The principles of OFDM

Multicarrier modulation techniques are rapidly moving from the textbook to the real world of modern communication systems

By Louis Litwin and Michael Pugel

The principles of orthogonal frequency division multiplexing (OFDM) modulation have been in existence for several decades. However, in recent years these techniques have quickly moved out of textbooks and research laboratories and into practice in modern communications systems. The techniques are employed in data delivery systems over the phone line, digital radio and television, and wireless networking systems. What is OFDM? And why has it recently become so popular? This article will review the fundamentals behind OFDM techniques, and also discuss common impairments and how, in some cases, OFDM mitigates their effect. Where applicable, the impairment effects and techniques will be compared to those in a single carrier system. A brief overview of some modern applications will conclude the article.

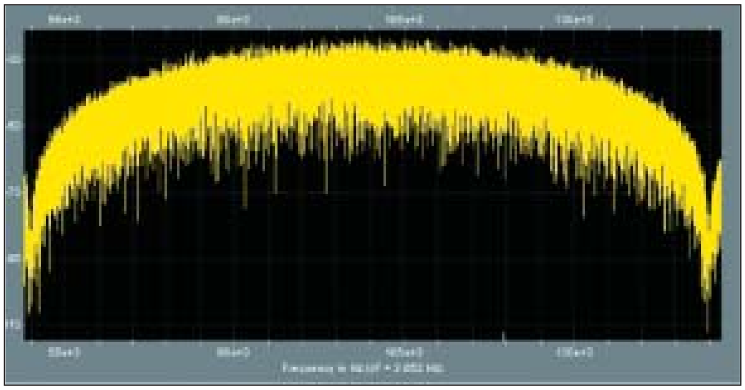

Figure 1. Single carrier spectrum example.

The single-carrier modulation system

A typical single-carrier modulation spectrum is shown in Figure 1. A single carrier system modulates information onto one carrier using frequency, phase, or amplitude adjustment of the carrier. For digital signals, the information is in the form of bits, or collections of bits called symbols, that are modulated onto the carrier. As higher bandwidths (data rates) are used, the duration of one bit or symbol of information becomes smaller. The system becomes more susceptible to loss of information from impulse noise, signal reflections and other impairments. These impairments can impede the ability to recover the information sent. In addition, as the bandwidth used by a single carrier system increases, the susceptibility to interference from other continuous signal sources becomes greater. This type of interference is commonly labeled as carrier wave (CW) or frequency interference.

Frequency division multiplexing modulation system

Frequency division multiplexing (FDM) extends the concept of single carrier modulation by using multiple subcarriers within the same single channel. The total data rate to be sent in the channel is divided between the various subcarriers. The data do not have to be divided evenly nor do they have to originate from the same information source. Advantages include using separate modulation/demodulation customized to a particular type of data, or sending out banks of dissimilar data that can be best sent using multiple, and possibly different, modulation schemes.

Current national television systems committee (NTSC) television and FM stereo multiplex are good examples of FDM. FDM offers an advantage over single-carrier modulation in terms of narrowband frequency interference since this interference will only affect one of the frequency subbands. The other subcarriers will not be affected by the interference. Since each subcarrier has a lower information rate, the data symbol periods in a digital system will be longer, adding some additional immunity to impulse noise and reflections.

FDM systems usually require a guard band between modulated subcarriers to prevent the spectrum of one subcarrier from interfering with another. These guard bands lower the system's effective information rate when compared to a single carrier system with similar modulation.

Orthogonality and OFDM

If the FDM system above had been able to use a set of subcarriers that were orthogonal to each other, a higher level of spectral efficiency could have been achieved. The guardbands that were necessary to allow individual demodulation of subcarriers in an FDM system would no longer be necessary. The use of orthogonal subcarriers would allow the subcarriers' spectra to overlap, thus increasing the spectral efficiency. As long as orthogonality is maintained, it is still possible to recover the individual subcarriers' signals despite their overlapping spectrums.

If the dot product of two deterministic signals is equal to zero, these signals are said to be orthogonal to each other. Orthogonality can also be

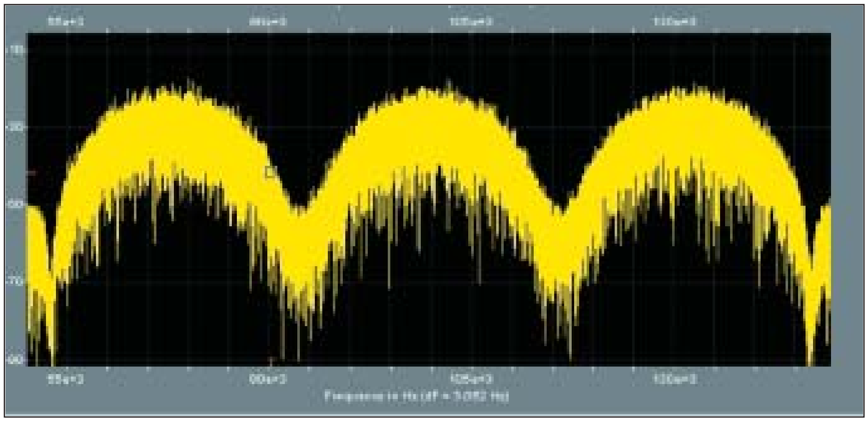

Figure 2. FDM signal spectrum example.

viewed from the standpoint of stochastic processes. If two random processes are uncorrelated, then they are orthogonal. Given the random nature of signals in a communications system, this probabilistic view of orthogonality provides an intuitive understanding of the implications of orthogonality in OFDM. Later in this article, we will discuss how OFDM is implemented in practice using the discrete fourier transform (DFT). Recall from signals and systems theory that the sinusoids of the DFT form an orthogonal basis set, and a signal in the vector space of the DFT can be represented as a linear combination of the orthogonal sinusoids. One view of the DFT is that the transform essentially correlates its input signal with each of the sinusoidal basis functions. If the input signal has some energy at a certain frequency, there will be a peak in the correlation of the input signal and the basis sinusoid that is at that corresponding frequency. This transform is used at the OFDM transmitter to map an input signal onto a set of orthogonal subcarriers, i.e., the orthogonal basis functions of the DFT. Similarly, the transform is used again at the OFDM receiver to process the received subcarriers. The signals from the subcarriers are then combined to form an estimate of the source signal from the transmitter. The orthogonal and uncorrelated nature of the subcarriers is exploited in OFDM with powerful results. Since the basis functions of the DFT are uncorrelated, the correlation performed in the DFT for a given subcarrier only sees energy for that corresponding subcarrier. The energy from other subcarriers does not contribute because it is uncorrelated. This separation of signal energy is the reason that the OFDM subcarriers' spectrums can overlap without causing interference.

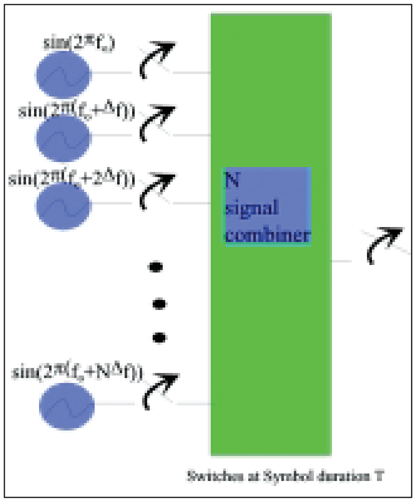

Figure 3. A Simple OFDM generator. N subcarriers transmitting 1 bit of information each, by turning on and off at time intervals T.

A simple OFDM example

Figure 3 shows a simple representation of an OFDM system. These types of systems have been built but the practicality of such construction quickly diminishes as the number of subcarriers increases. Each subcarrier carries one bit of information (N bits total) by its presence or absence in the output spectrum. The frequency of each subcarrier is selected to form an orthogonal signal set, and these frequencies are known at the receiver. Note that the output is updated at a periodic interval T that forms the symbol period as well as the time boundary for orthogonality. Figure 4 shows the resultant frequency spectrum. In the frequency domain, the resulting sin function side lobes produce overlapping spectra. The individual peaks of subbands all line up with the zero crossings of the other subbands. This overlap of spectral energy does not interfere with the system's ability to recover the original signal. The receiver multiplies (i.e., correlates) the incoming signal by the known set of sinusoids to produce the original set of bits sent. The digital implementation of an OFDM system will enhance these simple principles and permit more complex modulation.

Implementation of an OFDM system

The idea behind the analog implementation of OFDM can be extended to

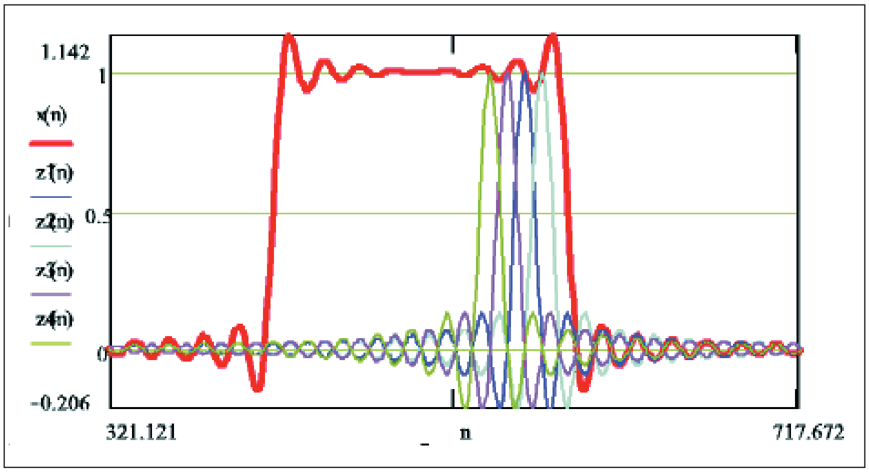

Figure 4. Overall spectrum of the simple OFDM signal shown with four subcarriers within. Note that the zero crossings all correspond to peaks of adjacent subcarriers.

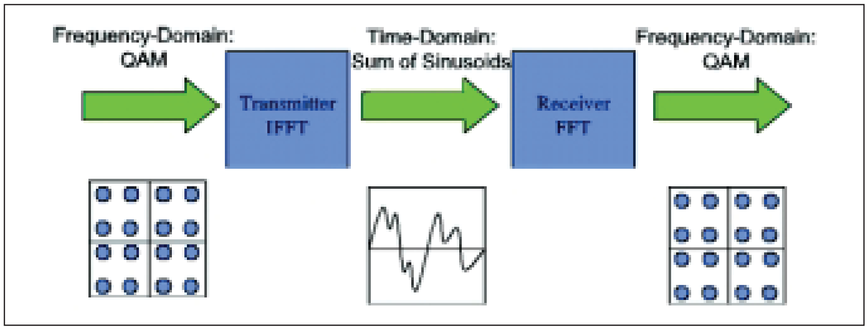

Figure 5. Block diagram of a simple OFDM system.

the digital domain by using the discrete Fourier Transform (DFT) and its counterpart, the inverse discrete Fourier Transform (IDFT). These mathematical operations are widely used for transforming data between the time-domain and frequency-domain. These transforms are interesting from the OFDM perspective because they can be viewed as mapping data onto orthogonal subcarriers. For example, the IDFT is used to take in frequency-domain data and convert it to time-domain data. In order to perform that operation, the IDFT correlates the frequency-domain input data with its orthogonal basis functions, which are sinusoids at certain frequencies. This correlation is equivalent to mapping the input data onto the sinusoidal basis functions.

In practice, OFDM systems are implemented using a combination of fast Fourier Transform (FFT) and inverse fast Fourier Transform (IFFT) blocks that are mathematically equivalent versions of the DFT and IDFT, respectively, but more efficient to implement. An OFDM system treats the source symbols (e.g., the QPSK or QAM symbols that would be present in a single carrier system) at the transmitter as though they are in the frequency-domain. These symbols are used as the inputs to an IFFT block that brings the signal into the time-domain. The IFFT takes in N symbols at a time where N is the number of subcarriers in the system. Each of these N input symbols has a symbol period of T seconds. Recall that the basis functions for an IFFT are N orthogonal sinusoids. These sinusoids each have a different frequency and the lowest frequency is DC. Each input symbol acts like a complex weight for the corresponding sinusoidal basis function. Since the input symbols are complex, the value of the symbol determines both the amplitude and phase of the sinusoid for that subcarrier. The IFFT output is the summation of all N sinusoids. Thus, the IFFT block provides a simple way to modulate data onto N orthogonal subcarriers. The block of N output samples from the IFFT make up a single OFDM symbol. The length of the OFDM symbol is NT where T is the IFFT input symbol period mentioned above.

After some additional processing, the time-domain signal that results from the IFFT is transmitted across the channel. At the receiver, an FFT block is used to process the received signal and bring it into the frequency-domain. Ideally, the FFT output will be the original symbols that were sent to the IFFT at the transmitter. When plotted in the complex plane, the FFT output samples will form a constellation, such as 16-QAM. However, there is no notion of a constellation for the time-domain signal. When plotted on the complex plane, the time-domain signal forms a scatter plot with no regular shape. Thus, any receiver processing that uses the concept of a constellation (such as symbol slicing) must occur in the frequency-domain. The block diagram in Figure 5 illustrates the switch between frequency-domain and time-domain in an OFDM system.

Multipath channels and the use of cyclic prefix

A major problem in most wireless systems is the presence of a multipath channel. In a multipath environment, the transmitted signal reflects off of several objects. As a result, multiple delayed versions of the transmitted signal arrive at the receiver. The multiple versions of the signal cause the received signal to be distorted. Many wired systems also have a similar problem where reflections occur due to impedance mismatches in the transmission line.

A multipath channel will cause two problems for an OFDM system. The first problem is intersymbol interference. This problem occurs when the received OFDM symbol is distorted by the previously transmitted OFDM symbol. The effect is similar to the intersymbol interference that occurs in a single-carrier system. However, in such systems, the interference is typically due to several other symbols instead of just the previous symbol; the symbol period in single carrier systems is typically much shorter than the time span of the channel, whereas the typical OFDM symbol period is much longer than the time span of the channel. The second problem is unique to multicarrier systems and is called Intrasymbol Interference. It is the result of interference amongst a given OFDM symbol's own subcarriers. The next sections illustrate how OFDM deals with these two types of interference.

Intersymbol interference

Assume that the time span of the channel is L_C samples long. Instead of a single carrier with a data rate of R symbols/second, an OFDM system has N subcarriers, each with a data rate of R/N symbols/second. Because the data rate is reduced by a factor of N, the OFDM symbol period is increased by a factor of N. By choosing an appropriate

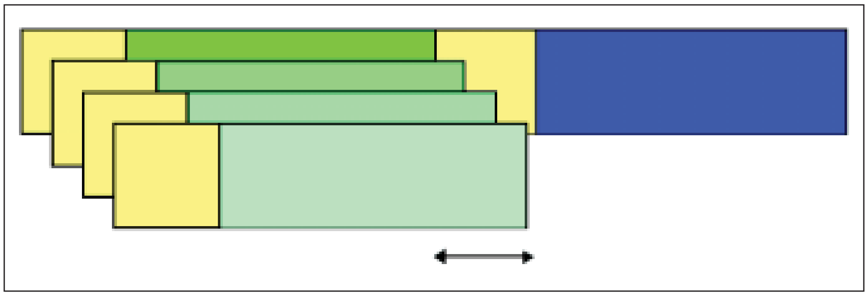

Figure 6. Example of intersymbol interference. The green symbol was transmitted first, followed by the blue symbol.

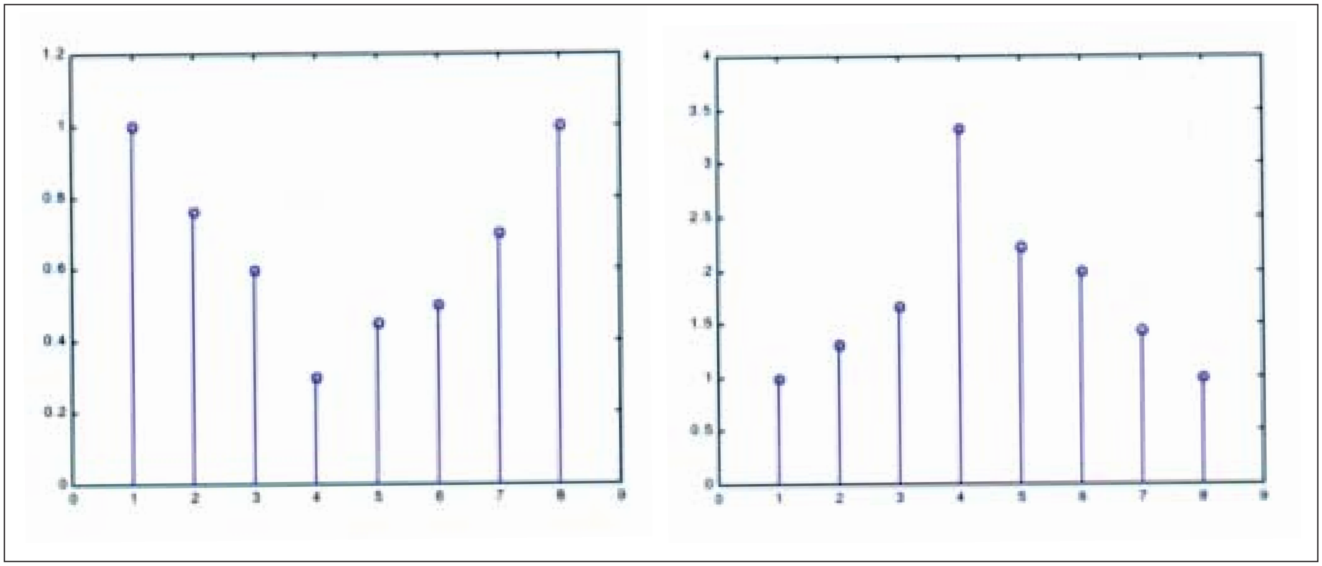

Figure 7. Left plot shows the frequency response of a channel, and the right plot shows the corresponding frequency-domain equalizer response. Note that the equalizer response is large when the channel response is small in order to counteract the effect of a channel null.

value for N, the length of the OFDM symbol becomes longer than the time span of the channel. Because of this configuration, the effect of intersymbol interference is the distortion of the first L_C samples of the received OFDM symbol. An example of this effect is shown in Figure 6. By noting that only the first few samples of the symbol are distorted, one can consider the use of a guard interval to remove the effect of intersymbol interference. The guard interval could be a section of all zero samples transmitted in front of each OFDM symbol. Since it does not contain any useful information, the guard interval would be discarded at the receiver. If the length of the guard interval is properly chosen such that it is longer than the time span of the channel, the OFDM symbol itself will not be distorted. Thus, by discarding the guard interval, the effects of intersymbol interference are thrown away as well.

Intrasymbol interference

The guard interval is not used in practical systems because it does not prevent an OFDM symbol from interfering with itself. This type of interference is called intrasymbol interference. The solution to the problem of intrasymbol interference involves a discrete-time property. Recall that in continuous-time, a convolution in time is equivalent to a multiplication in the frequency-domain. This property is true in discrete-time only if the signals are of infinite length or if at least one of the signals is periodic over the range of the convolution. It is not practical to have an infinite-length OFDM symbol, however, it is possible to make the OFDM symbol appear periodic. This periodic form is achieved by replacing the guard interval with something known as a cyclic prefix of length L_P samples. The cyclic prefix is a replica of the last L_P samples of the OFDM symbol where $L_P > L_C$. Since it contains redundant information, the cyclic prefix is discarded at the receiver. Like the case of the guard interval, this step removes the effects of intersymbol interference. Because of the way in which the cyclic prefix was formed, the cyclically-extended OFDM symbol now appears periodic when convolved with the channel. An important result is that the effect of the channel becomes multiplicative.

In a digital communications system, the symbols that arrive at the receiver have been convolved with the time-domain channel impulse response of length L_C samples. Thus, the effect of the channel is convolutional. In order to undo the effects of the channel, another convolution must be performed at the receiver using a time-domain filter known as an equalizer. The length of the equalizer needs to be on the order of the time span of the channel. The equalizer processes symbols in order to adapt its response in an attempt to remove the effects of the channel. Such an equalizer can be expensive to implement in hardware and often requires a large number of symbols in order to adapt its response to a good setting.

In OFDM, the time-domain signal is still convolved with the channel response. However, the data will ultimately be transformed back into the frequency-domain by the FFT in the receiver. Because of the periodic nature of the cyclically-extended OFDM symbol, this time-domain convolution will result in the multiplication of the spectrum of the OFDM signal (i.e., the frequency-domain constellation points) with the frequency response of the channel. The result is that each subcarrier's symbol will be multiplied by a complex number equal to the channel's frequency response at that subcarrier's frequency. Each received subcarrier experiences a complex gain (amplitude and phase distortion) due to the channel. In order to undo these effects, a frequency-domain equalizer is employed. Such an equalizer is much simpler than a time-domain equalizer. The frequency-domain equalizer consists of a single complex multiplication for each subcarrier. For the simple case of no noise, the ideal value of the equalizer's response is the inverse of the channel's frequency response. An example is shown in Figure 7. With such a setting, the frequency-domain equalizer would cancel out the multiplicative effect of the channel.

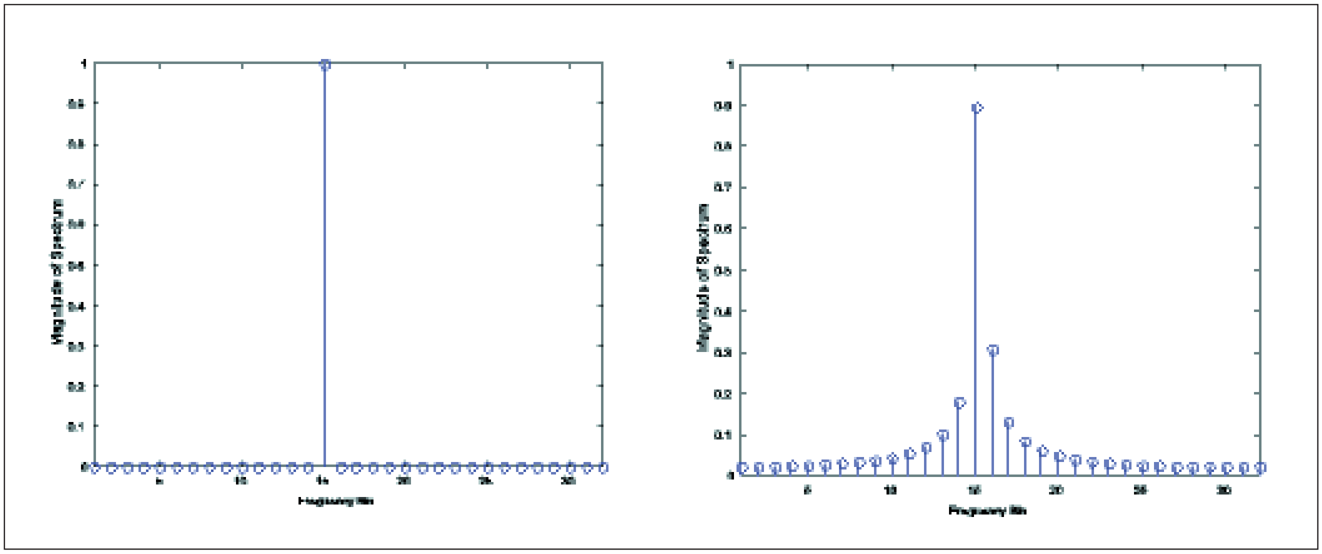

Figure 8. Received spectrum with one non-zero subcarrier. The left plot is for the case of no LO offset, and the right plot is for the presence of an LO offset.

COFDM: Coded OFDM

Coded OFDM, or COFDM, is a term used for a system in which the error control coding and OFDM modulation processes work closely together. An important step in a COFDM system is to interleave and code the bits prior to the IFFT. This step serves the purpose of taking adjacent bits in the source data and spreading them out across multiple subcarriers. One or more subcarriers may be lost or impaired due to a frequency null, and this loss would cause a contiguous stream of bit errors. Such a burst of errors would typically be hard to correct. The interleaving at the transmitter spreads out the contiguous bits such that the bit errors become spaced far apart in time. This spacing makes it easier for the decoder to correct the errors. Another important step in a COFDM system is to use channel information from the equalizer to determine the reliability of the received bits. The values of the equalizer response are used to infer the strength of the received subcarriers. For example, if the equalizer response had a large value at a certain frequency, it would correspond to a frequency null at that point in the channel. The equalizer response would have a large value at that point because it is trying to compensate for the weak received signal. This reliability information is passed on to the decoding blocks so that they can properly weight the bits when making decoding decisions. In the case of a frequency null, the bits would be marked as “low confidence” and those bits would not be weighted as heavily as bits from a strong subcarrier. COFDM systems are able to achieve excellent performance on frequency-selective channels because of the combined benefits of multicarrier modulation and coding.

Non-ideal effects in an OFDM system

This section will examine the effects of non-idealities in an OFDM system. These effects will include impairments and receiver offsets. Because the fourier transform is a fundamental operation in OFDM, the effects of several offsets can be intuitively understood by applying fourier transform theory.

Local oscillator frequency offset

At start-up, the local oscillator (LO) frequency at the receiver is typically different from the LO frequency at the transmitter. A carrier tracking loop is used to adjust the receiver’s LO frequency in order to match the transmitter’s LO frequency as closely as possible. The effect of having an LO frequency offset can be explained by Fourier Transform theory. The LO offset can be expressed mathematically by multiplying the received time-domain signal by a complex exponential whose frequency is equal to the LO offset amount. Recall from Fourier Transform theory that multiplication by a complex exponential in time is equivalent to a shift in frequency. The LO offset results in a frequency shift of the received signal spectrum. This shift causes a condition called “loss of orthogonality” to occur. The frequency shift causes the OFDM subcarriers to no longer be orthogonal. The orthogonality of the subcarriers is lost because the bins of the FFT will no longer line up with the peaks of the received signal’s since pulses. The result is a distortion called inter-bin interference or IBI. IBI occurs when energy from one bin spills over into adjacent bins and this energy distorts the affected subcarriers. In Fourier Transform theory this effect is called DFT leakage.

The left plot of Figure 8 shows the spectrum of a received OFDM signal with no LO offset. For the purpose of clarity, only one non-zero subcarrier was transmitted. Note that this subcarrier is not interfering with its adjacent subcarriers. The spectrum of the non-zero subcarrier actually extends over the entire range of the FFT, however, due to the orthogonal nature of the signal, the zero-crossings of the spectrum exactly line up with the other FFT bins. The right plot of Figure 8 shows the received spectrum of the same signal with one non-zero subcarrier, however, in this case there is an LO offset. This offset has resulted in a loss of orthogonality, and the zero-crossings of the non-zero subcarrier’s spectrum no longer line up with the FFT bins. The result is that energy from the non-zero subcarrier is spread out among all of the other subcarriers, with those sub-

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