HARRIS' SHOCK AND VIBRATION HANDBOOK

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Fifth Edition

McGRAW-HILL
New York Chicago San Francisco Lisbon London Madrid
Mexico City Milan New Delhi San Juan Seoul
Singapore Sydney Toronto

Library of Congress Cataloging-in-Publication Data

Harris' shock and vibration handbook / Cyril M. Harris, editor, Allan G. Piersol, editor.—5th ed.
p. cm.
ISBN 0-07-137081-1
1. Vibration—Handbooks, manuals, etc. 2. Shock (Mechanics)—Handbooks, manuals, etc. I. Harris, Cyril M., date. II. Piersol, Allan G.
TA355.H35 2002
620.3—dc21 2001044228

McGraw-Hill
A Division of The McGraw-Hill Companies

1 2 3 4 5 6 7 8 9 0 DOC/DOC 0 7 6 5 4 3 2 1

ISBN 0-07-137081-1

The sponsoring editor for this book was Kenneth P. McCombs, the editing supervisor was Stephen M. Smith, and the production supervisor was Sherri Souffrance. It was set in Times Roman by North Market Street Graphics.

Printed and bound by R. R. Donnelley & Sons Company.

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$$(\delta_r)_{\max} = \frac{1}{\omega_n} F(\omega_n)$$
$$(A_{eq})_r = \frac{\omega_n^2 (\delta_r)_{\max}}{g} = \frac{\omega_n}{g} F(\omega_n) \tag{23.54}$$

This result is clearly evident for the Fourier spectrum and undamped shock response spectrum of the acceleration impulse. The Fourier spectrum is the horizontal line (independent of frequency) shown in Fig. 23.3*A* and the shock response spectrum is the inclined straight line (increasing linearly with frequency) shown in Fig. 23.7*A*. Since the impulse exists only at $t = 0$, the entire response is residual. The undamped shock spectra in the impulsive region of the half-sine pulse and the decaying sinusoidal acceleration, Fig. 23.7*C* and *D*, respectively, also are related to the Fourier spectra of these shocks, Fig. 23.3*C* and *D*, in a similar manner. This results from the fact that the maximum response occurs in the residual motion for systems with small natural frequencies. Another example is the entire negative shock response spectrum with no damping for the half-sine pulse in Fig. 23.7*C*, whose values are ω_n/g times the values of the Fourier spectrum in Fig. 23.3*C*.

METHODS OF DATA REDUCTION

Even though preceding sections of this chapter include several analytic functions as examples of typical shocks, data reduction in general is applied to measurements of shock that are not definable by analytic functions. The following sections outline data reduction methods that are adapted for use with any general type of function, obtained in digital form in practice. Standard forms for presenting the analysis results are given in Ref. 8.

FOURIER SPECTRUM

The Fourier spectrum is computed using the discrete Fourier transform (DFT) defined in Eq. (14.6). The DFT is commonly computed using a fast Fourier transform (FFT) algorithm, as discussed in Chap. 14 (see Ref. 9 for details on FFT computations). Fourier spectra can be computed as a function of either radial frequency ω in radians/sec or cyclical frequency f in Hz, that is,

$$F_1(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad \text{or} \quad F_2(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{23.55}$$

where the two functions are related by $F_2(\omega) = 2\pi F_1(f)$.

SHOCK RESPONSE SPECTRUM

The shock response spectrum can be computed by the following techniques: (*a*) direct numerical or recursive integration of the Duhamel integral in Eq. (23.33), or (*b*) convolution or recursive filtering procedures. One of the most widely used programs for computing the shock response spectrum is the "ramp invariant method" detailed in Ref. 10. Any of these computational procedures can be modified to count

FINITE FOURIER TRANSFORMS

Since frequency domain descriptions of vibrations are generally of the greatest engineering value, the Fourier transform plays a major role in both the theoretical definitions of properties and the analysis algorithms for vibration data. The finite Fourier transform of a sample record $x(t)$ is defined as

$$X(f,T) = \int_0^T x(t)e^{-j2\pi ft}dt = \int_0^T x(t)\cos(2\pi ft)dt - j\int_0^T x(t)\sin(2\pi ft)dt \qquad (22.3)$$

where $j = \sqrt{-1}$. Three properties of the definition in Eq. (22.3) should be noted, as follows:

1. The finite Fourier transform is generally a complex number that is defined for both positive and negative frequencies, that is, $X(f,T)$; $-\infty < f < \infty$. However, $X(-f,T) = X^*(f,T)$, where the asterisk denotes the complex conjugate, meaning that values at mathematically negative frequencies are redundant and provide no information beyond that provided by the values at positive frequencies. Since engineers typically think of frequency as a positive value, it is common to present finite Fourier transforms as $2X(f,T)$; $0 < f < \infty$.
2. Fourier transforms are often defined as a function of radial frequency ω in radians/sec, as opposed to cyclical frequency f in Hz, particularly for analytical applications. However, data analysis is usually accomplished in terms of cyclical frequency f, as defined in Eq. (22.3). The two definitions are interrelated by $X(f,T) = 2\pi\, X(\omega,T)$.
3. The finite Fourier transform $X(f,T)$ is equivalent to the Fourier series of $x(t)$ assumed to have a period T.

STATIONARY DETERMINISTIC VIBRATIONS

Stationary deterministic vibration environments generally fall into one of two categories, namely, periodic vibrations or almost-periodic vibrations.

Periodic Vibrations. Periodic vibrations are those with time-histories that exactly repeat themselves after a time interval T_P, that is, $x(t) = x(t + iT_P)$; $i = 1, 2, 3, \ldots$, where T_P is called the *period* of the vibration. All periodic vibrations can be decomposed into a Fourier series, which consists of a collection of commensurately related sine waves,[1,2] that is,

$$x(t) = a_0 + \sum_k a_k \sin(2\pi k f_1 t + \theta_k) \qquad k = 1, 2, 3, \ldots \qquad (22.4)$$

where a_0 is the mean value, kf_1 is the kth frequency component (harmonic), and a_k and θ_k are the amplitude and phase angle associated with the kth frequency component of the periodic vibration. The $k = 1$ component is called the *fundamental frequency* of the periodic vibration, and is given by $f_1 = 1/T_P$. The magnitude of the frequency components in Eq. (22.4) are given by

$$L_x(f) = \frac{2|X(f,T_P)|}{T_P} \qquad 0 < f \qquad (22.5)$$

If $f_1 = 0$, the first zero crossing of the autocorrelation function occurs at a delay $\tau = 1/f_2$.

The average relation between two variables $x(t)$ and $y(t)$ is represented by the *cross-correlation* $R_{xy}(\tau,t_1)$ defined by

$$R_{xy}(\tau,t_1) = \overline{x(t_1)y(t_1+\tau)} \tag{11.18}$$

For variables of a stationary process, the cross-correlation is a function only of the delay τ. However, the maximum value does not necessarily occur at $\tau = 0$. The cross-correlation function can be approximated by the time average:

$$R_{xy}(\tau) \simeq \frac{1}{T}\int_0^T x(t)y(t+\tau)\,dt \tag{11.19}$$

POWER SPECTRAL DENSITY

The frequency content of a random variable $x(t)$ is represented by the *power spectral density* $W_x(f)$, defined as the mean-square response of an ideal narrow-band filter to $x(t)$, divided by the bandwidth Δf of the filter in the limit as $\Delta f \to 0$ at frequency f (Hz):

$$W_x(f) = \lim_{\Delta f \to 0} \frac{\overline{x_{\Delta f}^2}}{\Delta f} \tag{11.20}$$

This is illustrated in Fig. 22.5. By this definition the sum of the power spectral components over the entire frequency range must equal the total mean-square value of x:

$$\overline{x^2} = \int_0^\infty W_x(f)\,df \tag{11.21}$$

The term *power* is used because the dynamical power in a vibrating system is proportional to the square of the vibration amplitude.

An alternate approach to the power spectral density of stationary variables uses the *Fourier series* representation of $x(t)$ over a finite time period $0 \le t \le T$, defined in Eq. (22.4) as

$$x(t) = \bar{x} + \sum_{n=1}^{\infty} A_n \cos(2\pi f_n t) + \sum_{n=1}^{\infty} B_n \sin(2\pi f_n t) \tag{11.22}$$

where $f_n = n/T$. The coefficients of the Fourier series are found by

$$A_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi f_n t)\,dt$$

$$B_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi f_n t)\,dt \tag{11.23}$$

Comparing this to Eq. (11.19), it follows that the coefficients of the Fourier series are a measure of the correlation of $x(t)$ with the cosine and sine waves at a particular frequency.