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The dynamics of towed flexible cylinders
Part 1. Neutrally buoyant elements

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The transverse vibrations of a thin, flexible cylinder under nominally constant towing conditions are investigated. The cylinder is neutrally buoyant, of radius a_A with a free end and very small bending stiffness. As the cylinder is towed with velocity U, the tangential drag causes the tension in the cylinder to increase from zero at its free end to a maximum at the towing point. Transverse vibrations of the cylinder are opposed by a normal viscous drag force. Both the normal and tangential viscous forces can be described conveniently in terms of drag coefficients C_N and C_T. The ratio C_N/C_T has a crucial effect on the motion of the cylinder. The form of the transverse displacement is found to be greatly influenced by the existence of a critical point at which the effect of tension in the cylinder is cancelled by a fluid loading term. Matched asymptotic expansions are used to extend the solution across this critical point to apply the downstream boundary condition. Displacements well upstream of the critical point have a simple form, while nearer to the critical point the solution depends on whether the normal drag coefficient C_N is greater or less than one-half C_T.

The typical acoustic streamer geometry considered is found to be stable to transverse displacements at all towing speeds. Forced perturbations of frequency ω are investigated. At low frequencies they propagate effectively along the cylinder with speed U. At higher frequencies they are attenuated.

The effect of a rope drogue of length l_R, radius a_R, is investigated. Provided $\omega l_R a_R/U a_A$ is very small, the drogue has the same effect as a small increase in the length of the cylinder. However at higher frequencies and for small values of the ratio C_N/C_T attaching a drogue may be disadvantageous because it reduces the attenuation of high-frequency disturbances as they propagate down the cylinder.

1. Introduction

Towed instrumentation packages in the form of long flexible cylinders are used extensively to detect and analyse acoustic signals in the ocean. A typical geometry is illustrated in figure 1. It consists of a heavier-than-water cable attached at one end to a ship and at the other to a neutrally buoyant slender cylinder containing a sonar array. This cylinder is sometimes referred to as an acoustic 'streamer' or 'array'. There may possibly be a rope at the downstream end of the cylinder acting as a drogue. If such an arrangement is to give good resolution of the acoustic signals it detects, the instantaneous shape of the acoustic streamer must be known. When the ship maintains a constant velocity, the cylinder is straight and horizontal. However, changes in the ship's path will make it deform. In these two papers we analyse linear departures from the ideal case due, for example, to changes in ship speed or heading.

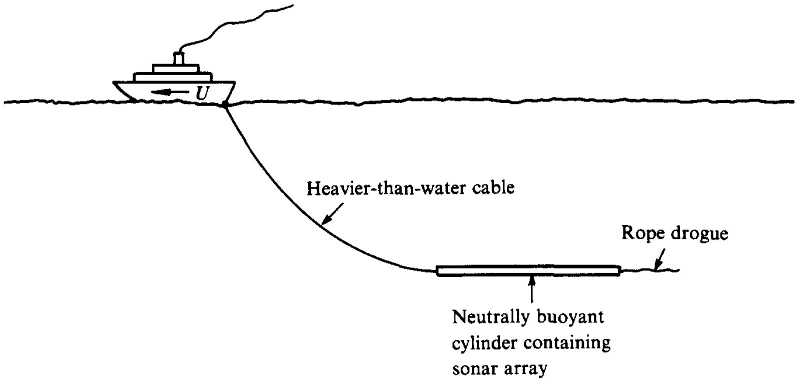

FIGURE 1. Typical geometry for a ship towing an array.

The aim of this work is to provide a simple means by which the shape of the towed system can be predicted either from the ship's path or from an accelerometer at the leading edge of the cylinder. Part 1 deals with the displacements of the neutrally buoyant elements, while Part 2 investigates the propagation of disturbances along the negatively buoyant cable. The results of Part 1 provide the downstream boundary conditions for the cable in Part 2.

Computer programs have been developed to calculate the three-dimensional path of a towed system as a ship manoeuvres (see for example Ivers & Mudie 1973, 1975; Huston & Kamman 1981; Sanders 1982; Ablow & Schechter 1983). In general these packages require considerable computing resources and, if they are to run in real time, certain simplifying assumptions must be made. Since we are investigating small departures from constant velocity, we adopt a different approach, and linearize the transverse equations of motion. Païdoussis (1966, 1968) derived a linearized form of the transverse momentum equation for neutrally buoyant flexible cylinders with an axial flow. A term has been omitted from these early versions and Païdoussis (1973) gives the correct form of the equation of motion. Disturbances of frequency ω satisfy a linear fourth-order differential equation. The coefficient of the fourth derivative depends on the bending stiffness of the cylinder. Perturbations of cylinders whose response depends on their bending stiffness have been extensively studied in the literature (see for example Hawthorne 1961; Païdoussis 1966, 1968, 1973; Lee 1978; Prokhorovich, Prokhorovich & Smirnov 1982).

However, acoustic streamers are very long in comparison with their radius a_A, and, for motions with wavelengths comparable with the cylinder length, the restoring force due to bending stiffness is exceedingly small. It is therefore appropriate to recognize this and neglect the effect of the bending stiffness over most of the cylinder. This approximation has been made by Ortloff & Ives (1969), Kennedy (1980), Kennedy & Strahan (1981) and Lee & Kennedy (1985). The differential equation then reduces to second order, the coefficient of the highest derivative being $T(x)-\rho_0 \pi a_A^2 U^2$, where $T(x)$ is the tension in the cylinder and varies along its length. U is the mean flow velocity and ρ_0 the density of the surrounding fluid; $\rho_0 \pi a_A^2 U^2$ arises due to the effect of fluid loading.

In §2–4 we consider a thin, neutrally buoyant, flexible cylinder with its downstream end unrestrained. $T(x)$ then vanishes at this free end, and increases along the cylinder due to tangential drag. It is well known that the transverse displacements of a tensioned string *in vacuo* satisfy a hyperbolic differential equation. When fluid loading is included, the equation remains hyperbolic over most of its length, but is elliptic near the free end, having a regular singular point at x_c, a critical position at which $T(x_c) = \rho_0 \pi a_A^2 U^2$. Ortloff & Ives and Kennedy & Strahan base their work on Païdoussis' early erroneous equation of motion, and find one of the solutions of their linear, second-order equation to be unbounded at x_c. They therefore reject this solution and the downstream boundary condition and describe the response of the cylinder in terms of the other (finite) solution. However, when the correct form of Païdoussis' equation is used, and for reasonable values of the drag coefficients, both solutions are finite at the critical position, although one solution has a branch point there. Hence, before the downstream boundary condition can be applied, further investigation is needed to see how this solution behaves as x crosses x_c. In the region of x_c the response is controlled by the bending stiffness of the cylinder. We therefore use the fourth-order equation in the region of x_c, and the method of matched asymptotic expansions to join these 'inner' bending solutions to the 'outer' tension-dominated response. In this way, the general solution for the vibration of a fluid-loaded cylinder, in the limit of small bending stiffness, can be found. Application of the free-end boundary condition then leads to an analytical expression for transverse displacements of the towed cylinder at frequency ω. The displacements are found to have a simple form well upstream of x_c. Nearer to x_c the expression is more complicated and depends on whether the normal drag coefficient C_N is greater or less than half the tangential drag coefficient C_T.

In §3 we investigate the stability of a neutrally buoyant towed cylinder by seeing whether there are any free modes that grow in time. A practical streamer geometry is found to be stable at all towing speeds.

Since the towed cylinder is stable, it is appropriate to determine its response to forcing at its upstream end. At low frequencies for which $\omega l_A/U$ is small the disturbances propagate along the cylinder, virtually unchanged in amplitude and with a phase speed U, while for higher frequencies the disturbances decay in amplitude along the streamer. The first form of motion is often described as 'worm-in-a-hole' because all points of the cylinder take the same track. This motion is compatible with observations of low-frequency oscillations of towed arrays.

So far we have assumed the end of the cylinder to be free. However in many practical situations it is attached to a rope drogue. In §5 we investigate the effect of a rope drogue. We use the work in the earlier sections to describe the transverse motions of the rope which has a free end, and apply continuity of displacement and force at the junction of the drogue and cylinder. If $\omega l_R a_R/U a_A$ is very small, the main effect of the drogue is found to be the same as an increase in length of the cylinder by an amount $l_R a_R/a_A$, where a_R and l_R are the radius and length of the rope drogue respectively. At higher frequencies a drogue can have an adverse effect if the ratio C_N/C_T is small. Then attaching a drogue reduces the attenuation of high-frequency transverse disturbances as they propagate down the cylinder.

2. The transverse motion of a neutrally buoyant flexible cylinder

Consider an acoustic streamer consisting of a long flexible cylinder of length l_A, radius a_A, towed in the negative x-direction at a constant speed U. If the cylinder is neutrally buoyant its mean position is horizontal. We will investigate linear departures from this arrangement and choose a frame of reference in which the distant fluid has a velocity $(U, 0, 0)$, with the origin at the mean position of the

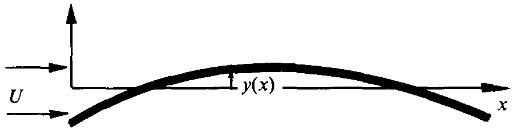

FIGURE 2. Linear perturbations of the towed cylinder, viewed in a frame of reference in which the distant fluid has velocity $(U, 0, 0)$.

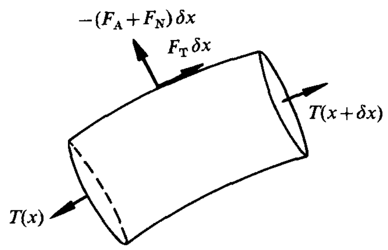

FIGURE 3. Forces acting on a small length, δx, of the neutrally buoyant cylinder.

upstream end of the cylinder as shown in figure 2. It follows from the neutral buoyancy of the cylinder, and the linearity of the disturbances, that perturbations in the $(y = 0)$- and $(z = 0)$-planes satisfy identical uncoupled equations. It is therefore sufficient just to investigate the motion in one plane, $(z = 0)$ say.

The equation of motion of the cylinder may be derived by considering the balance of forces on a small length as shown in figure 3. Let $T(x)$ be the variable tension in the cylinder, F_N and F_T the viscous forces acting on the cylinder per unit length in the local normal and tangential directions respectively, and F_A the inviscid force due to the acceleration of the virtual mass of the cylinder. Resolving in the x-direction gives, to zeroth order in the perturbations,

$$F_T + \frac{\partial T}{\partial x} = 0. \tag{2.1}$$

The transverse momentum balance gives, to first order in the disturbances,

$$m\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right) - F_A - F_N + F_T\frac{\partial y}{\partial x} - B\frac{\partial^4 y}{\partial x^4}, \tag{2.2}$$

where m is the mass of the cylinder per unit length and B is its bending stiffness. This is Païdoussis' equation. The term $F_T\,\partial y/\partial x$ was omitted in Païdoussis' early work (see for example Païdoussis 1966, 1968). This error was later corrected (Païdoussis 1973 and Païdoussis & Yu 1976) but unfortunately the earlier erroneous form has been adopted in much of the towed-array literature (Ortloff & Ives 1969; Kennedy 1980; Kennedy & Strahan 1981). As Païdoussis (1973) points out, omitting the term $F_T\,\partial y/\partial x$ is equivalent to taking the tangential viscous force to act in the x-direction rather than in the instantaneous tangential direction.

F_A is the force required to accelerate the neighbouring fluid as the cylinder deforms. Provided the flow does not separate, the expression derived by Lighthill (1960) may be used:

$$F_A = \rho_0 \pi a_A^2 \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 y, \tag{2.3}$$

where ρ_0 is the fluid density.

The viscous forces acting on a long, thin flexible cylinder are discussed by Taylor (1952). He proposes the form

$$F_N = \tfrac{1}{2}\rho_0 V^2\{2a_A C_D \sin^2 i + 2\pi a_A C_N \sin i\}, \tag{2.4a}$$

$$F_T = \rho_0 V^2 \pi a_A C_T \cos i, \tag{2.4b}$$

where V is the magnitude of the relative velocity between the cylinder and the distant flow, and i is the angle between this relative velocity and the local tangent. For linear perturbations of a neutrally buoyant element, V is equal to U and i is small,

$$\sin i \approx i \approx \frac{1}{U}\frac{\partial y}{\partial t} + \frac{\partial y}{\partial x}, \quad \cos i \approx 1.$$

These expressions therefore reduce to

$$\mathrm{F}_N = \rho_0 \pi a_A U C_N \left(\frac{\partial y}{\partial t} + U\frac{\partial y}{\partial x}\right), \tag{2.5a}$$

$$F_T = \rho_0 \pi a_A U^2 C_T. \tag{2.5b}$$

With this form for the tangential drag the longitudinal momentum equation (2.1) may be integrated immediately to give

$$T(x) = T(l_A) + \rho_0 \pi a_A U^2 C_T (l_A - x), \tag{2.6}$$

$T(l_A)$ is the tension at the downstream end of the array, and vanishes if the end is free. Then $T(0)$, the tension at the upstream end, is directly proportional to the drag coefficient C_T. Hence C_T may be inferred from measurements of $T(0)$. Data from large-scale experiments suggest $C_T = 0.0025$ (Andrew private communication 1984). Ni & Hansen (1978) obtained similar values of C_T for a range of Reynolds numbers in their rig experiments. There is less evidence about the appropriate value of C_N. Taylor discusses in some detail how the value of C_N would vary in the range $0 \leqslant C_N \leqslant C_T$ depending on the type of roughness on the cylinder. We will therefore investigate the effect of varying C_N within this range.

When the expressions for F_A and F_N in (2.3) and (2.5) are substituted into the transverse momentum equation (2.2) they lead to

$$\rho_0 \pi a_A^2 \frac{\partial^2 y}{\partial t^2} = (T(x) - \rho_0 \pi a_A^2 U^2)\frac{\partial^2 y}{\partial x^2} - \rho_0 \pi a_A^2 \left(\frac{\partial^2 y}{\partial t^2} + 2U\frac{\partial^2 y}{\partial x \partial t}\right) - \rho_0 \pi a_A U C_N \left(\frac{\partial y}{\partial t} + U\frac{\partial y}{\partial x}\right) - B\frac{\partial^4 y}{\partial x^4}. \tag{2.7}$$

The coefficient of the second derivative of y vanishes at a position x_c, where $T(x_c)$ is equal to the fluid-loading term $\rho_0 \pi a_A^2 U^2$. Using (2.6) to rewrite $T(x)$ shows that

$$T(x) - \rho_0 \pi a_A^2 U^2 = \rho_0 \pi a_A U^2 C_T (x_c - x), \tag{2.8}$$

where

$$x_c = l_A + \frac{T(l_A)}{\rho_0 \pi a_A U^2 C_T} - \frac{a_A}{C_T}. \tag{2.9}$$

x_c lies on the cylinder if

$$l_A \geqslant x_c \geqslant 0,$$

i.e. if

$$1 \geqslant \frac{T(l_A)}{\rho_0 \pi a_A^2 U^2} \geqslant 1 - \frac{l_A C_T}{a_A}. \tag{2.10}$$

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