

Google's PageRank and Beyond:  
The Science of Search Engine Rankings

**EXHIBIT 2099**

*Facebook, Inc. et al.*

v.



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# Google's PageRank and Beyond: The Science of Search Engine Rankings

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