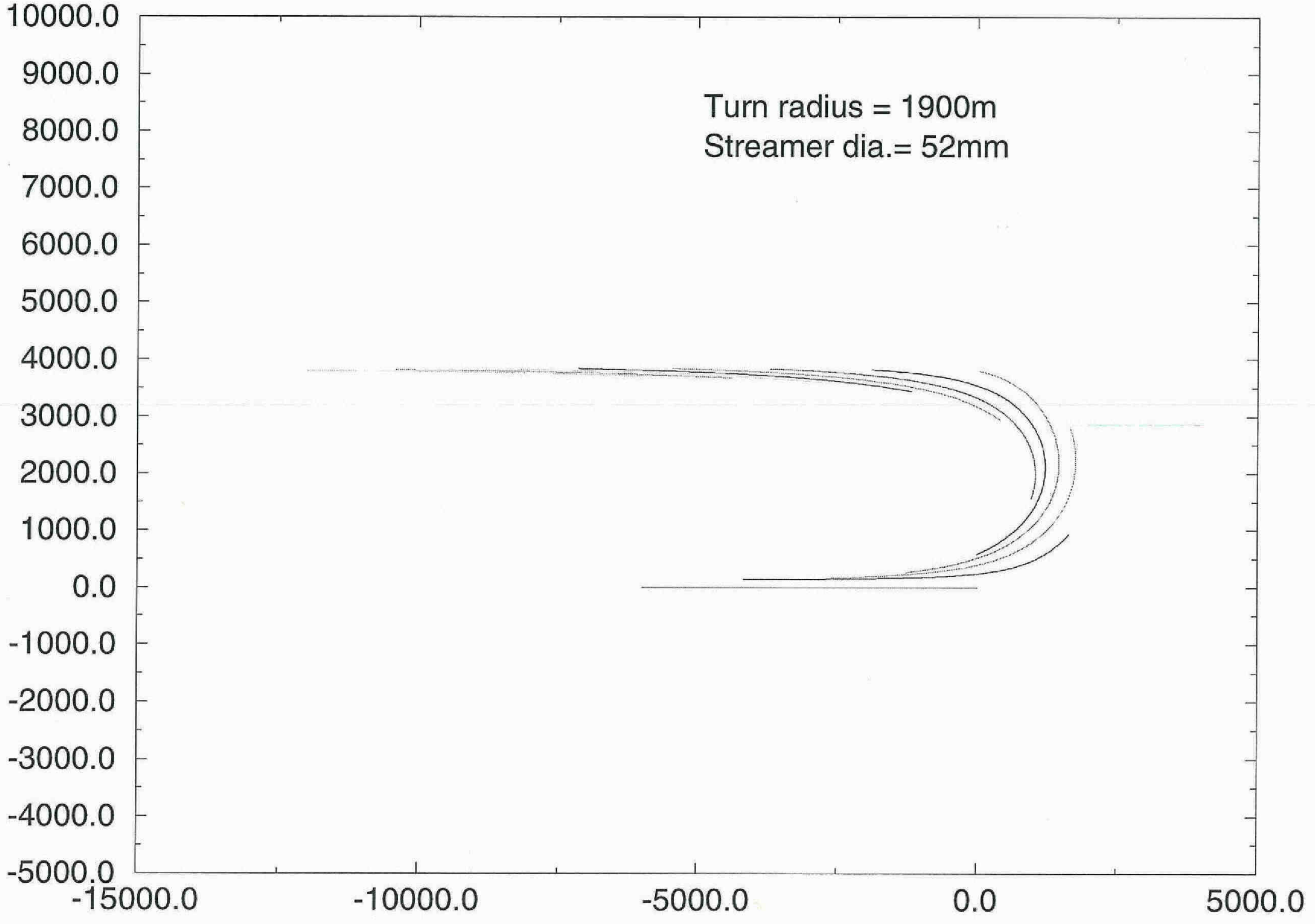
TURN WITH 9KM STREAMER (WITHOUT BIRDS)

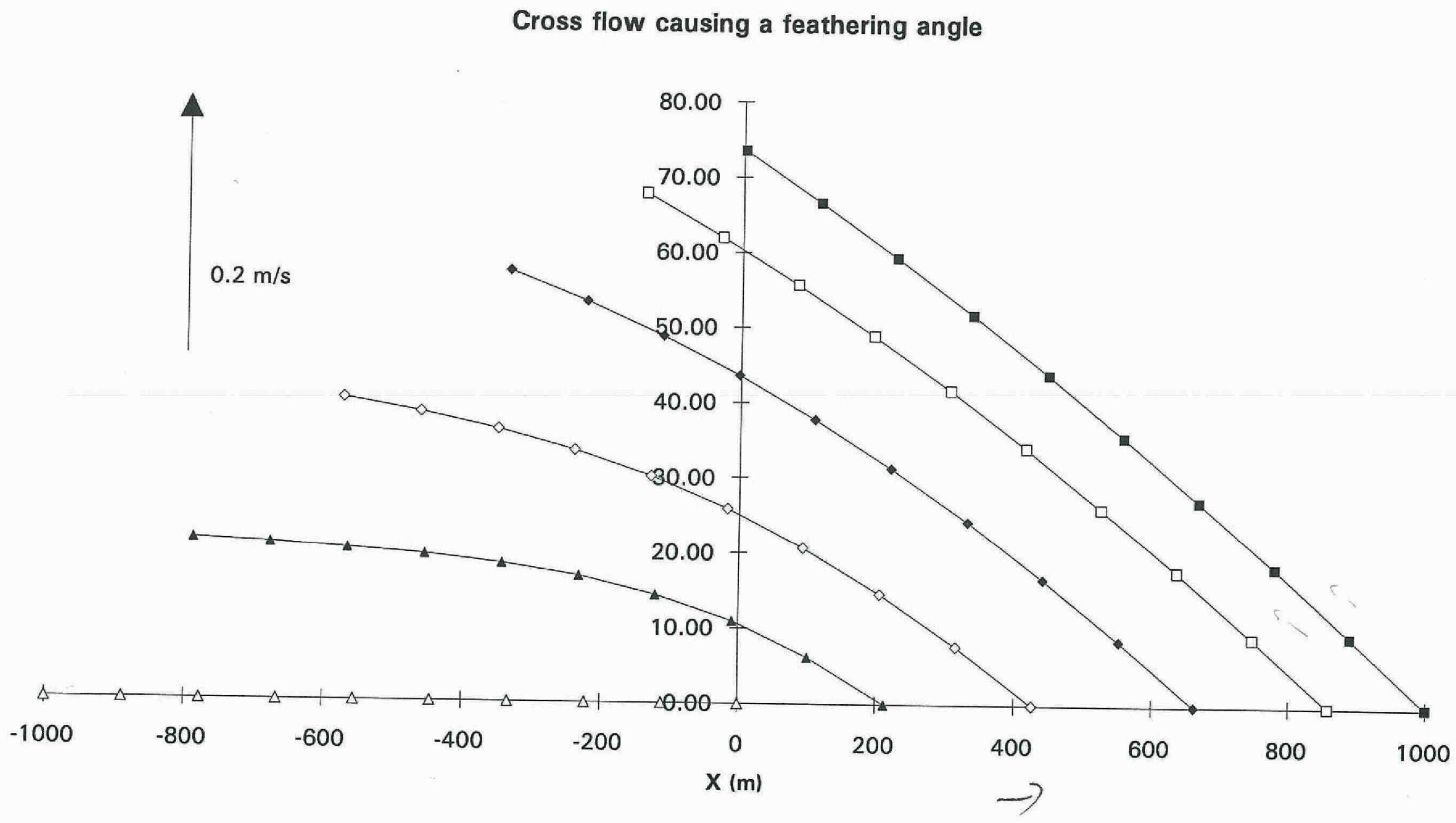


24th 2 wks GR
25th 3 dys ØH
18th 5 dys. KØ.

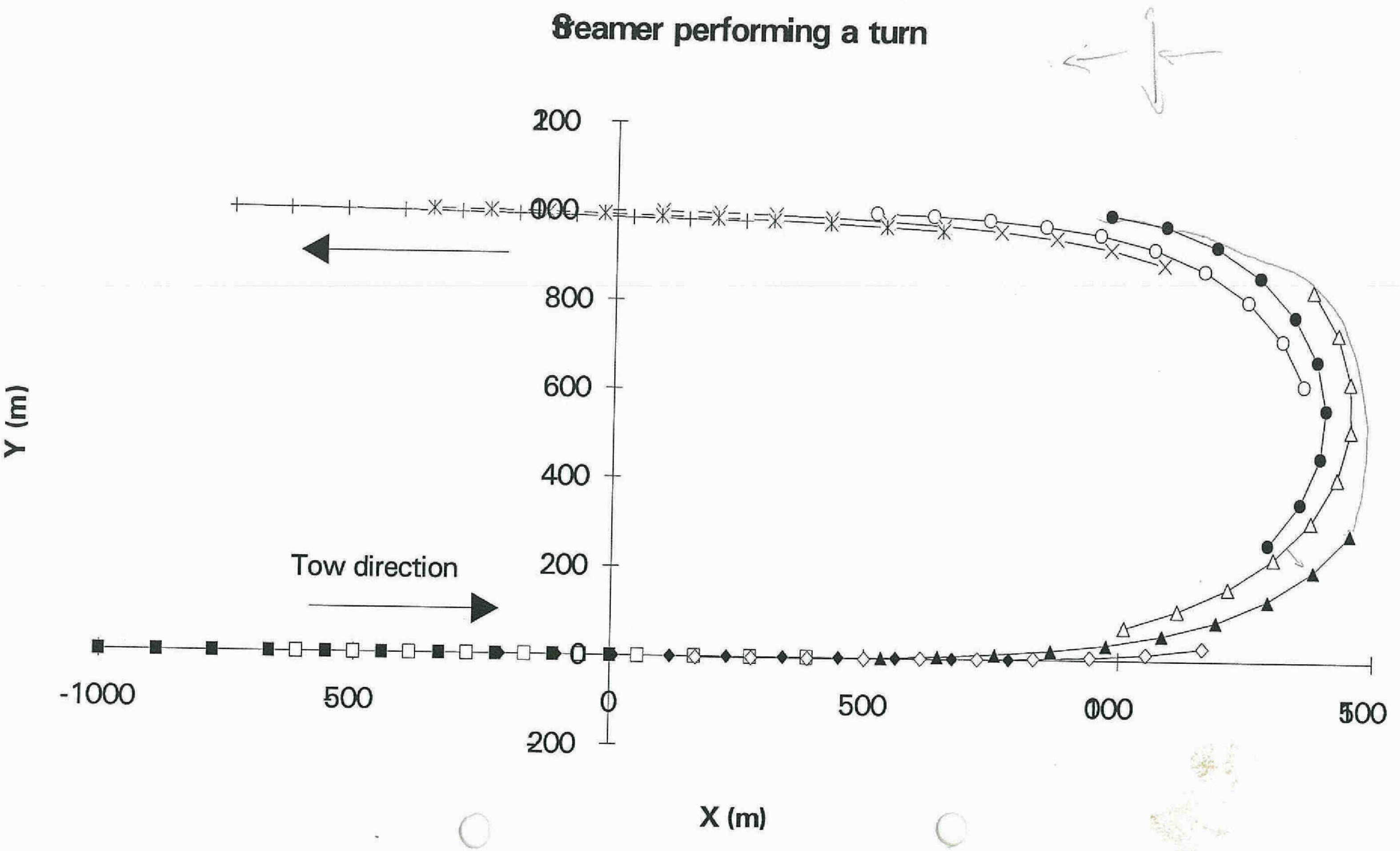
7th aug. 2 – 3 wks.

TURN WITH 6KM STREAMER (WITHOUT BIRDS)

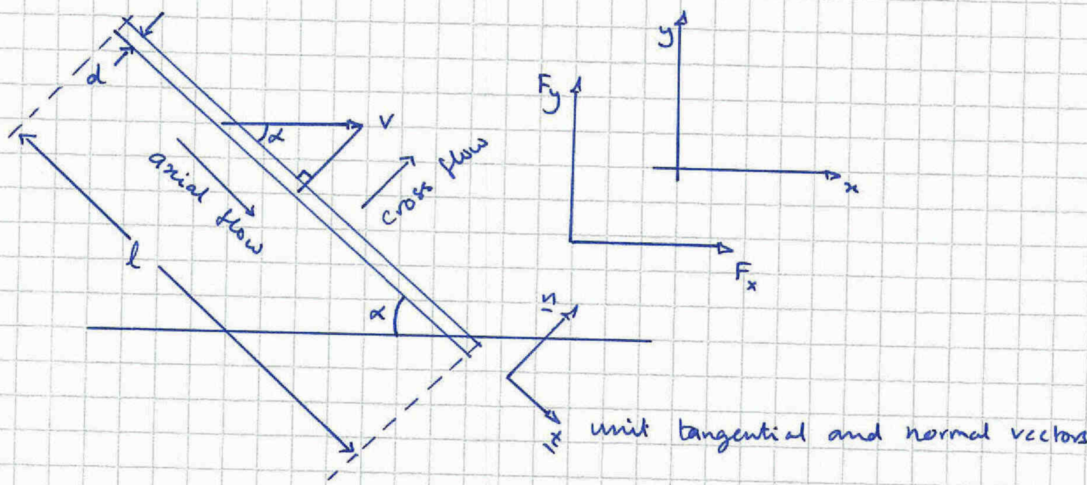



Cross flow causing a feathering angle
0.2 m/s
Y (m)
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
-1000
-800
-600
-400
-200
0
200
400
600
800
1000
X (m)

Streamer performing a turn



Notes on streamer balancing program.



Force due to cross flow $v\sin\alpha$

$$\underline{F_c} = C_D \tfrac{1}{2}\rho\, v^2 \sin^2\alpha \, dL \; \underline{n}$$

Contribution in cartesians

$$\underline{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} C_D \frac{1}{2}\rho v^2 \sin^2\alpha \, dL \,.\, \sin\alpha \\ C_D \frac{1}{2}\rho v^2 \sin^2\alpha \, dL \,.\, \cos\alpha \end{pmatrix}$$

Friction force along streamer due to axial flow $v\cos\alpha$

$$\underline{F_a} = \pi d L\, C_f \tfrac{1}{2}\rho\, v^2 \cos^2\alpha$$

Defn of $C_f = \dfrac{\tau_\omega}{\frac{1}{2}\rho v^2}$

Force $= \tau_\omega \times$ surface area

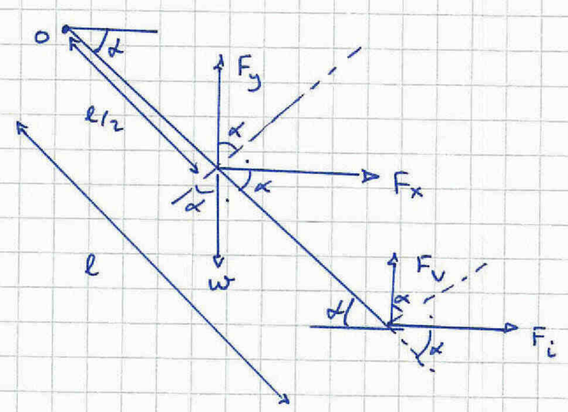
Contribution in cartesians

$$\underline{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} \pi d L\, C_f \frac{1}{2}\rho v^2 \cos^2\alpha \,.\, \cos\alpha \\ \pi d L\, C_f \frac{1}{2}\rho v^2 \cos^2\alpha \,.\, \sin\alpha \end{pmatrix}$$

$$\therefore \quad \underline{F} = \begin{pmatrix} \frac{1}{2}\rho v^2 dL \,(C_D \sin^3\alpha + \pi C_f \cos^3\alpha) \\ \frac{1}{2}\rho v^2 dL \,(C_D \sin^2\alpha\cos\alpha + \pi C_f \cos^2\alpha \sin\alpha) \end{pmatrix}$$

$$\underline{F} = \tfrac{1}{2}\rho v^2 dL \begin{pmatrix} C_D \sin^3\alpha + \pi C_f \cos^3\alpha \\ C_D \sin^2\alpha\cos\alpha + \pi C_f \cos^2\alpha \sin\alpha \end{pmatrix}$$

Moments balance about O

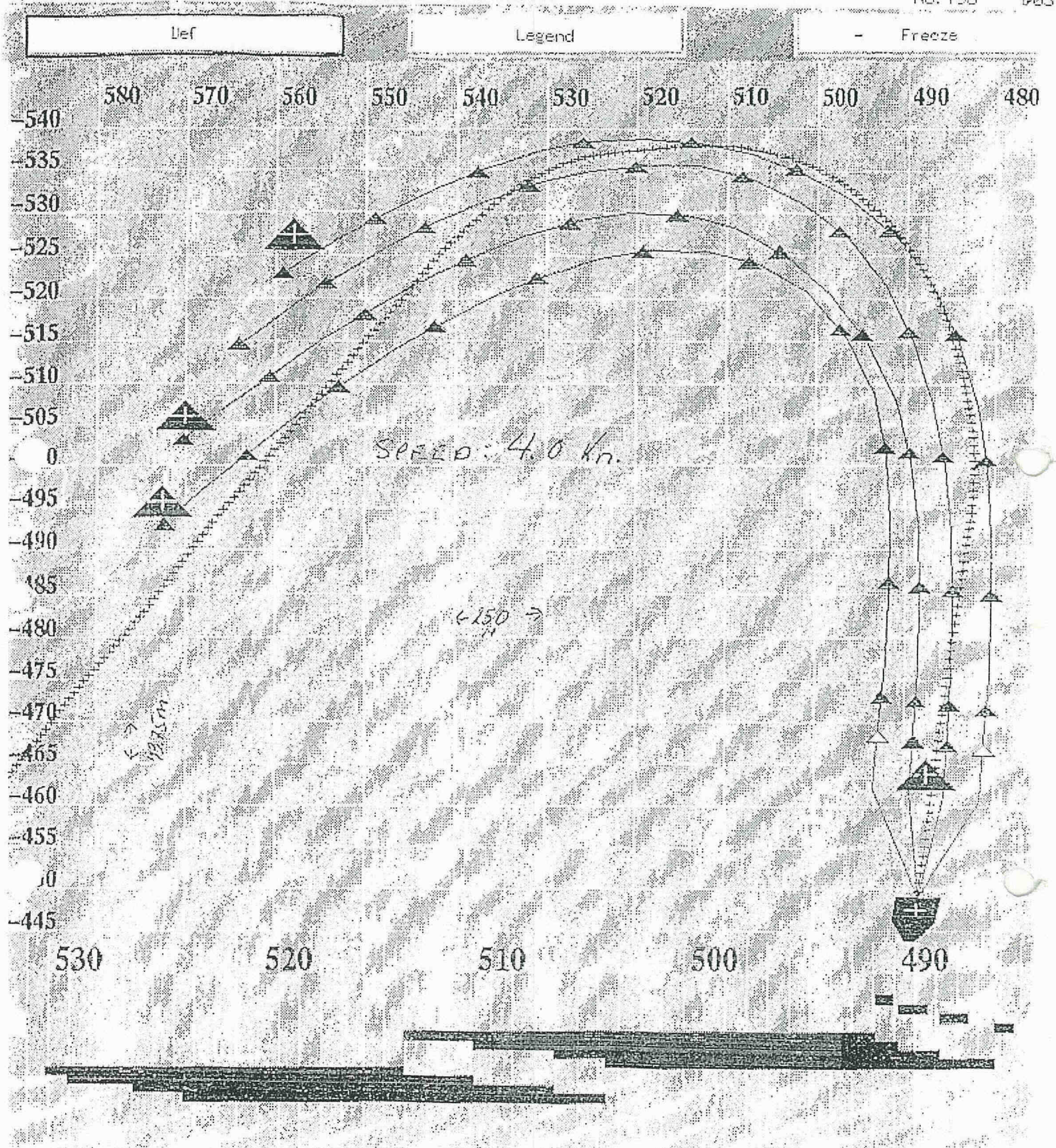


$$F_y \cos\alpha \cdot \frac{l}{2} + F_x \sin\alpha \cdot \frac{l}{2} - w \cos\alpha \frac{l}{2} + F_v \cos\alpha \cdot l + F_i \sin\alpha \cdot l = 0$$

$$\left(\frac{F_x}{2} + F_i \right) \sin\alpha = \left(\frac{F_y}{2} - \frac{w}{2} + F_v \right) \cos\alpha$$

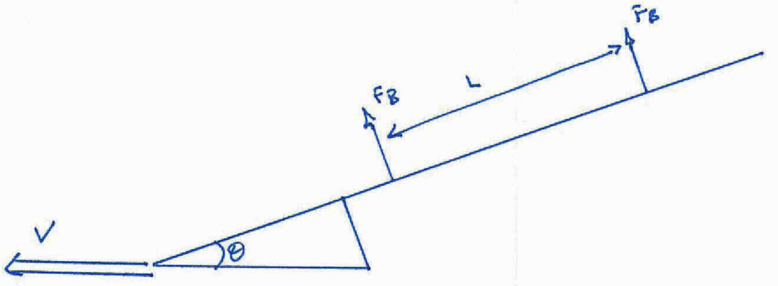
$$\tan\alpha = \frac{\frac{F_y}{2} - \frac{w}{2} + F_v}{\frac{F_x}{2} + F_i}$$

25/07/93 15:04 GECO DIAMOND → 47 2 475537 NO. 796 P03



```
Input file for dynamic.C dynamic streamer program - bird deflection
-------------------------------------------------------------------

Basic conditions
----------------
water_density = 1020.0      // rho (Kg/m3)
diameter      = 60.0e-3     // diameter (m)
length        = 3000.0      // length of streamer (m)

theat0        = 3.14159     // Initial angle to horizontal
ucross        = 0.0         // Cross flow velocity (m/s)
tail_tension  = 10.0        // Tail Tension (N)

Bird control
------------
nbird         = 2           // number of birds (evenly distributed)

Bird_functions
1500.0  300.0 0.0 0.0 0.0
2000.0 -300.0 0.0 0.0 0.0

Boat control
------------
time        x_boat       yboat
Boat_positions
0.0         0.0          0.0
10.0        25.0         0.0
100.0       250.0        0.0

Numerical constants
-------------------
nsave         = 8           // number of points at which to output streamer po
number_nodes  = 101         // number of points along streamer
number_tsteps = 200         // number time steps (increase this if calcs crash
itmax         = 50          // maximum iterations for the non-linear iterator
epsnlit       = 1.0e-6      // epsilon for the non-linear solver

Fluid mechanics constants
-------------------------
Ct            = 0.007       // Ct Tangential drag coefficient
Cn            = 1.1         // Cn Normal drag coefficient
```

SIMPLE ESTIMATE FOR HORIZONTAL STEERING



Boat speed = V
Cross flow = $V \sin\theta$
Bird Force = F_B
Distance between birds = L

θ = Feathering Angle
ρ = water density
d = streamer diameter
C_n = Drag coeff normal to streamer.

Assume forces normal to streamer balance. F_B distributed evenly over L. ∴

$$\frac{1}{2} \rho d C_n V^2 \sin^2\theta \, L = F_B$$

∴

$$\sin\theta = \left(\frac{2 F_B}{\rho d C_n V^2 L} \right)^{1/2}$$

Typical F_B = 500 N, ρ = 1000 Kg/m², d = 60 × 10⁻³, C_n = 1
V = 3 ms⁻¹, L = 300 m

θ = 4.5 degrees.

```
// FILE:  blocksolve.cpp

// block tri-diagonal solver
//
//   ( b[1] c[1]                   ) ( y[1] )       ( d[1] )
//   ( a[2] b[2] c[2]              ) ( y[2] )       ( d[2] )
//   (      a[3] b[3] c[3]         ) ( y[3] )       ( d[3] )
//   ( ........................... ) ( .... )   =   ( .... )
//   (                   b[n] c[n] ) ( y[n] )       ( d[n] )
//
// where a[i],b[i],c[i] are m x m matricies
// y, d at m vectors

#include <stdio.h>
#include <math.h>
#define  NRANSI
#include "nrutil.h"

void blocksolve(float ***a, float ***b, float ***c, float **y,
                float **d, int m, int n)
// a[1..n][1..m][1..m] , similarly b and c
// y[1..n][1..m]
// d[1..n][1..m]
{
    int    i;
    float  ***e, **f, **minv, **mtemp, *vtemp;

    void matinv(float **ainv, float **a, int n);
    void matmul(float **ab, float **a, float **b, int n);
    void mscal(float **a, float s, int m);
    void matvec(float *v1, float **a, float *v2, int m);
    void vscal(float *v1, float s, int m);
    void matadd(float **a, float **b, int m);
    void veccop(float *a, float *b, int m);
    void vecadd(float *a, float *b, int m);

    e     = f3tensor(1, n, 1, m, 1, m);
    f     = matrix(1, n, 1, m);
    minv  = matrix(1, m, 1, m);
    mtemp = matrix(1, m, 1, m);
    vtemp = vector(1, m);

    matinv(minv, b[1], m);
    matmul(e[1], minv, c[1], m);
    mscal(e[1], -1.0, m);
    matvec(f[1], minv, d[1], m);

    for (i=2; i<=n; i++) {

        matmul(mtemp, a[i], e[i-1], m);
        matadd(mtemp, b[i], m);
        matinv(minv, mtemp, m);
        matmul(e[i], minv, c[i], m);
        mscal(e[i], -1.0, m);

        matvec(vtemp, a[i], f[i-1], m);
        vscal(vtemp, -1.0, m);
        vecadd(vtemp, d[i], m);
        matvec(f[i], minv, vtemp, m);

    }

    veccop(y[n], f[n], m);
    for (i=n-1; i>=1; i--) {
```

```
        matvec(y[i], e[i], y[i+1], m);
        vecadd(y[i], f[i], m);

    }

    free_vector(vtemp, 1, m);
    free_matrix(mtemp, 1, m, 1, m);
    free_matrix(minv, 1, m, 1, m);
    free_matrix(f, 1, n, 1, m);
    free_f3tensor(e, 1, n, 1, m, 1, m);
}
```