OPTI 517
Image Quality

Richard Juergens

Senior Engineering Fellow

Raytheon Missile Systems

520-794-0917

rcjuergens@raytheon.com

Why is Image Quality Important?

- Resolution of detail
  - Smaller blur sizes allow better reproduction of image details
  - Addition of noise can mask important image detail


Original


Blur added


Noise added


Pixelated

Step One - What is Your Image Quality (IQ) Spec?

- There are many metrics of image quality
  - Geometrical based (e.g., spot diagrams, RMS wavefront error)
  - Diffraction based (e.g., PSF, MTF)
  - Other (F-theta linearity, uniformity of illumination, etc.)
- It is imperative that you have a specification for image quality when you are designing an optical system
  - Without it, you don't know when you are done designing!

You vs. the Customer

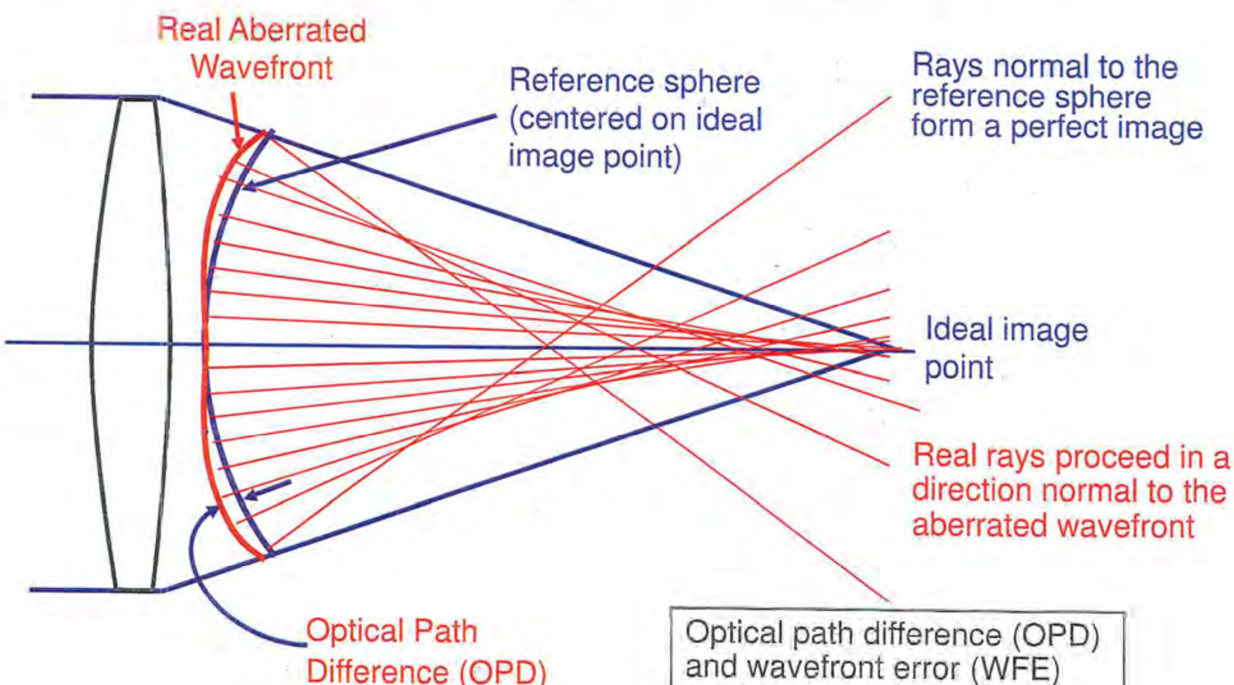
- Different kinds of image quality metrics are useful to different people
- Customers usually work with performance-based specifications
  - MTF, ensquared energy, distortion, etc.
- Designers often use IQ metrics that mean little to the customer
  - E.g., ray aberration plots and field plots
  - These are useful in the design process, but are not end-product specs
- In general, you will be working to an end-product specification, but will probably use other IQ metrics during the design process
  - Often the end-product specification is difficult to optimize to or may be time consuming to compute
- Some customers do not express their image quality requirements in terms such as MTF or ensquared energy
  - They know what they want the optical system to do
- It is up to the optical engineer (in conjunction with the system engineer) to translate the customer's needs into a numerical specification suitable for optimization and image quality analysis

When to Use Which IQ Metric

- The choice of appropriate IQ metric usually depends on the application of the optical system
  - Long-range targets where the object is essentially a point source
    - Example might be an astronomical telescope
    - Ensquared energy or RMS wavefront error might be appropriate
  - Ground-based targets where the details of the object are needed to determine image features
    - Example is any kind of image in which you need to see detail
    - MTF would be a more appropriate metric
  - Laser scanning systems
    - A different type of IQ metric such as the variation from F-theta distortion
- The type of IQ metric may be part of the lens specification or may be a derived requirement flowed down to the optical engineer from systems engineering
  - Do not be afraid to question these requirements
  - Often the systems engineering group doesn't really understand the relationship between system performance and optical metrics

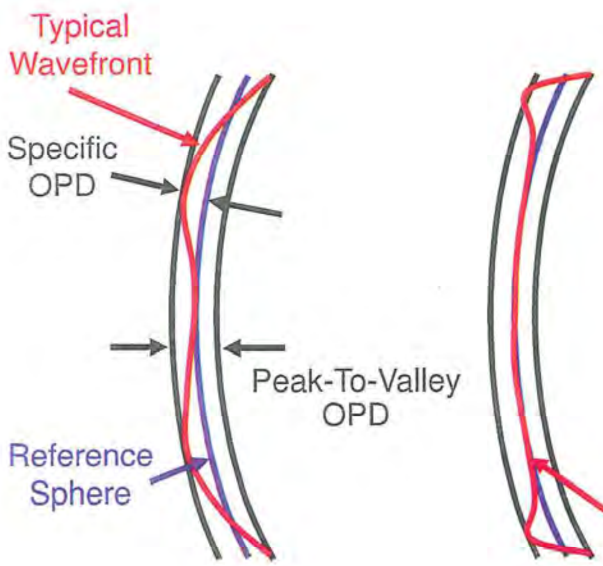
Wavefront Error

- Aberrations occur when the converging wavefront is not perfectly spherical



Optical path difference (OPD) and wavefront error (WFE) are just two different names for the same error

Optical Path Difference



Peak-to-valley OPD is the difference between the longest and the shortest paths leading to a selected focus

RMS wavefront error is given by:

$$\left\langle W^n \right\rangle = \frac{\iint [W(x,y)]^n dxdy}{\iint dxdy}$$

$$W_{rms} = \left\langle W^2 \right\rangle - \left\langle W \right\rangle^2$$

For n discrete rays across the pupil

$$RMS = \sqrt{\sum OPD^2/n}$$

This wavefront has the same P-V wavefront error as the example at the left, but it has a lower RMS

Peak-to-Valley vs. RMS

- The ratio of P-V to RMS is not a fixed quantity
- Typical ratios of P-V to RMS (from Shannon's book)
  - Defocus 3.5
  - 3rd order spherical 13.4
  - 5th order spherical 57.1
  - 3rd order coma 8.6
  - 3rd order astigmatism 5.0
  - Smooth random errors ~5
- In general, for a mixture of lower order aberrations, P-V/RMS ≈ 4.5
- When generating wavefront error budgets, RMS errors from different sources can be added in an RSS fashion
  - P-V errors cannot be so added
- In general, Peak-to-Valley wavefront error is a poor choice to use for error budgeting
  - However, Peak-to-Valley surface error or wavefront error is still commonly used as a surface error specification for individual optical components and even for complete optical systems

Rayleigh Criterion

- Lord Rayleigh observed that when the maximum wavefront error across a wavefront did not exceed $\lambda/4$ peak-to-valley, the image quality was "not sensibly degraded"
  - This quarter-wave limit is now called the Rayleigh Criterion
- This is approximately equivalent to the RMS wavefront error being about 0.07 wave or less (using the value for defocus)
- The Strehl Ratio is a related measure of image quality
  - It can be expressed (for RMS wavefront error < 0.1 wave) as

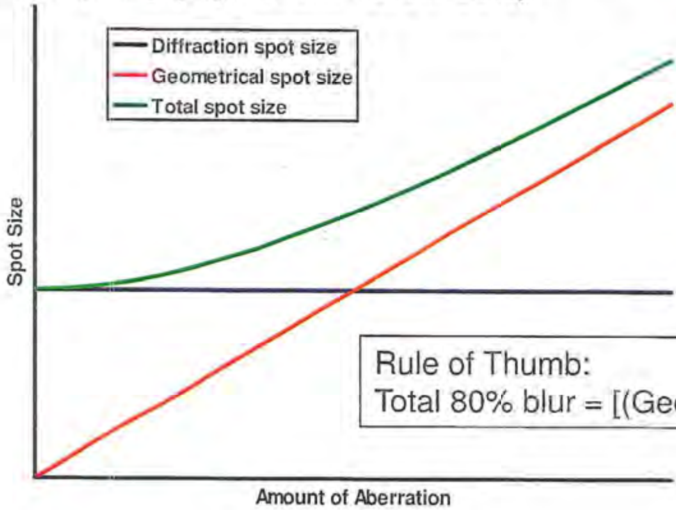
$$\text{Strehl Ratio} = e^{-(2\pi\Phi)^2} \approx 1-(2\pi\Phi)^2$$

    where Φ is the RMS wavefront error (in waves)

  - For $\Phi = 0.07$ wave, the Strehl Ratio ≈ 0.8
- Requiring the Strehl Ratio to be 0.80 or greater for acceptable image quality is often called the Maréchal Criterion

Diffraction-limited Performance

- Many systems have "diffraction-limited" performance as a specification
  - Taken literally, this might mean that no aberrations are allowed
  - As a practical matter, it means that diffraction dominates the image and that the geometric aberrations are small compared to the Airy disk
- There is a distinction between the best possible performance, as limited by diffraction, and performance that is below this limit but produces acceptable image quality (e.g., Strehl Ratio > 80%)



Rule of Thumb:
Total 80% blur = $[(\text{Geo 80\% blur})^2 + (\text{Airy diameter})^2]^{1/2}$

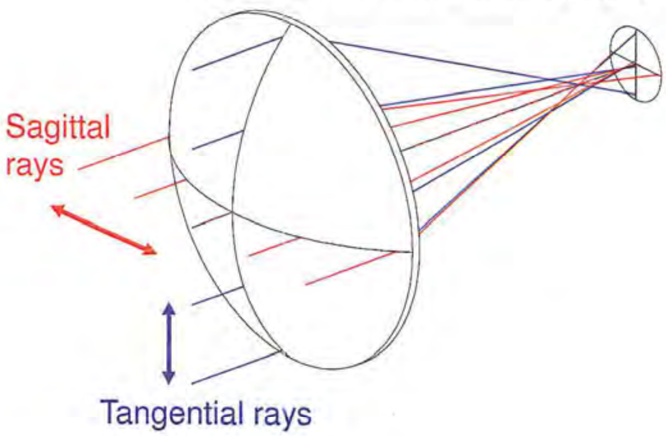
10

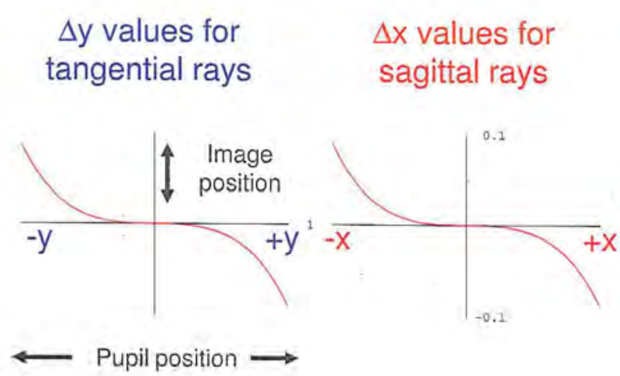
Image Quality Metrics

- The most commonly used geometrical-based image quality metrics are
  - Ray aberration curves
  - Spot diagrams
  - Seidel aberrations
  - Encircled (or ensquared) energy
  - RMS wavefront error
  - Modulation transfer function (MTF)
- The most commonly used diffraction-based image quality metrics are
  - Point spread function (PSF)
  - Encircled (or ensquared) energy
  - MTF
  - Strehl Ratio

Ray Aberration Curves

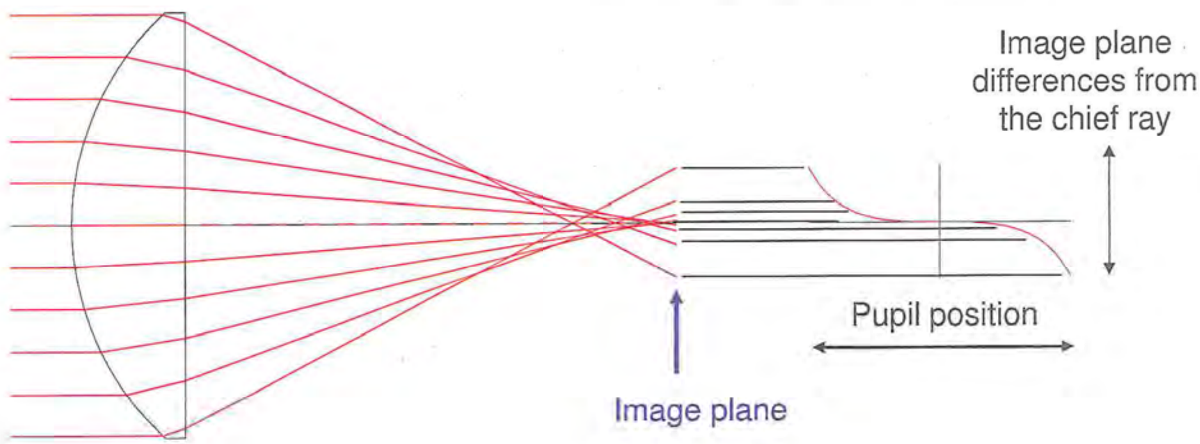
- These are by far the image quality metric most commonly used by optical designers during the design process
- Ray aberration curves trace fans of rays in two orthogonal directions
  - They then map the image positions of the rays in each fan relative to the chief ray vs. the entrance pupil position of the rays





Graphical Description of Ray Aberration Curves

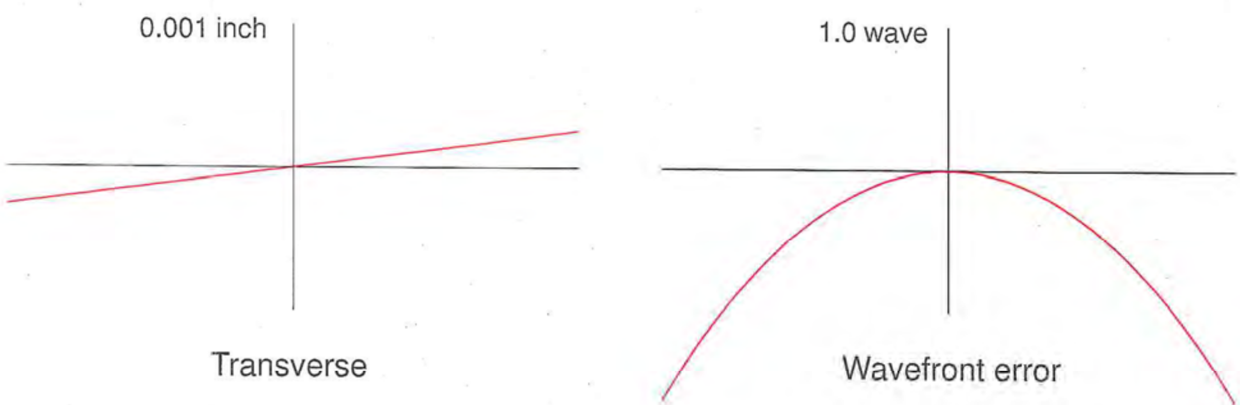
- Ray aberration curves map the image positions of the rays in a fan
  - The plot is image plane differences from the chief ray vs. position in the fan



- Ray aberration curves are generally computed for a fan in the YZ plane and a fan in the XZ plane
  - This omits skew rays in the pupil, which is a failing of this IQ metric

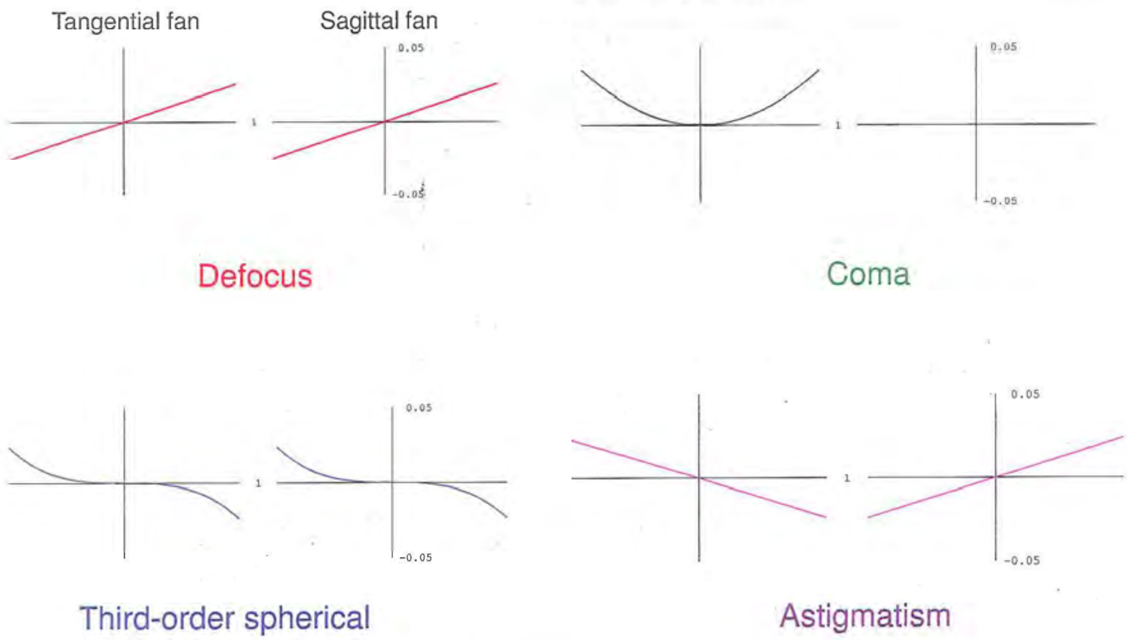
Transverse vs. Wavefront Ray Aberration Curves

- Ray aberration curves can be transverse (linear) aberrations in the image vs. pupil position or can be OPD across the exit pupil vs. pupil position
  - The transverse curve is a scaled derivative of the wavefront curve
- Example curves for pure defocus:



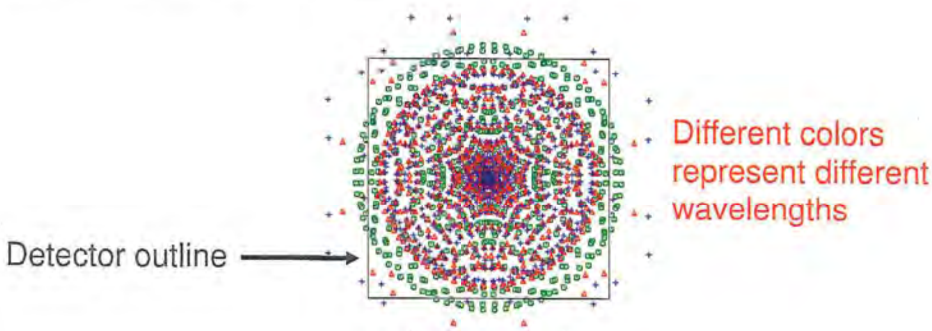
More on Ray Aberration Curves

- The shape of the ray aberration curve can tell what type of aberration is present in the lens for that field point (transverse curves shown)



The Spot Diagram

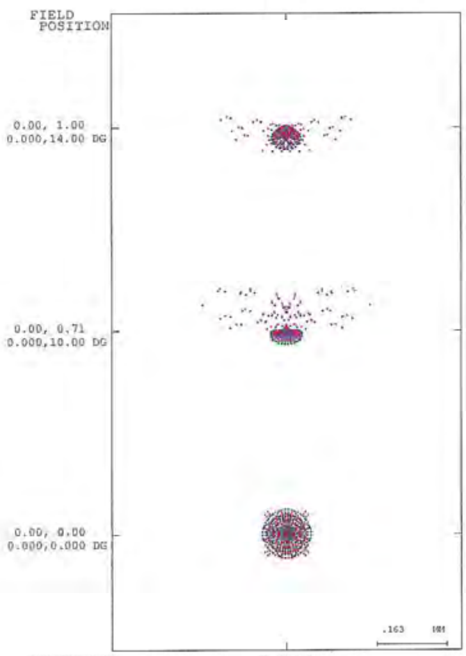
- The spot diagram is readily understood by most engineers
- It is a diagram of how spread out the rays are in the image
  - The smaller the spot diagram, the better the image
  - This is geometrical only; diffraction is ignored
- It is useful to show the detector size (and/or the Airy disk diameter) superimposed on the spot diagram

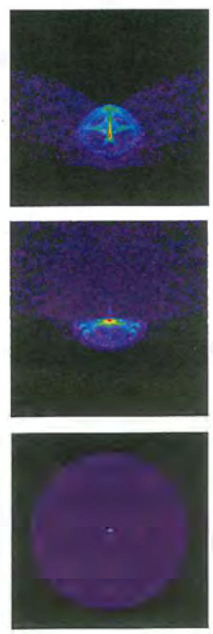


- The shape of the spot diagram can often tell what type of aberrations are present in the image

Main Problem With Spot Diagrams

- The main problem is that spots in the spot diagram don't convey intensity
  - A ray intersection point in the diagram does not tell the intensity at that point

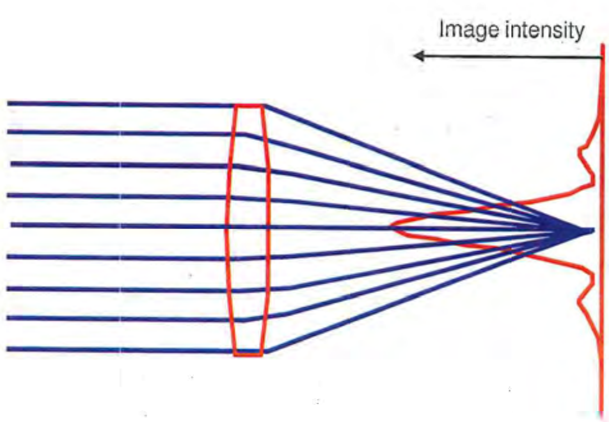


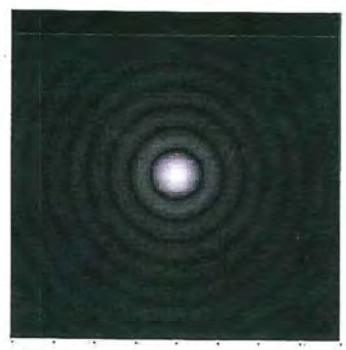


The on-axis image appears spread out in the spot diagram, but in reality it has a tight core with some surrounding low-intensity flare

Diffraction

- Some optical systems give point images (or near point images) of a point object when ray traced geometrically (e.g., a parabola on-axis)
- However, there is in reality a lower limit to the size of a point image
- This lower limit is caused by diffraction
  - The diffraction pattern is usually referred to as the Airy disk

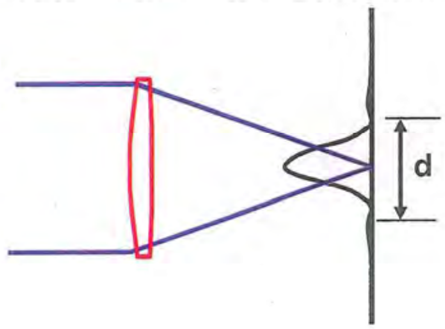


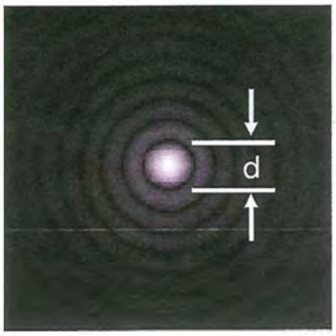


Diffraction pattern of a perfect image

Size of the Diffraction Image

- The diffraction pattern of a perfect image has several rings
  - The center ring contains ~84% of the energy, and is usually considered to be the "size" of the diffraction image



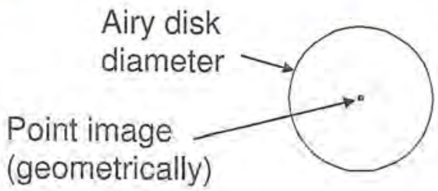


Very important !!!!

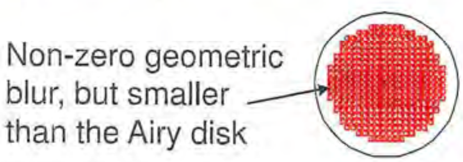
- The diameter of the first ring is given by $d \approx 2.44\ \lambda\ f/\#$
  - This is independent of the focal length; it is only a function of the wavelength and the f/number
  - The angular size of the first ring $\beta = d/F \approx 2.44\ \lambda/D$
- When there are no aberrations and the image of a point object is given by the diffraction spread, the image is said to be <u>diffraction-limited</u>

Spot Size vs. the Airy Disk

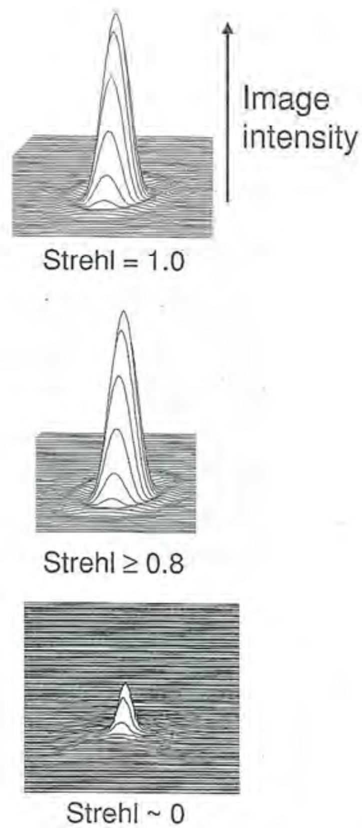
- Regime 1 – Diffraction-limited



- Regime 2 – Near diffraction-limited

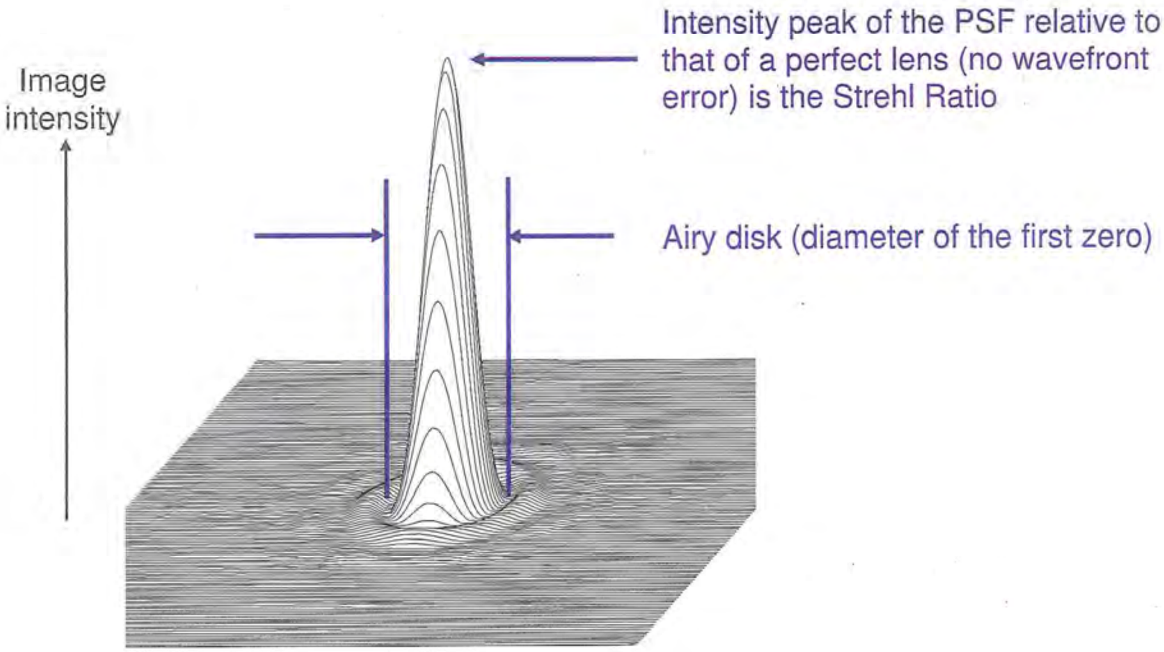


- Regime 3 – Far from diffraction-limited



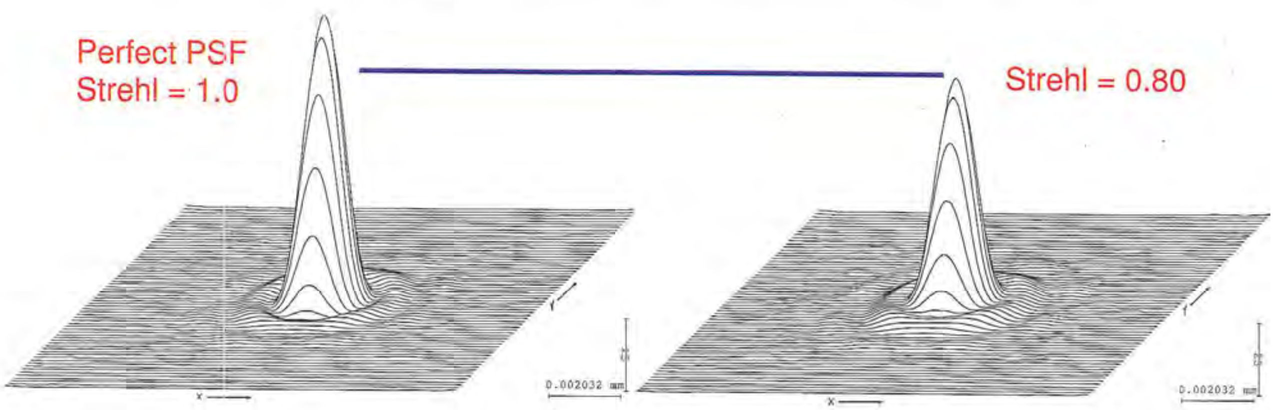
Point Spread Function (PSF)

- This is the image of a point object including the effects of diffraction and all aberrations



Diffraction Pattern of Aberrated Images

- When there is aberration present in the image, two effects occur
  - Depending on the aberration, the shape of the diffraction pattern may become skewed
  - There is less energy in the central ring and more in the outer rings



PSF vs. Defocus

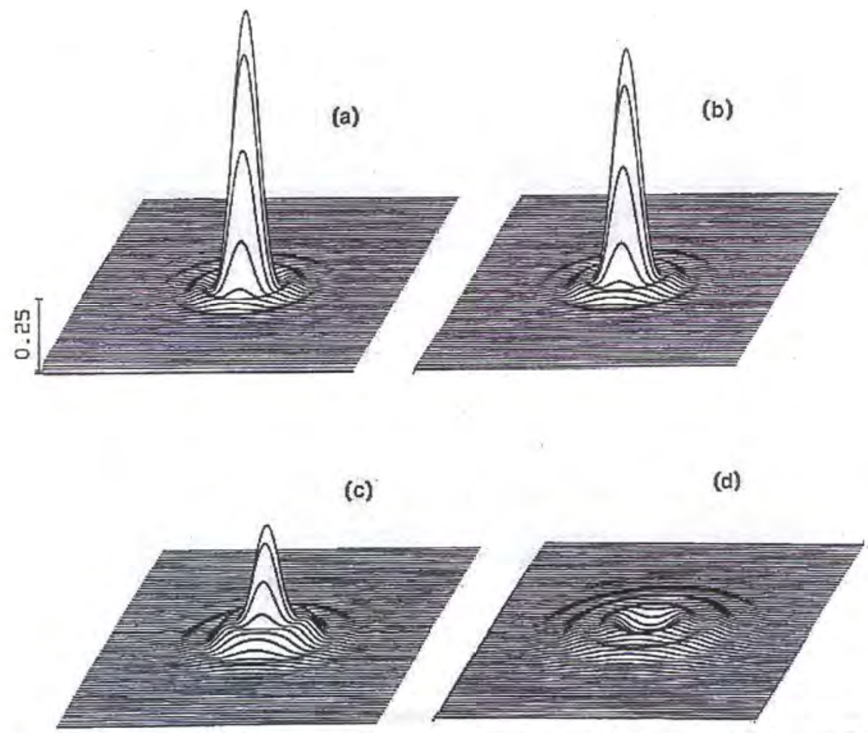


Figure 11.24 Point spread functions for different amounts of defocus. (a) 0.125 wave (P-V); 0.037 wave rms; 0.95 Strehl. (b) 0.25 wave (P-V); 0.074 wave rms; 0.80 Strehl. (c) 0.50 wave (P-V); 0.148 wave rms; 0.39 Strehl. (d) 1.00 wave (P-V); 0.297 wave rms; 0.00 Strehl.

PSF vs. Third-order Spherical Aberration

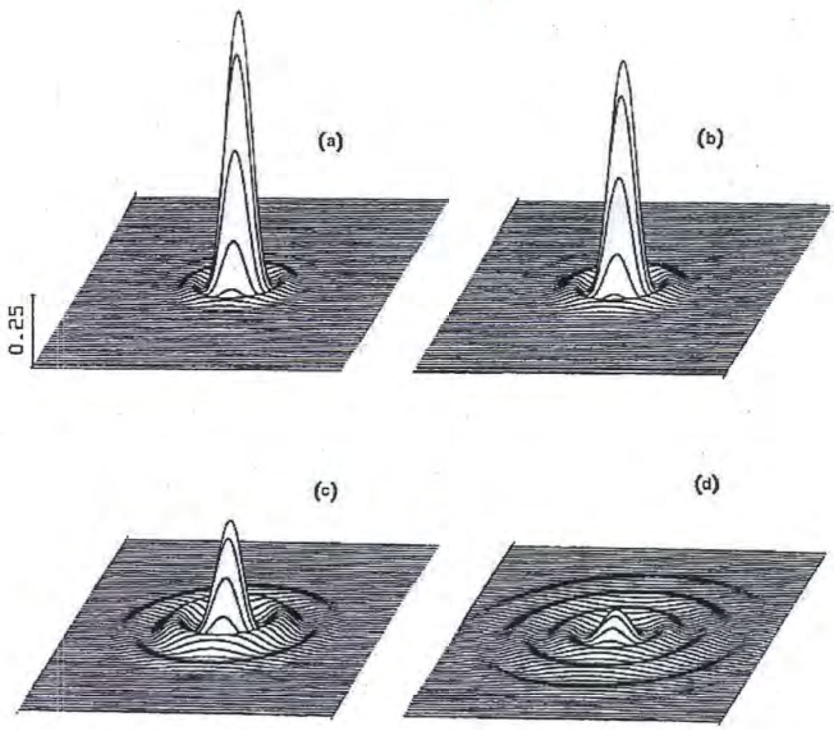


Figure 11.25 Point spread functions for different amounts of third-order spherical aberration. (a) 0.125 wave (P-V); 0.040 wave rms; 0.94 Strehl. (b) 0.25 wave (P-V); 0.080 wave rms; 0.78 Strehl. (c) 0.50 wave (P-V); 0.159 wave rms; 0.37 Strehl. (d) 1.00 wave (P-V); 0.318 wave rms; 0.08 Strehl. *Note:* Reference sphere centered at $0.5LA_m$ (midway between marginal and paraxial foci).

PSF vs. Third-order Coma

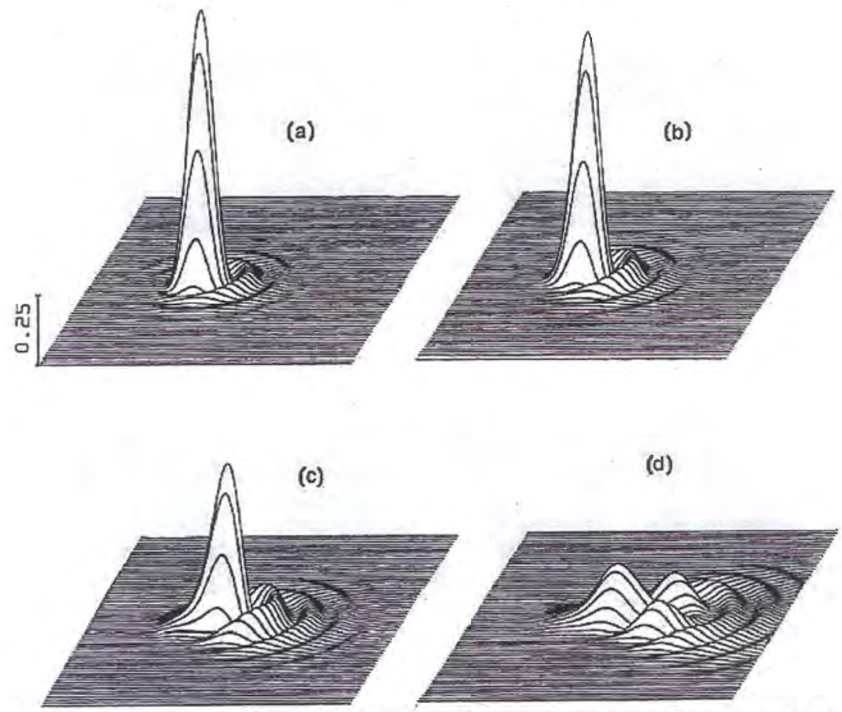


Figure 11.26 Point spread functions for different amounts of third-order coma. (a) 0.125 wave (P-V); 0.031 wave rms; 0.96 Strehl. (b) 0.25 wave (P-V); 0.061 wave rms; 0.86 Strehl. (c) 0.50 wave (P-V); 0.123 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.25 wave rms; 0.18 Strehl. *Note:* P-V OPD reference sphere centered at 0.25Coma_T from chief ray intersection point. rms OPD reference sphere centered at 0.226Coma_T from chief ray intersection point.

PSF vs. Astigmatism

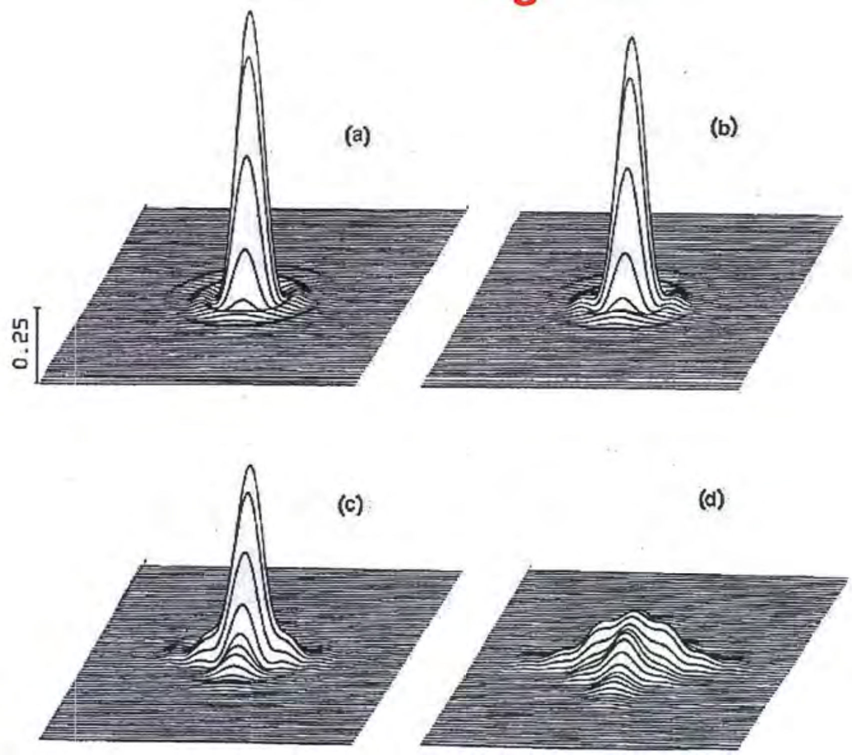


Figure 11.27 Point spread functions for different amounts of astigmatism. (a) 0.125 wave (P-V); 0.026 wave rms; 0.97 Strehl. (b) 0.25 wave (P-V); 0.052 wave rms; 0.90 Strehl. (c) 0.50 wave (P-V); 0.104 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.207 wave rms; 0.18 Strehl. *Note:* Reference sphere centered midway between sagittal and tangential foci.

PSF for Strehl = 0.80

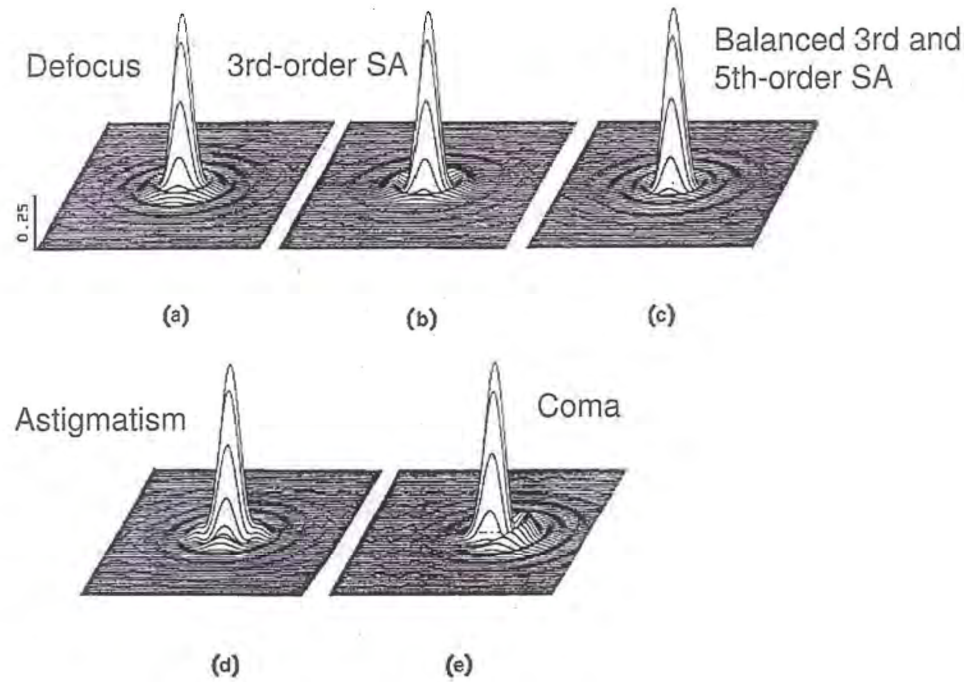


Figure 11.29 Point spread functions for five different aberrations, each with a Strehl ratio of 0.80 (the Marechal criterion). In each case the center of the reference sphere is located to minimize the rms OPD, which is 0.075 wave for all five aberrations. (a) Defocus: 0.25 wave (P-V). (b) Third-order spherical: 0.235 wave (P-V). (c) Balanced third- and fifth-order spherical: 0.221 wave (P-V). (d) Astigmatism: 0.359 wave (P-V). (e) Coma: 0.305 wave (P-V).

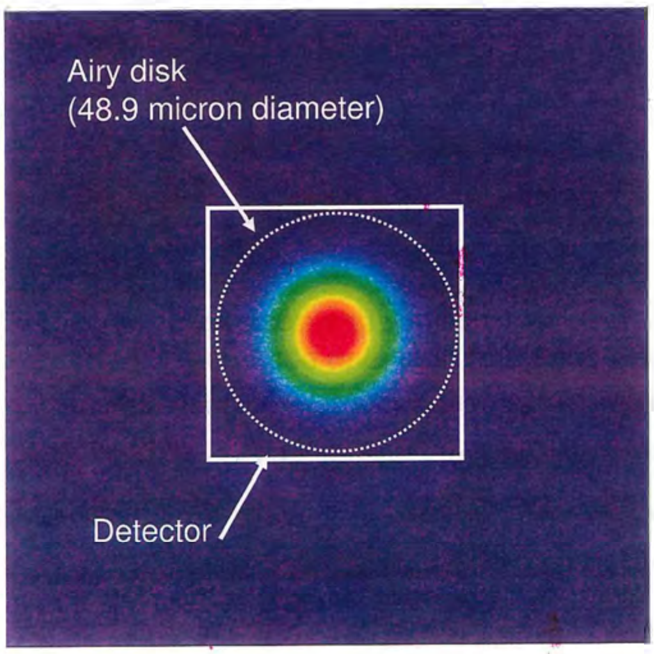
Encircled or Ensquared Energy

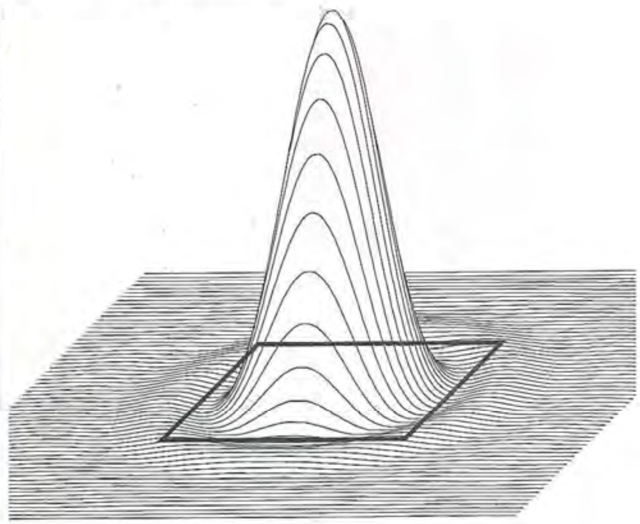
- Encircled or ensquared energy is the ratio of the energy in the PSF that is collected by a single circular or square detector to the total amount of energy that reaches the image plane from that object point
  - This is a popular metric for system engineers who, reasonably enough, want a certain amount of collected energy to fall on a single pixel
  - It is commonly used for systems with point images, especially systems which need high signal-to-noise ratios
- For %EE specifications of 50-60% this is a reasonably linear criterion
  - However, the specification is more often 80%, or even worse 90%, energy within a near diffraction-limited diameter
  - At the 80% and 90% levels, this criterion is highly non-linear and highly dependent on the aberration content of the image, which makes it a poor criterion, especially for tolerancing

Ensquared Energy Example

Ensquared energy on a detector of same order of size as the Airy disk

Perfect lens, f/2, 10 micron wavelength, 50 micron detector

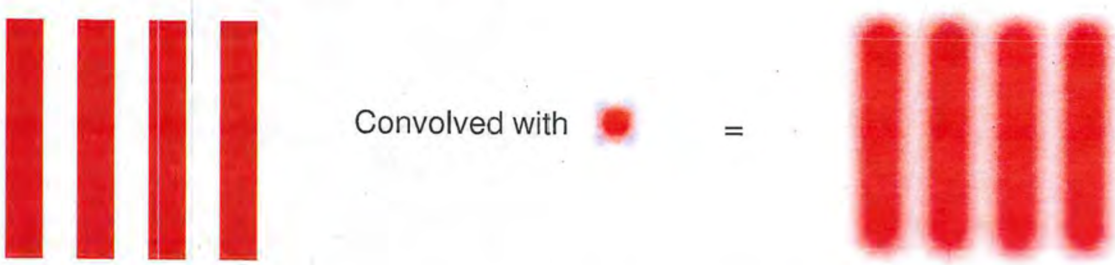




Approximately 85% of the energy is collected by the detector

Modulation Transfer Function (MTF)

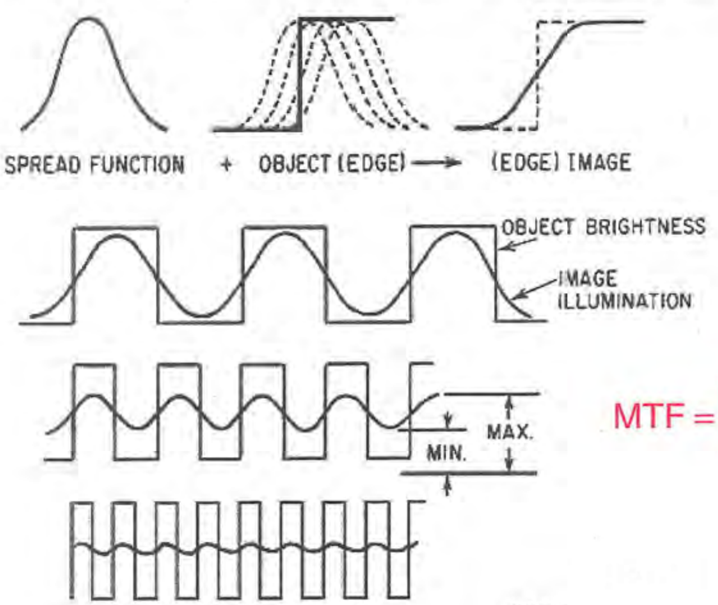
- MTF is the Modulation Transfer Function
- Measures how well the optical system images objects of different sizes
  - Size is usually expressed as spatial frequency (1/size)
- Consider a bar target imaged by a system with an optical blur
  - The image of the bar pattern is the geometrical image of the bar pattern convolved with the optical blur



- MTF is normally computed for sine wave input, and not square bars to get the response for a pure spatial frequency
- Note that MTF can be geometrical or diffraction-based

Computing MTF

- The MTF is the amount of modulation in the image of a sine wave target
  - At the spatial frequency where the modulation goes to zero, you can no longer see details in the object of the size corresponding to that frequency
- The MTF is plotted as a function of spatial frequency (1/sine wave period)



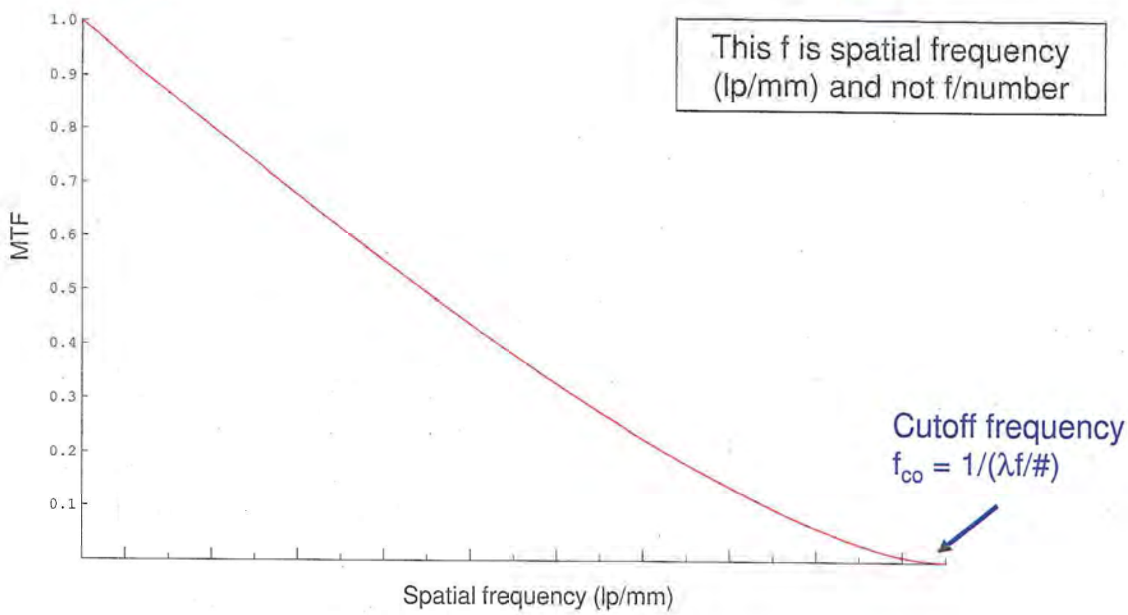
$$MTF = \frac{Max - Min}{Max + Min}$$

OPTI 517

MTF of a Perfect Image

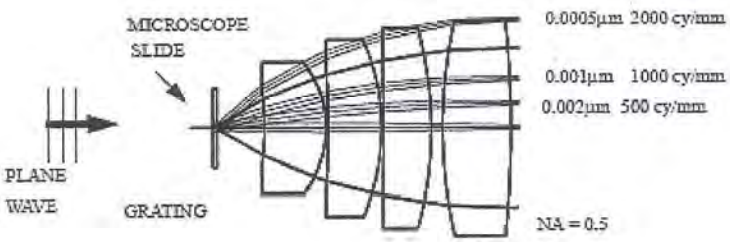
- For an aberration-free image and a round pupil, the MTF is given by

$$\mathrm{MTF}(f) = \frac{2}{\pi}[\varphi - \cos\varphi \sin\varphi] \qquad \varphi = \cos^{-1}(f / f_{co}) = \cos^{-1}(\frac{\lambda f}{2NA})$$



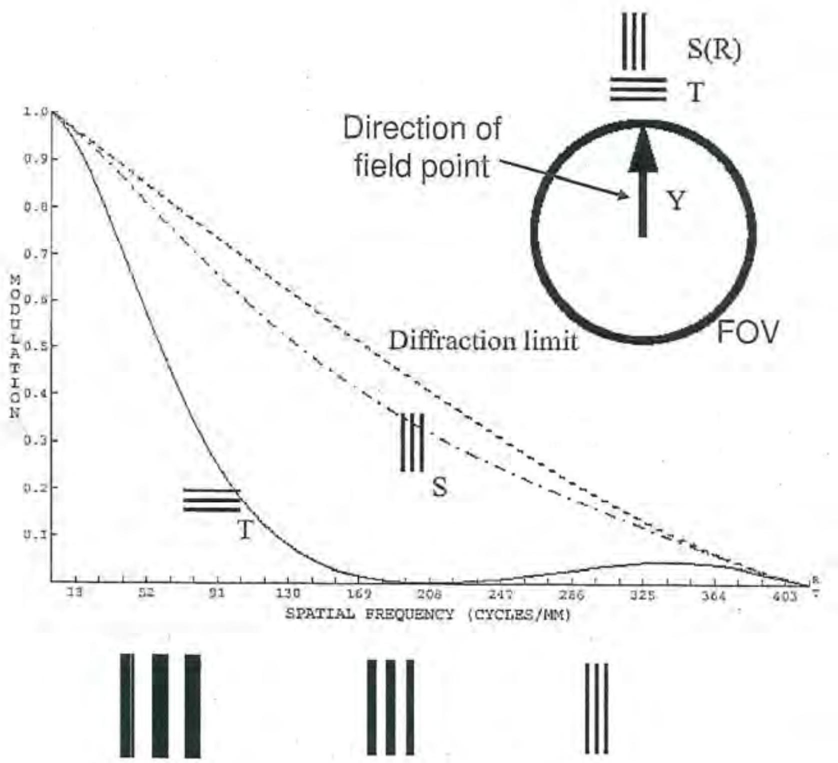
Abbe's Construct for Image Formation

- Abbe developed a useful framework from which to understand the diffraction-limiting spatial frequency and to explain image formation in microscopes



- If the first-order diffraction angle from the grating exceeds the numerical aperture (NA = 1/(2f/#)), no light will enter the optical system for object features with that characteristic spatial period

Example MTF Curve



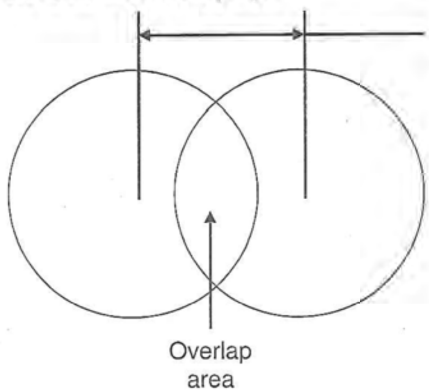
MTF depends on target orientation

S = Sagittal
R = Radial

T = Tangential

MTF as an Autocorrelation of the Pupil

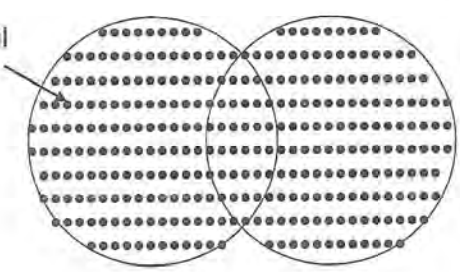
- The MTF is usually computed by lens design programs as the autocorrelation of the OPD map across the exit pupil



Relative spatial frequency = spacing between shifted pupils
(cutoff frequency = pupil diameter)

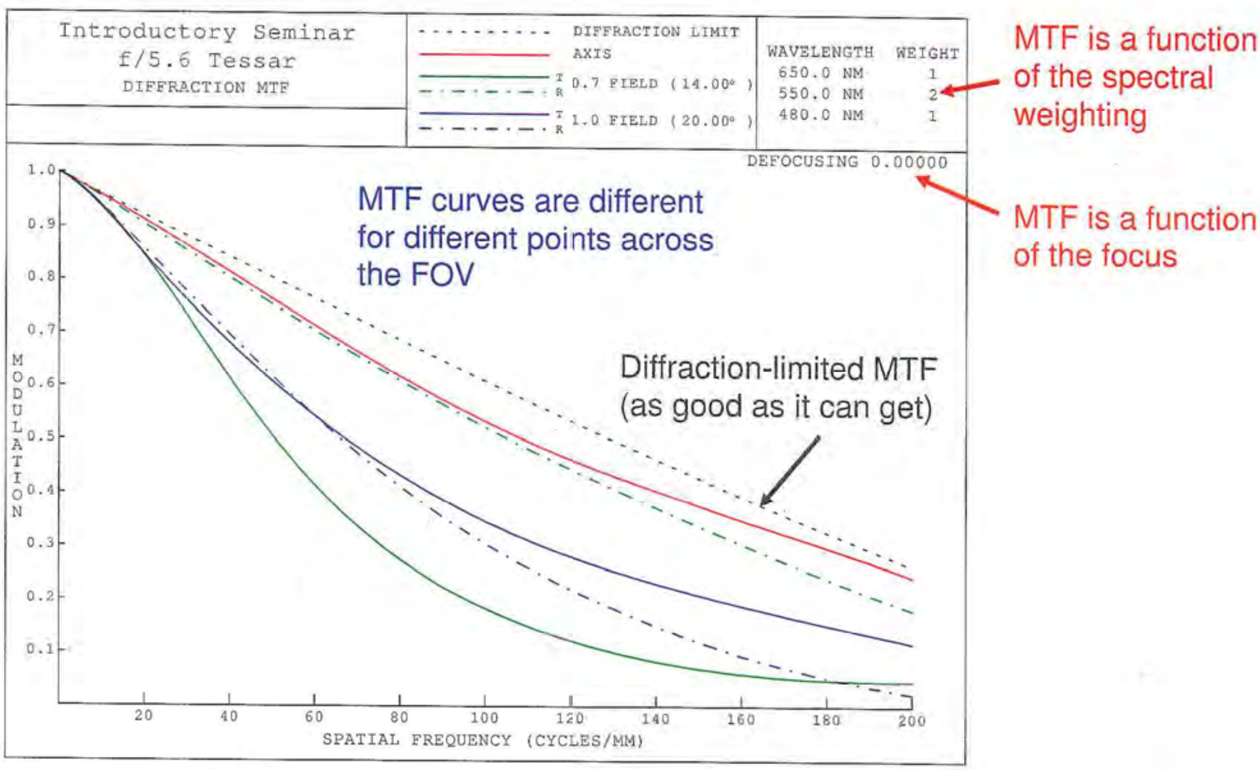
Perfect MTF = overlap area / pupil area

Complex OPD computed for many points across the pupil



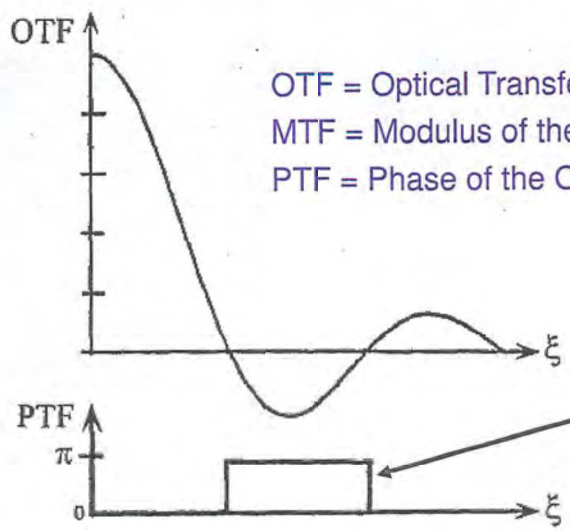
MTF is computed as the normalized integral over the overlap region of the difference between the OPD map and its shifted complex conjugate

Typical MTF Curves



Phase Shift of the OTF

- Since OPD relates to the phase of the ray relative to the reference sphere, the pupil autocorrelation actually gives the OTF (optical transfer function), which is a complex quantity
  - MTF is the real part (modulus) of the OTF



OTF = Optical Transfer Function

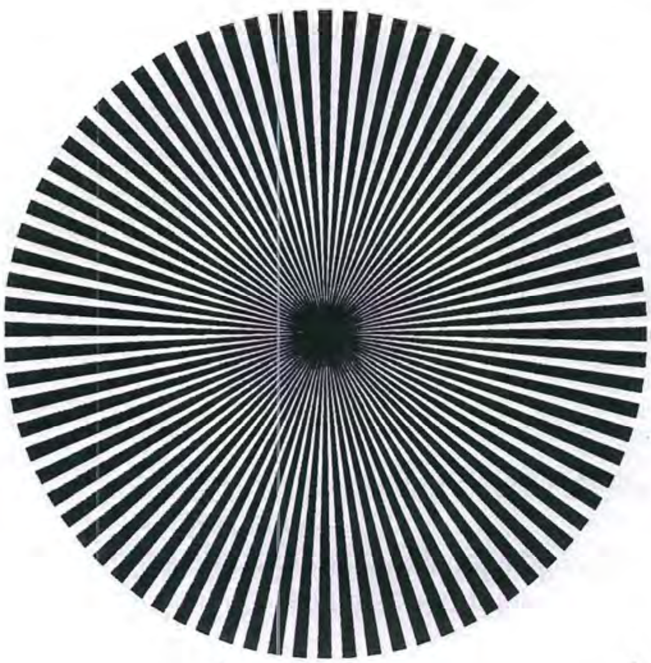
MTF = Modulus of the OTF

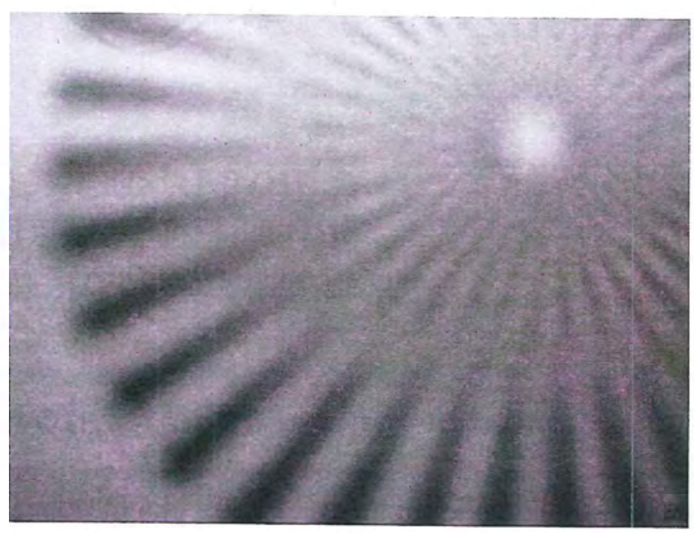
PTF = Phase of the OTF

When the OTF goes negative, the phase is π radians

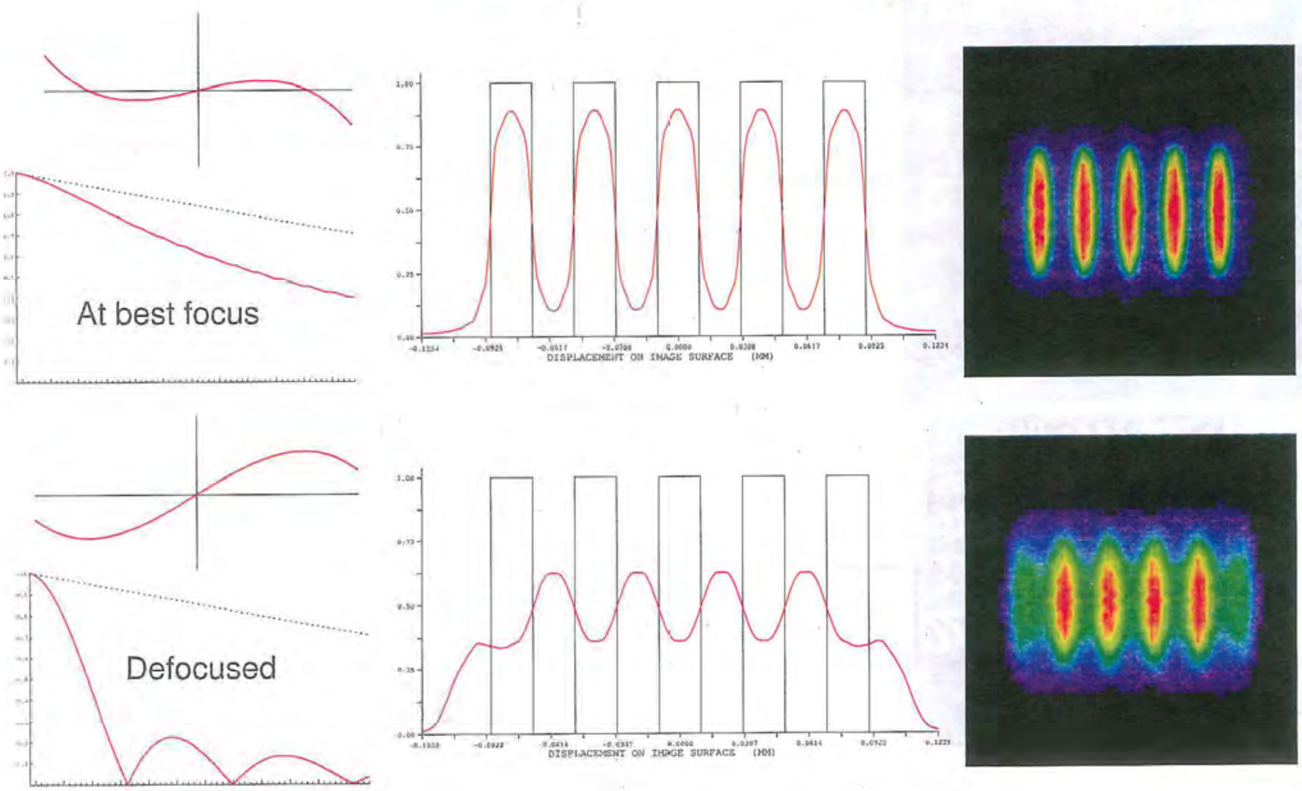
What Does OTF < 0 Mean?

- When the OTF goes negative, it is an example of contrast reversal

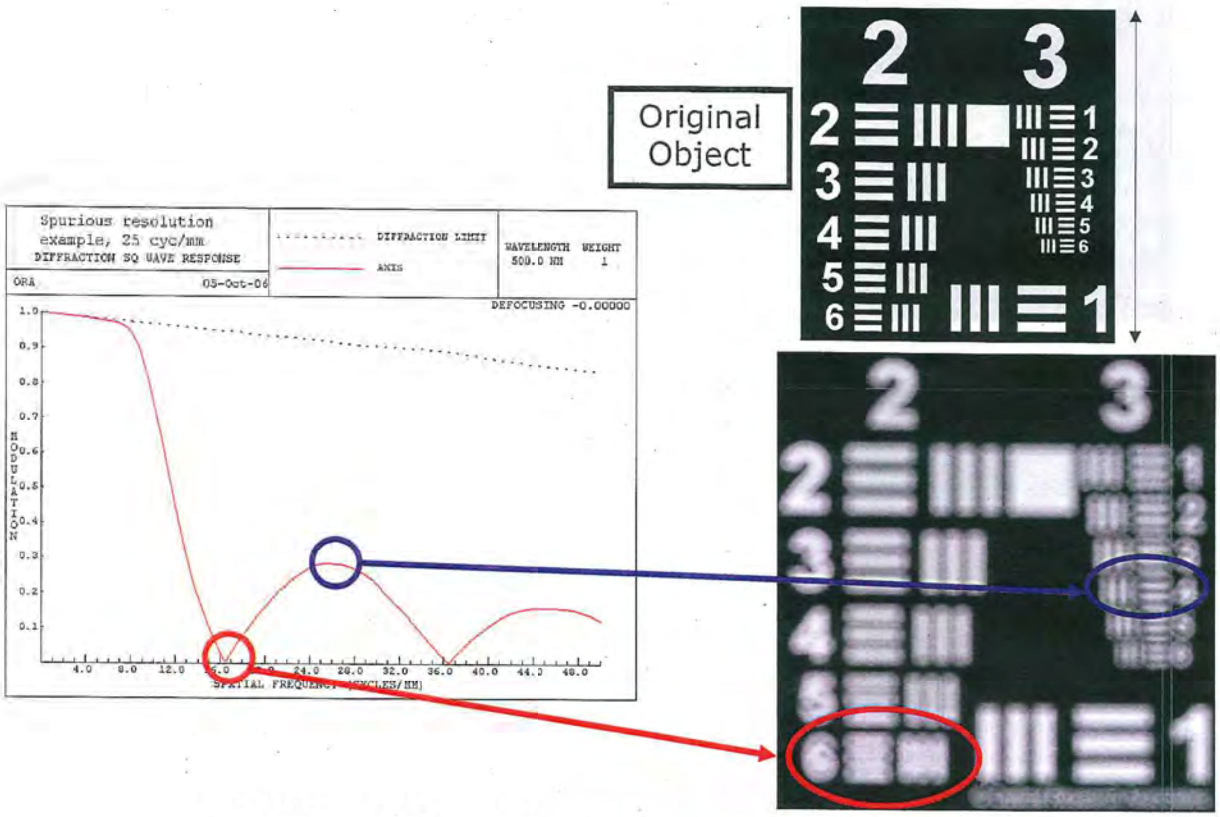




Example of Contrast Reversal



More on Contrast Reversal

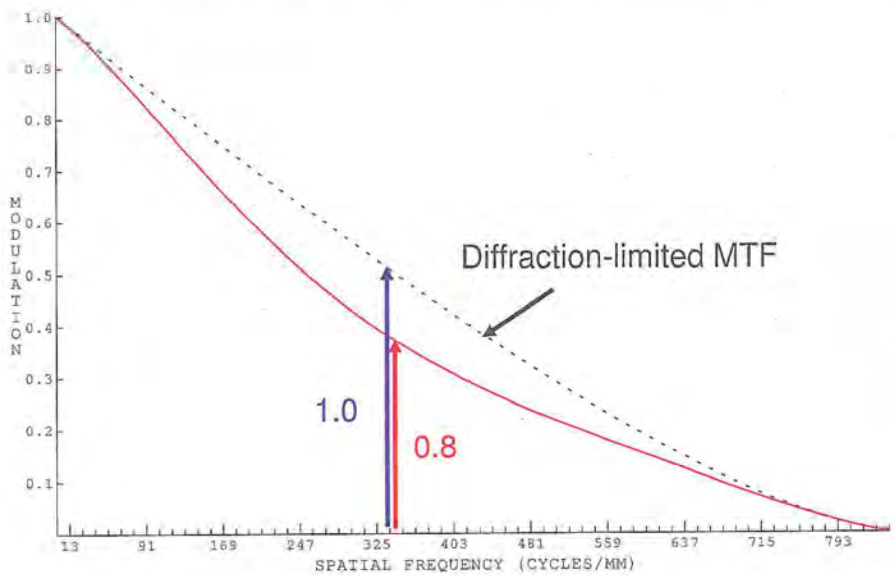


OPTI 517

40

Effect of Strehl = 0.80

- When the Strehl Ratio = 0.80 or higher, the image is considered to be equivalent in image quality to a diffraction-limited image
- The MTF in the mid-range spatial frequencies is reduced by the Strehl ratio



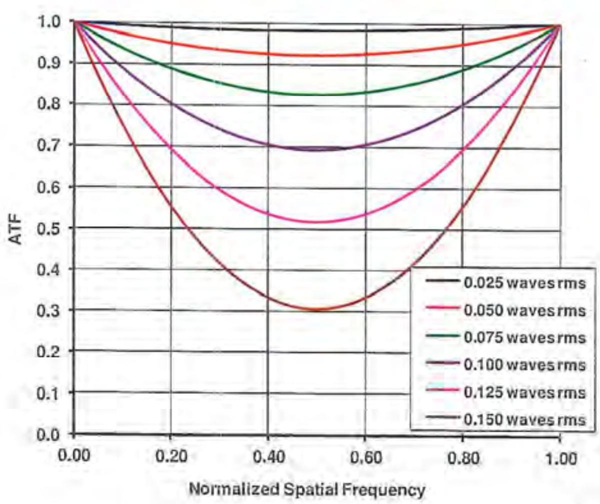
Aberration Transfer Function

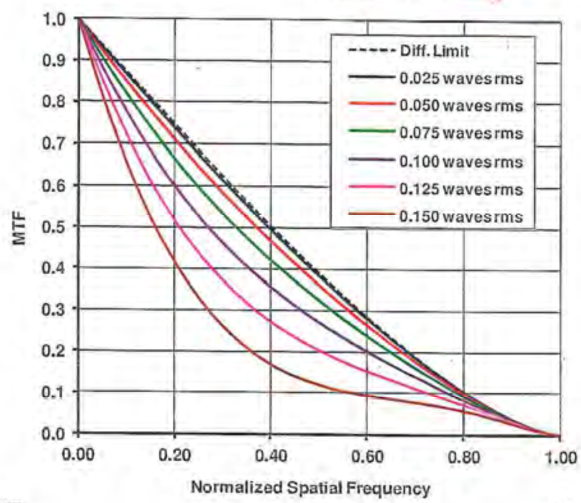
- Shannon has shown that the MTF can be approximated as a product of the diffraction-limited MTF (DTF) and an aberration transfer function (ATF)

$$DTF(\nu) = \frac{2}{\pi}\left[\cos^{-1}\nu - \nu\sqrt{1-\nu^2}\right] \qquad \nu = f / f_{co}$$

$$ATF(\nu) = 1 - \left(\frac{W_{rms}}{0.18}\right)^2 \left(1 - 4(\nu - 0.5)^2\right)$$

Bob Shannon

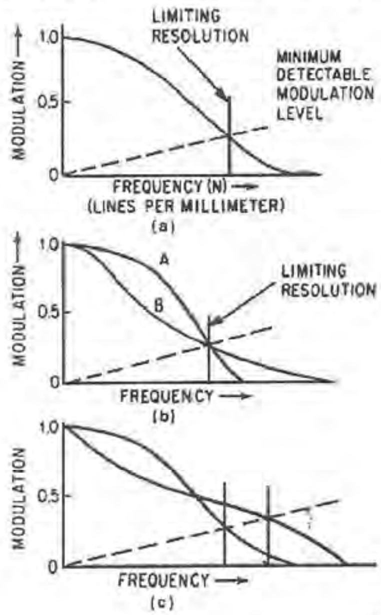




OPTI 517 42

Demand Contrast Function

- The eye requires more modulation for smaller objects to be able to resolve them
  - The amount of modulation required to resolve an object is called the demand contrast function
  - This and the MTF limits the highest spatial frequency that can be resolved

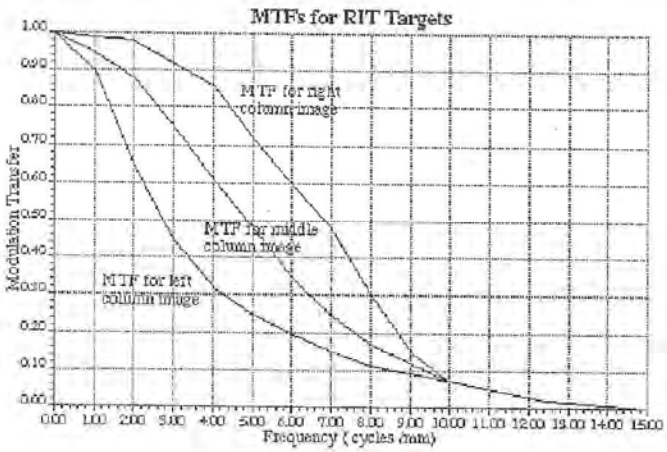


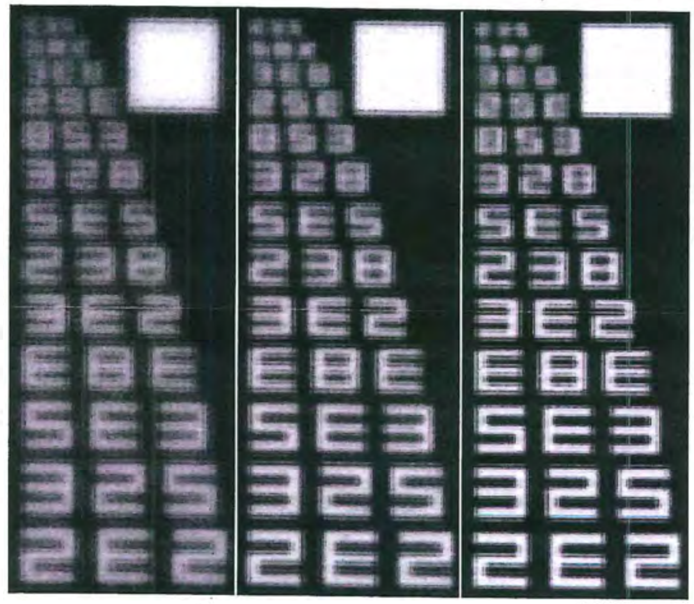
The limiting resolution is where the Demand Contrast Function intersects the MTF

System A will produce a superior image although it has the same limiting resolution as System B

System A has a lower limiting resolution than System B even though it has higher MTF at lower frequencies

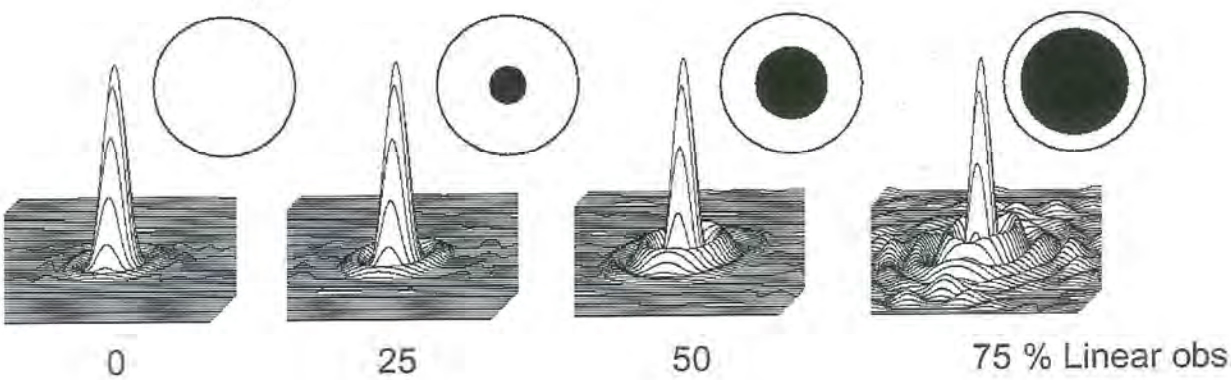
Example of Different MTFs on RIT Target





Central Obscurations

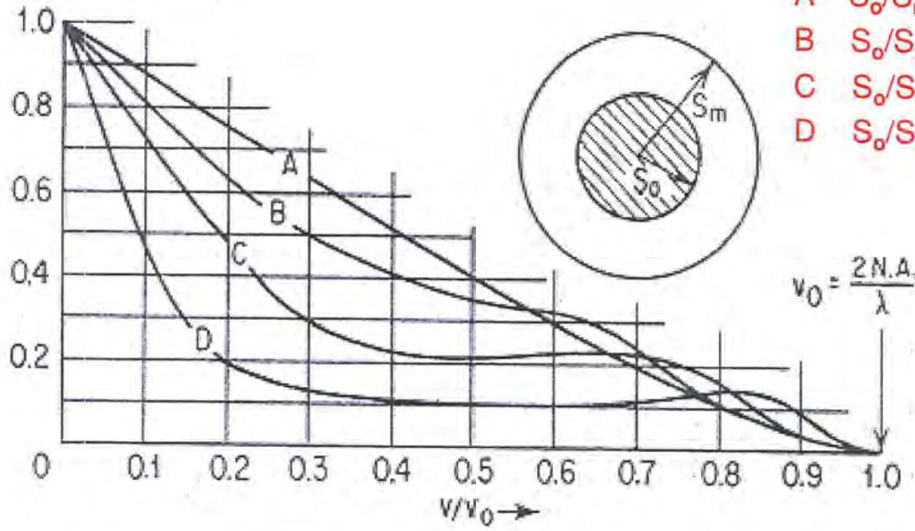
- In on-axis telescope designs, the obscuration caused by the secondary mirror is typically 30-50% of the diameter
  - Any obscuration above 30% will have a noticeable effect on the Airy disk, both in terms of dark ring location and in percent energy in a given ring (energy shifts out of the central disk and into the rings)
- Contrary perhaps to expectations, as the obscuration increases the diameter of the first Airy ring decreases (the peak is the same, and the loss of energy to the outer rings has to come from somewhere)



Central Obscurations

- Central obscurations, such as in a Cassegrain telescope, have two deleterious effects on an optical system
  - The obscuration causes a loss in energy collected (loss of area)
  - The obscuration causes a loss of MTF

A $S_o/S_m = 0.00$
B $S_o/S_m = 0.25$
C $S_o/S_m = 0.50$
D $S_o/S_m = 0.75$



Coherent Illumination

- Incoherent illumination fills the whole entrance pupil
- Partially coherent illumination fills only part of the entrance pupil
  - Coherent illumination essentially only fills a point in the entrance pupil

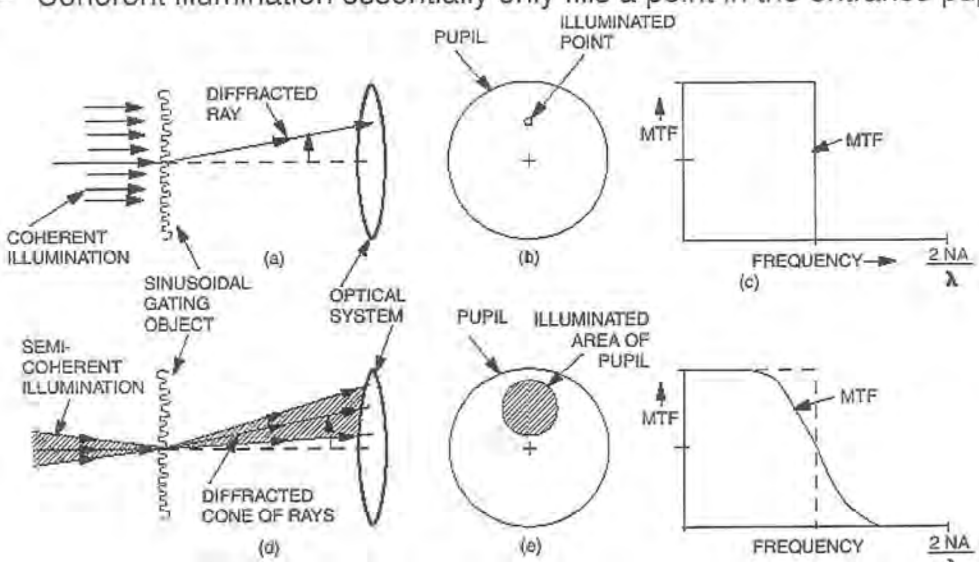


Figure 11.20 (a–c) The MTF with coherent illumination. (d–f) The MTF with semicoherent illumination (which partially fills the pupil).

OPTI 517

47

MTF of Partially Coherent Illumination

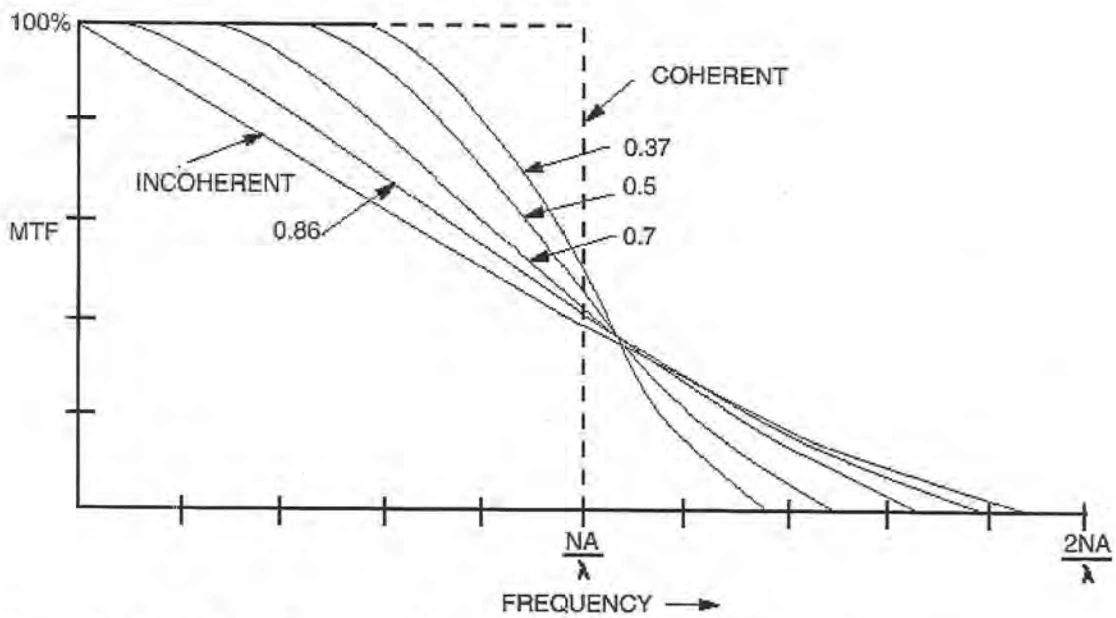
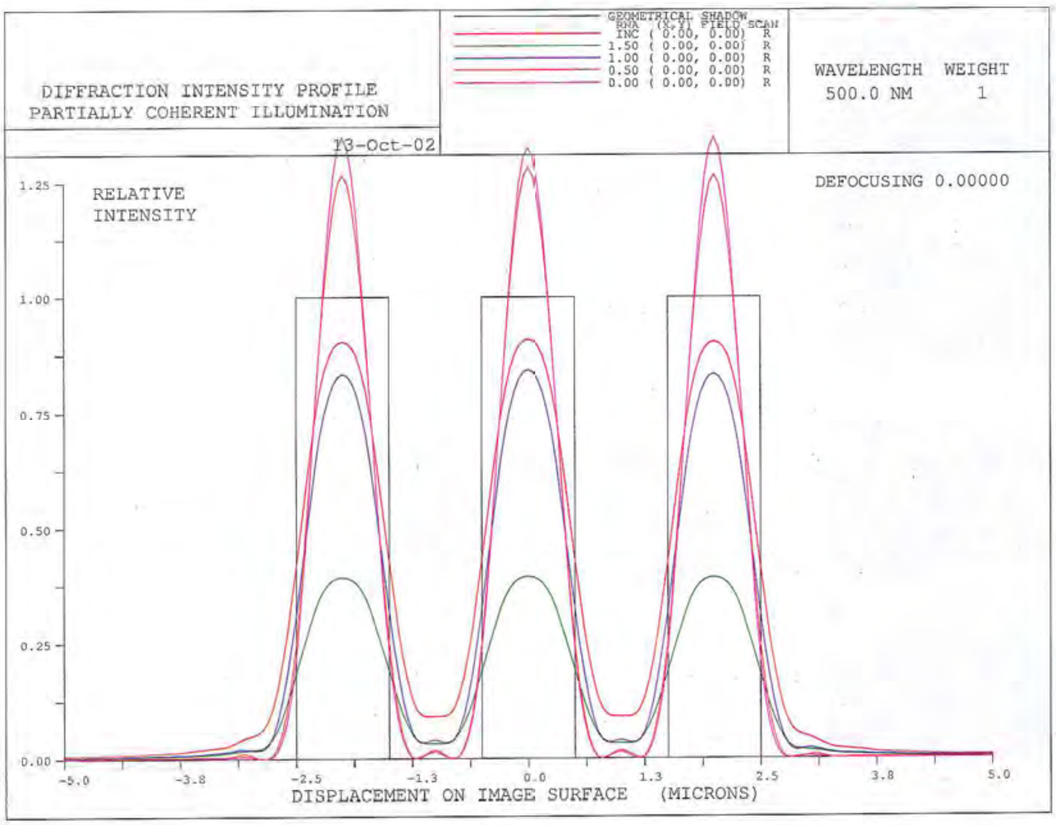
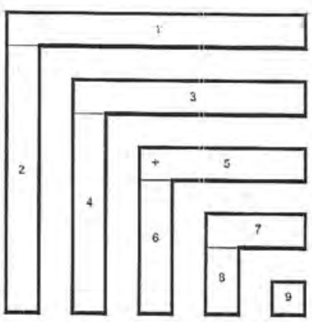


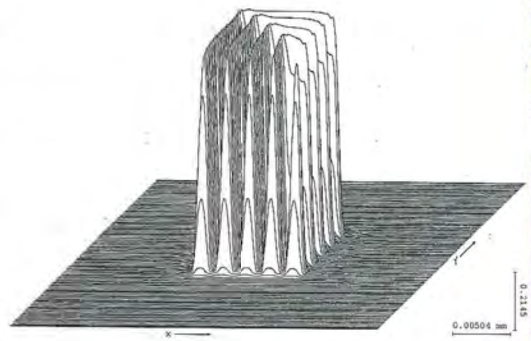
Figure 11.21 MTF vs. frequency for a partially filled pupil (semicoherent illumination). Numbers are the ratio of illuminating system NA to optical system NA.

Partial Coherent Image of a 3-Bar Target

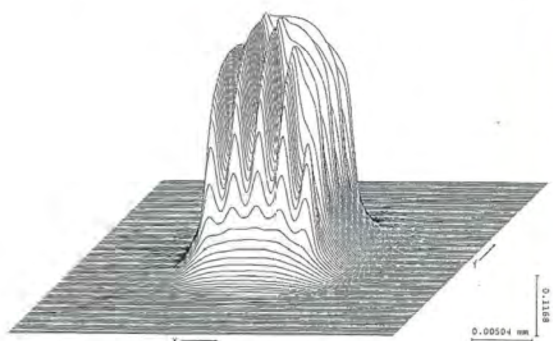


Example of Elbows Imaged in Partially Coherent Light





With 1 wave of spherical aberration

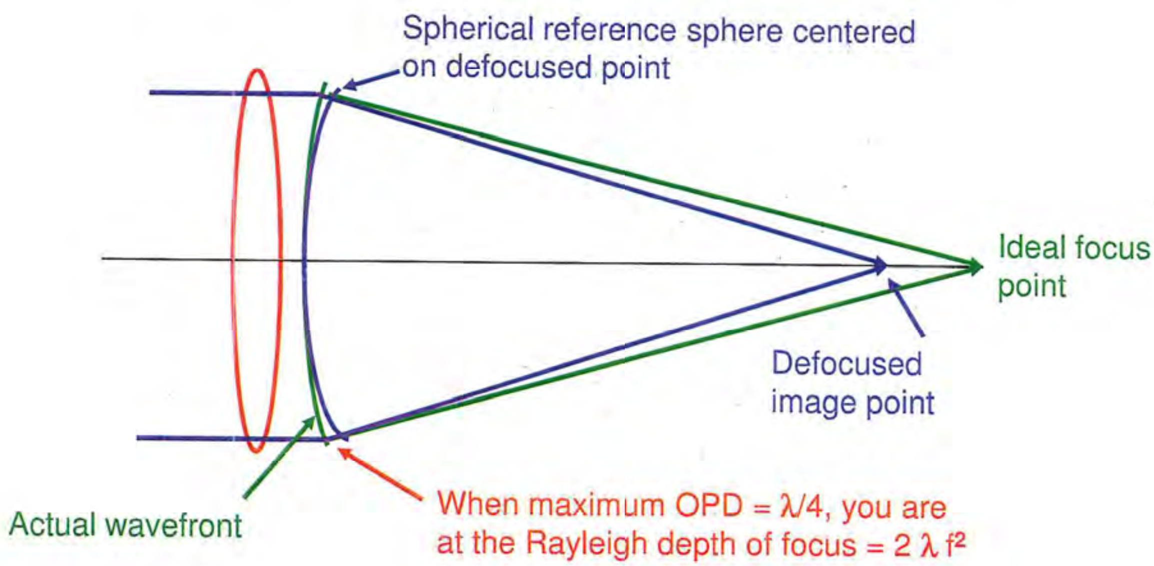


The Main Aberrations in an Optical System

- Defocus – the focal plane is not located exactly at the best focus position
- Chromatic aberration – the axial and lateral shift of focus with wavelength
- The Seidel aberrations
  - Spherical Aberration
  - Coma
  - Astigmatism
  - Distortion
  - Curvature of field

Defocus

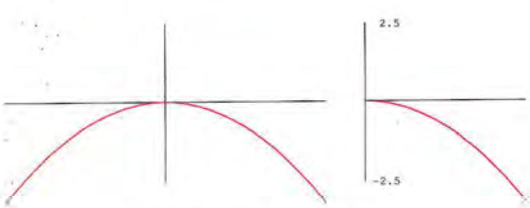
- Technically, defocus is not an aberration in that it can be corrected by simply refocusing the lens
- However, defocus is an important effect in many optical systems



Defocus Ray Aberration Curves

Wavefront map

Spot diagram

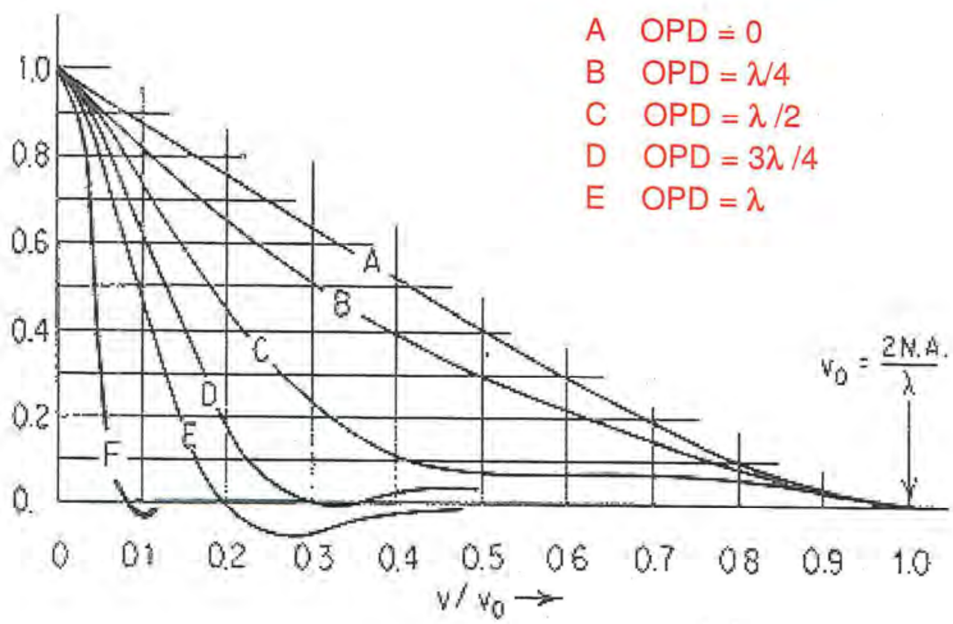


Wavefront error

Transverse ray aberration

MTF of a Defocused Image

- As the amount of defocus increases, the MTF drops accordingly

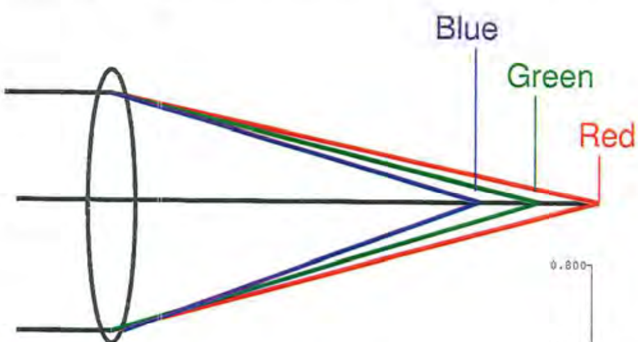


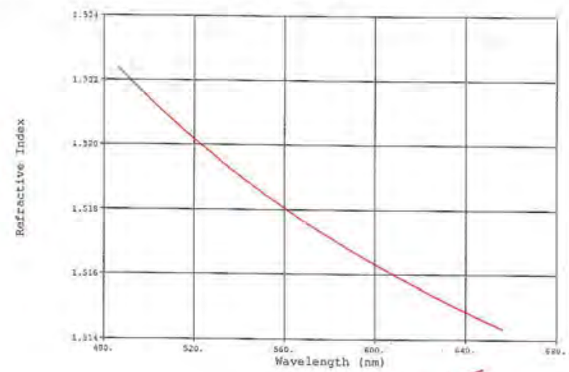
Sources of Defocus

- One obvious source of defocus is the location of the object
  - For lenses focused at infinity, objects closer than infinity have defocused images
  - There's nothing we can do about this (unless we have a focus knob)
- Changes in temperature
  - As the temperature changes, the elements and mounts change dimensions and the refractive indices change
  - This can cause the lens to go out of focus
  - This can be reduced by design (material selection)
- Another source is the focus procedure
  - There are two possible sources of error here
    - Inaccuracy in the measurement of the desired focus position
    - Resolution in the positioning of the focus (e.g., shims in 0.001 inch increments)
  - The focus measurement procedure and focus position resolution must be designed to not cause focus errors which can degrade the image quality beyond the IQ specification

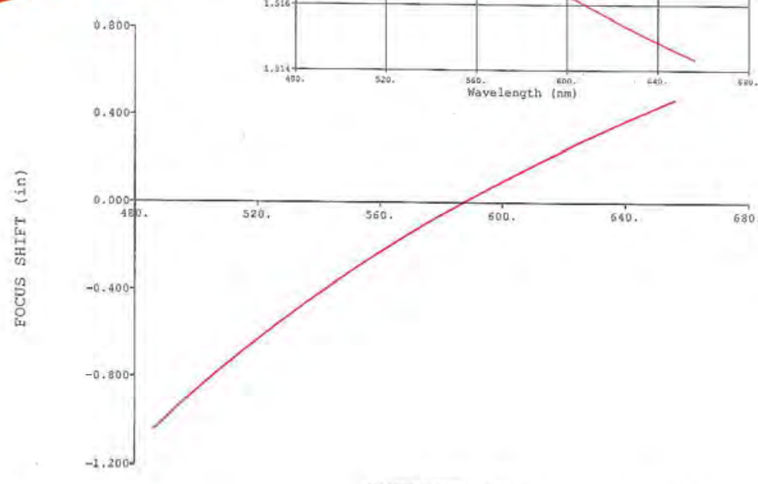
Chromatic Aberration

- Chromatic aberration is caused by the lens's refractive index changing with wavelength





The shorter wavelengths focus closer to the lens because the refractive index is higher for the shorter wavelengths

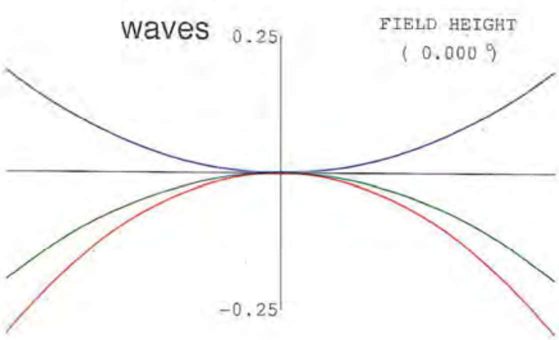


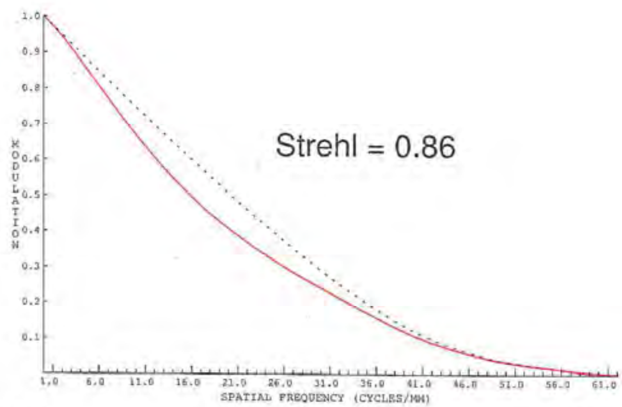
Computing Chromatic Aberration

- The chromatic aberration of a lens is a function of the dispersion of the glass
  - Dispersion is a measure of the change in index with wavelength
- It is commonly designated by the Abbe V-number for three wavelengths
  - For visible glasses, these are F (486.13), d (587.56), C (656.27)
  - For infrared glasses they are typically 3, 4, 5 or 8, 10, 12 microns
  - $V = (n_{middle}-1) / (n_{short} - n_{long})$
- For optical glasses, V is typically in the range 35-80
- For infrared glasses they vary from 50 to 1000
- The axial (longitudinal) spread of the short wavelength focus to the long wavelength focus is F/V
  - Example 1: N-BK7 glass has a V-value of 64.4. What is the axial chromatic spread of an N-BK7 lens of 100 mm focal length?
    - Answer: 100/64.4 = 1.56 mm
    - Note that if the lens were f/2, the diffraction DOF = $\pm 2\lambda f^2 = \pm 0.004$ mm
  - Example 2: Germanium has a V-value of 942 (for 8 – 12 μ). What is the axial chromatic spread of a germanium lens of 100 mm focal length?
    - Answer: 100/942 = 0.11 mm   Note: DOF(f/2) = $\pm 2\lambda f^2 = \pm 0.08$ mm

Chromatic Aberration Example - Germanium Singlet

- We want to use an f/2 germanium singlet over the 8 to 12 micron band
- Question - What is the longest focal length we can have and not need to color correct? (assume an asphere to correct any spherical aberration)
- Answer
  - Over the 8-12 micron band, for germanium V = 942
  - The longitudinal defocus = F / V = F / 942
  - The 1/4 wave depth of focus is $\pm 2\lambda f^2$
  - Equating these and solving gives $F = 4*942*\lambda*f^2 = 150$ mm

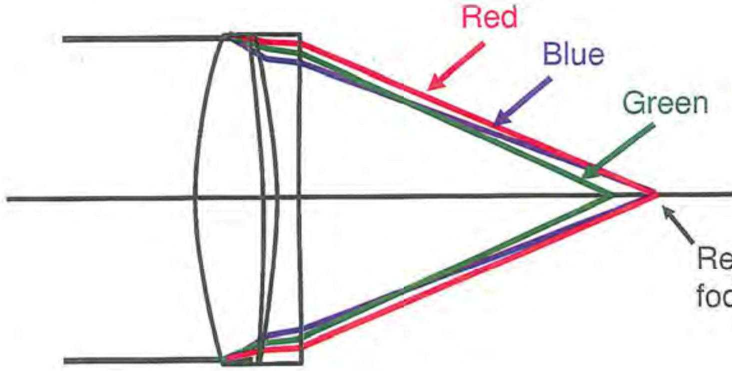




OPTI 517 58

Correcting Chromatic Aberration

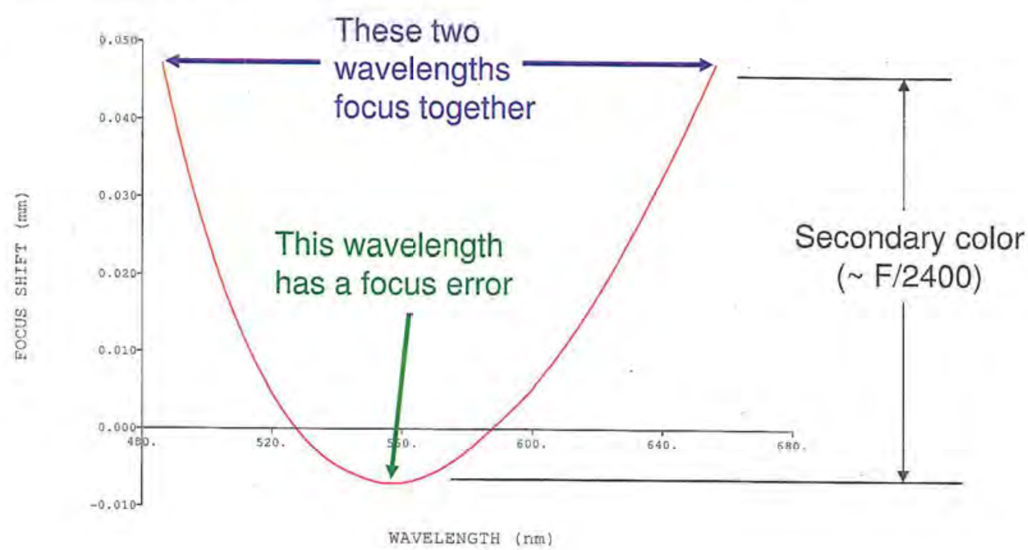
- Chromatic aberration is corrected by a combination of two glasses
  - The positive lens has low dispersion (high V number) and the negative lens has high dispersion (low V number)



  - This will correct primary chromatic aberration
    - The red and blue wavelengths focus together
    - The green (or middle) wavelength still has a focus error
      - This residual chromatic spread is called secondary color

Secondary Color

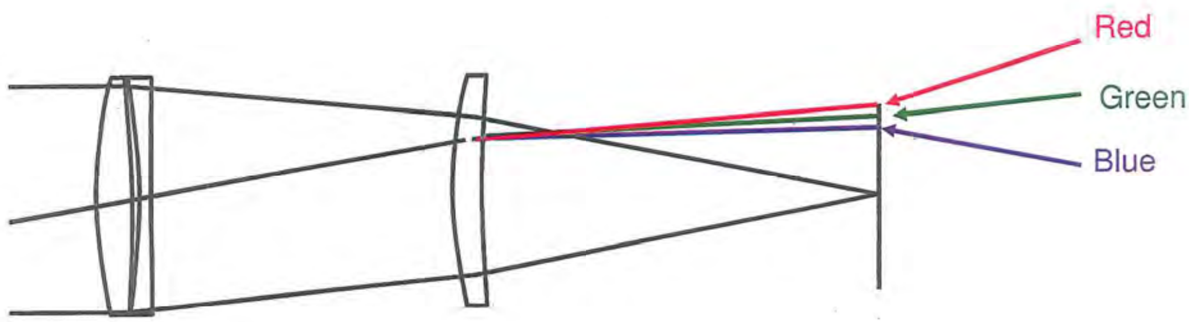
- Secondary color is the residual chromatic aberration left when the primary chromatic aberration is corrected



- Secondary color can be reduced by selecting special glasses
  - These glasses cost more (naturally)

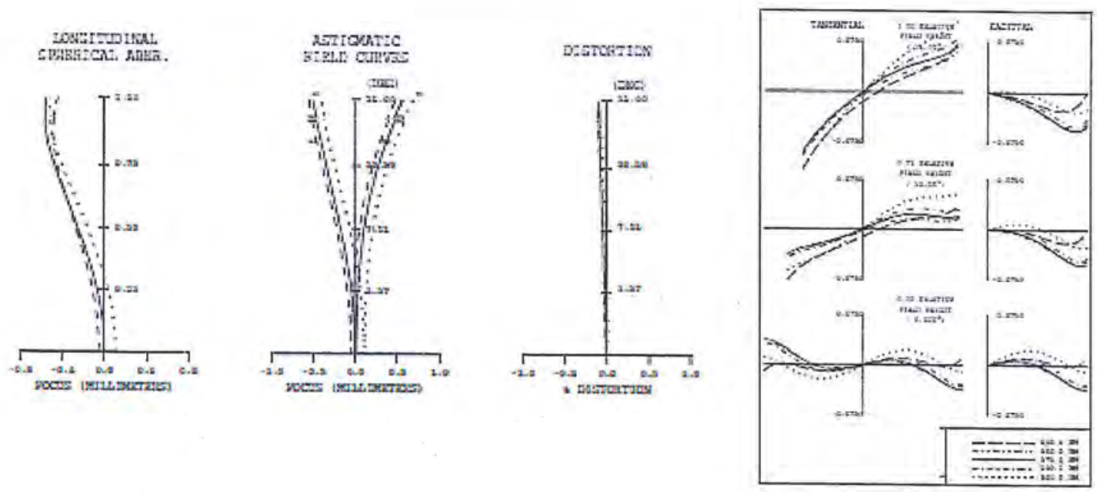
Lateral Color

- Lateral color is a change in focal length (or magnification) with wavelength
  - This results in a different image size with wavelength
  - The effect is often seen as color fringes at the edge of the FOV
  - This reduces the MTF for off-axis images



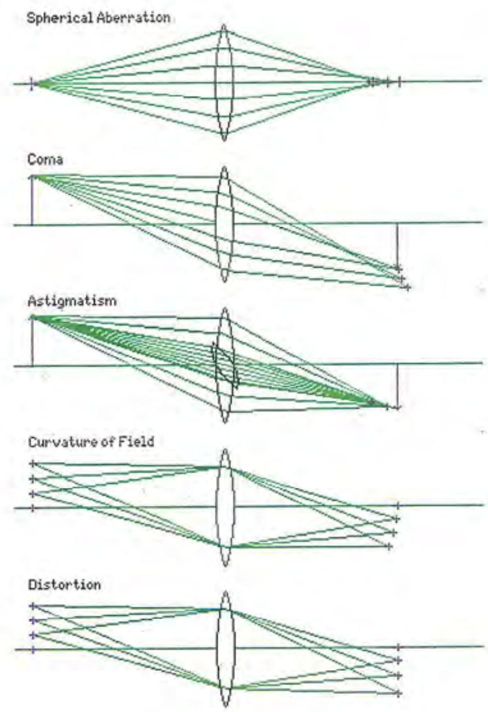
Higher-order Chromatic Aberrations

- For broadband systems, the chromatic variation in the third-order aberrations are often the most challenging aberrations to correct (e.g., spherochromatism, chromatic variation of coma, chromatic variation of astigmatism)
  - These are best studied with ray aberration curves and field plots



The Seidel Aberrations

- These are the classical aberrations in optical design
  - Spherical aberration
  - Coma
  - Astigmatism
  - Distortion
  - Curvature of field
- These aberrations, along with defocus and chromatic aberrations, are the main aberrations in an optical system



The Importance of Third-order Aberrations

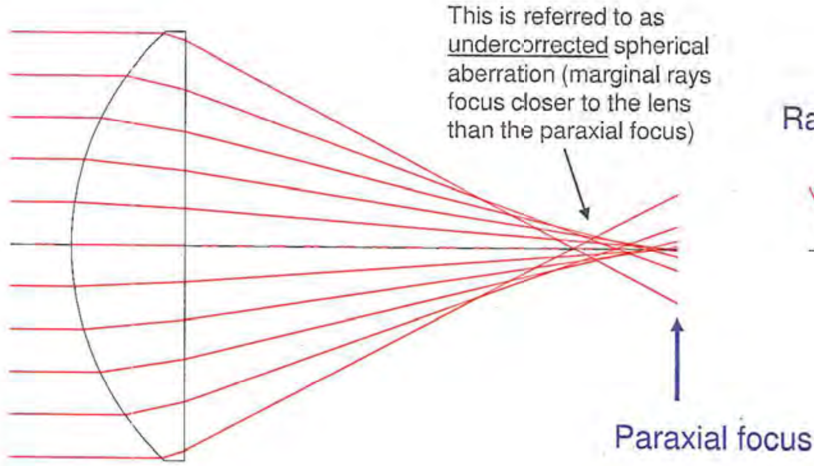
- The ultimate performance of any unconstrained optical design is almost always limited by a specific aberration that is an intrinsic characteristic of the design form
- A familiarity with aberrations and lens forms is an important ingredient in a successful optimization that makes optimal use of the time available to accomplish the design
- A knowledge of the aberrations
  - Allows "spotting" lenses that are at the end of the road with respect to optimization
  - Gives guidance in what direction to "kick" a lens that has strayed from the optimal solution

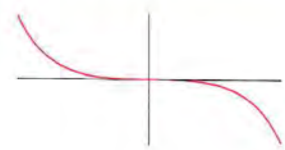
Orders of Aberrations

	Third-order	Fifth-order
Spherical	$(F\#)^{-3}$, $(\Theta)^{0}$	$(F\#)^{-5}$, $(\Theta)^{0}$
Coma	$(F\#)^{-2}$, $(\Theta)^{1}$	$(F\#)^{-4}$, $(\Theta)^{1}$
Astigmatism	$(F\#)^{-1}$, $(\Theta)^{2}$	$(F\#)^{-1}$, $(\Theta)^{4}$
Field Curvature	$(F\#)^{-1}$, $(\Theta)^{2}$	$(F\#)^{-1}$, $(\Theta)^{4}$
Distortion	$(F\#)^{0}$, $(\Theta)^{3}$	$(F\#)^{0}$, $(\Theta)^{5}$

Spherical Aberration

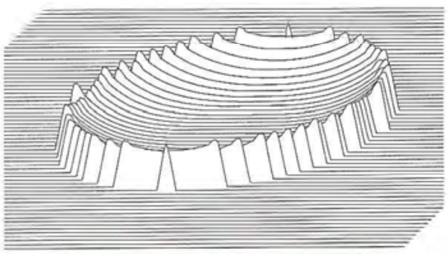
- Spherical aberration is an on-axis aberration
- Rays at the outer parts of the pupil focus closer to or further from the lens than the paraxial focus





- The magnitude of the (third-order) spherical aberration goes as the cube of the aperture (going from f/2 to f/1 increases the SA by a factor of 8)

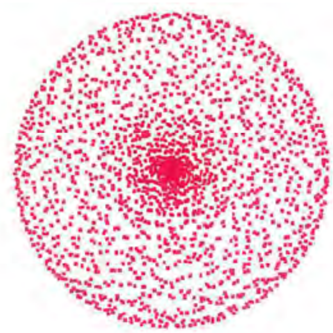
Third-order SA Ray Aberration Curves

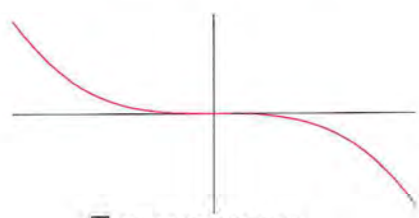


Wavefront map

Wavefront error

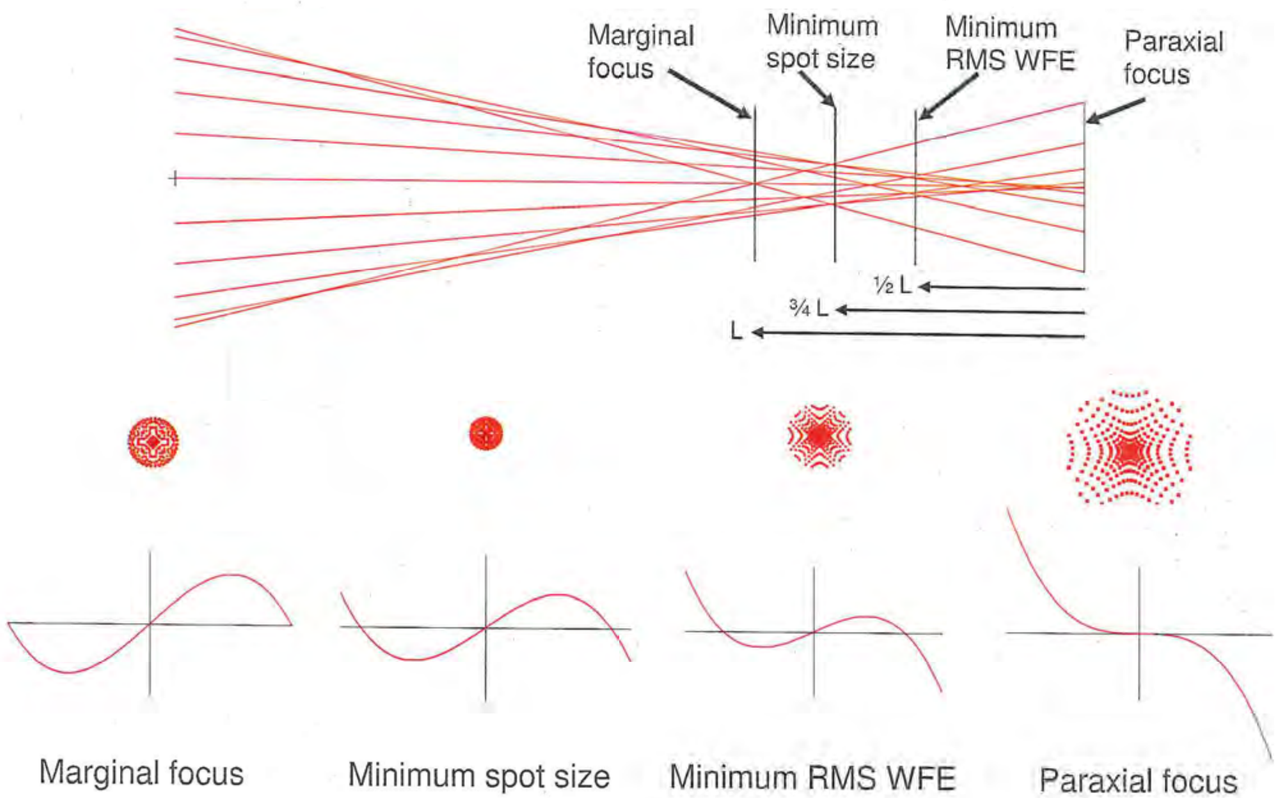
Spot diagram





Transverse ray aberration curve

Spherical Aberration

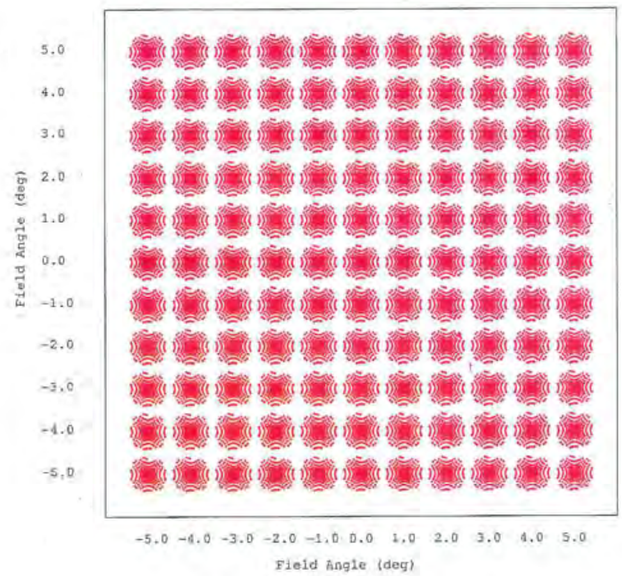


Scaling Laws for Spherical Aberration

$\propto (f/\#)^{-3}$

Spot size goes as the cube of the EPD (or inverse cube of the f/#)

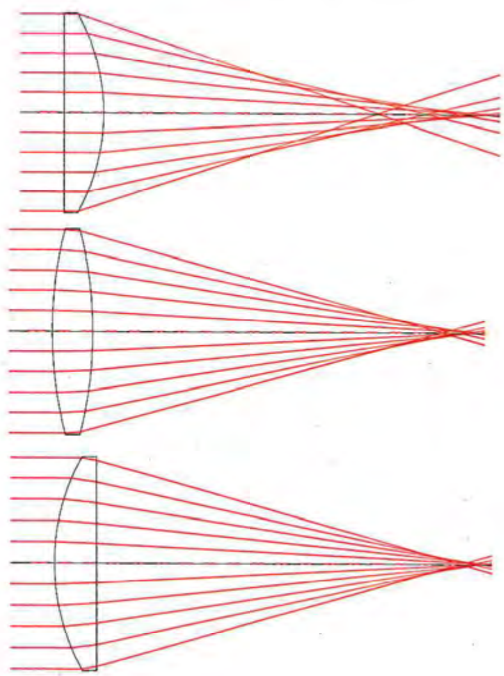
$\propto \theta^0$



Spot size not dependent on field position

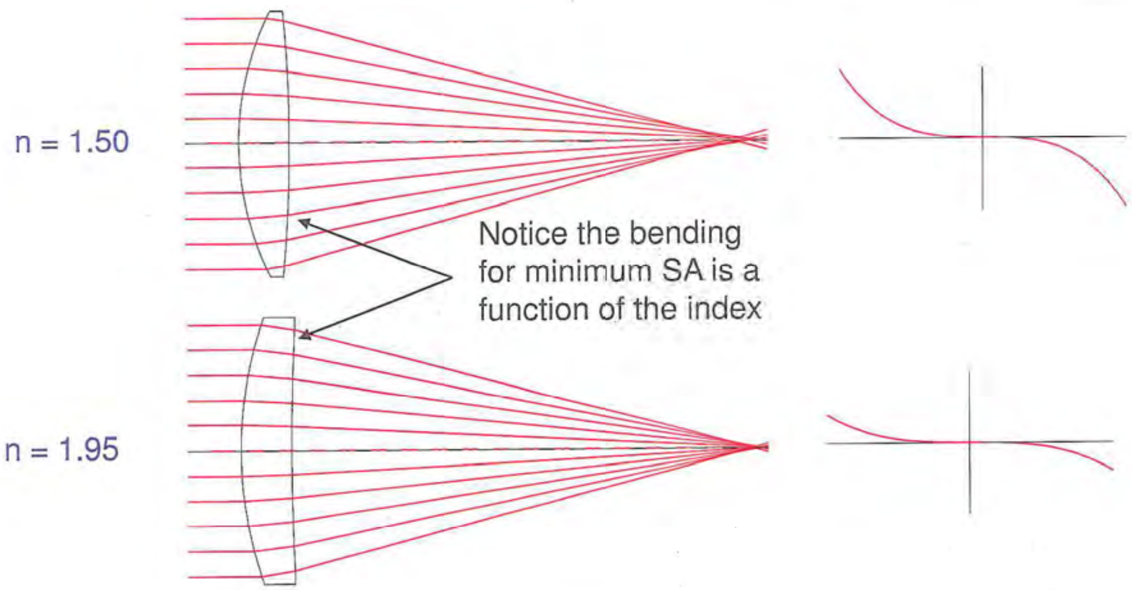
Spherical Aberration vs. Lens Shape

- The spherical aberration is a function of the lens bending, or shape of the lens



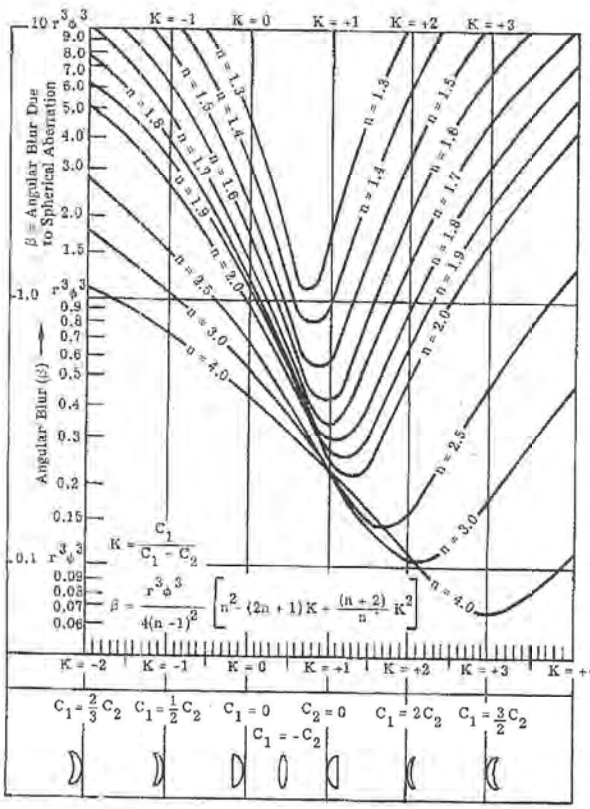
Spherical Aberration vs. Refractive Index

- Spherical aberration is reduced with higher index materials
  - Higher indices allows shallower radii, allowing less variation in incidence angle across the lens

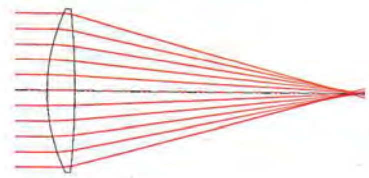


Spherical Aberration vs. Index and Bending

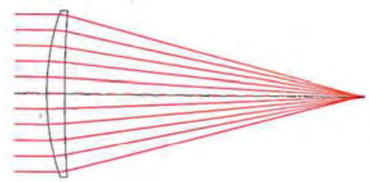
$$\beta \text{ at } K_{min} = r^3\varphi^3 \frac{4n^2 - n}{16(n-1)^2(n+2)}$$



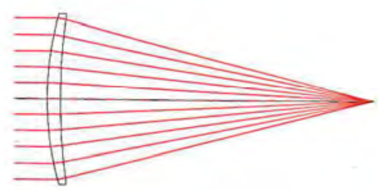
n = 1.5



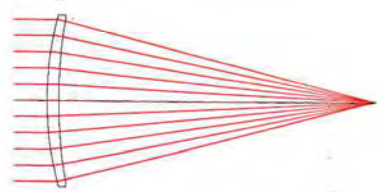
n = 2.0



n = 3.0

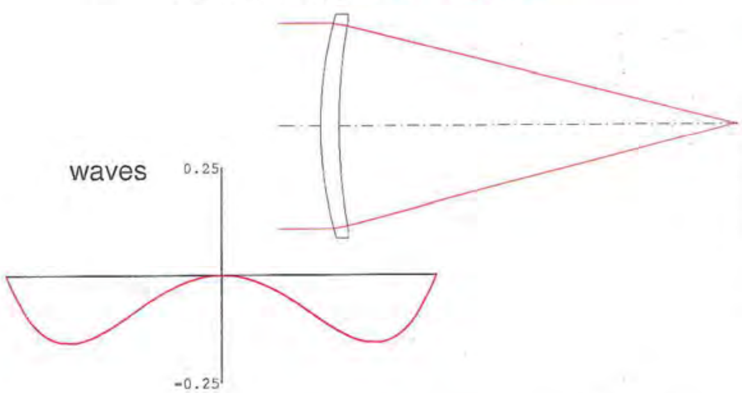


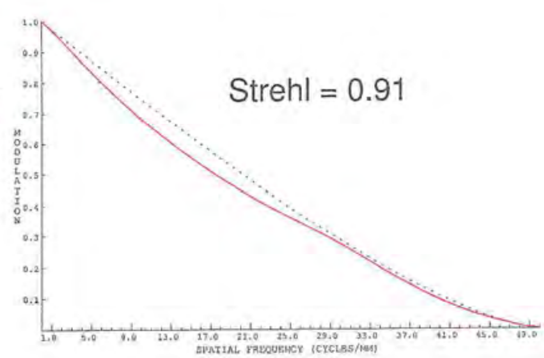
n = 4.0



Example - Germanium Singlet

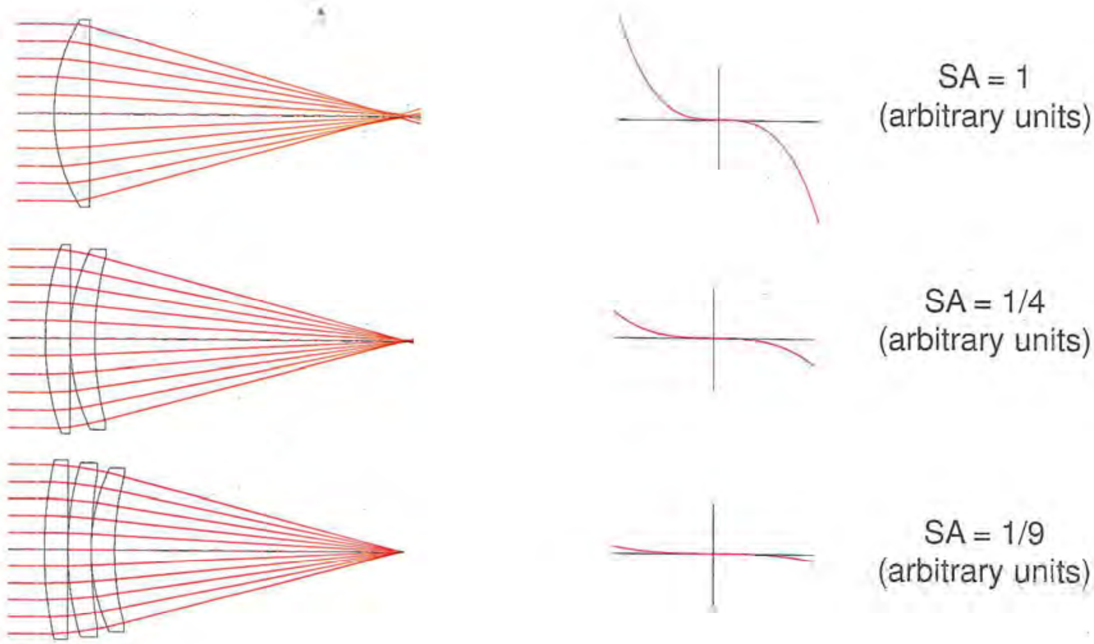
- We want an f/2 germanium singlet to be used at 10 microns (0.01 mm)
- Question - What is the longest focal length we can have and not need aspherics to correct spherical aberration?
- Answer
  - Diffraction Airy disk angular size is $\beta_{diff} = 2.44\ \lambda/D$
  - Spherical aberration angular blur is $\beta_{sa} = 0.00867 / f^3$
  - Equating these gives $D = 2.44\ \lambda\ f^3 / 0.00867 = 22.5$ mm
  - For f/2, this gives F = 45 mm





Spherical Aberration vs. Number of Lenses

- Spherical aberration can be reduced by splitting the lens into more than one lens



Spherical Aberration vs. Number of Lenses

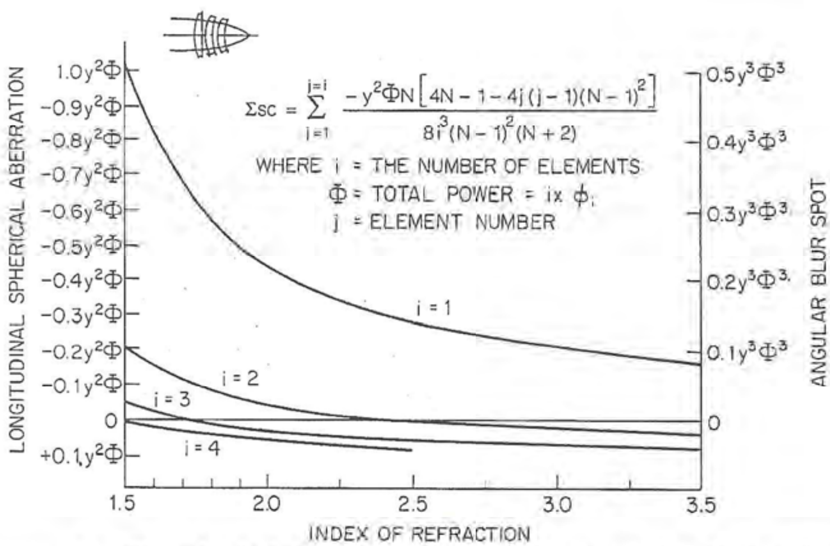
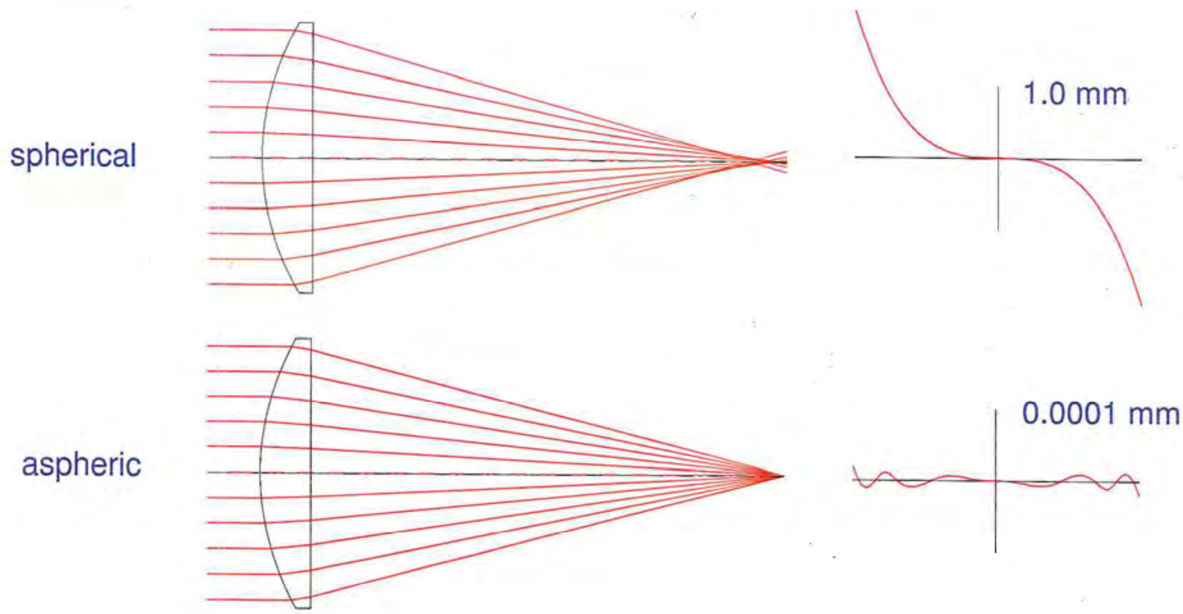


Figure 3.2 The spherical aberration of one, two, three, and four thin positive elements, each bent for minimum spherical aberration, plotted as a function of the index of refraction, and showing the reduction in the amount of aberration produced by splitting a single element into two or more elements (of the same total power). Each plot is labeled with *i*, the number of elements in the set. (The object is at infinity.)

Spherical Aberration and Aspherics

- The spherical aberration can be reduced, or even effectively eliminated, by making one of the surfaces aspheric



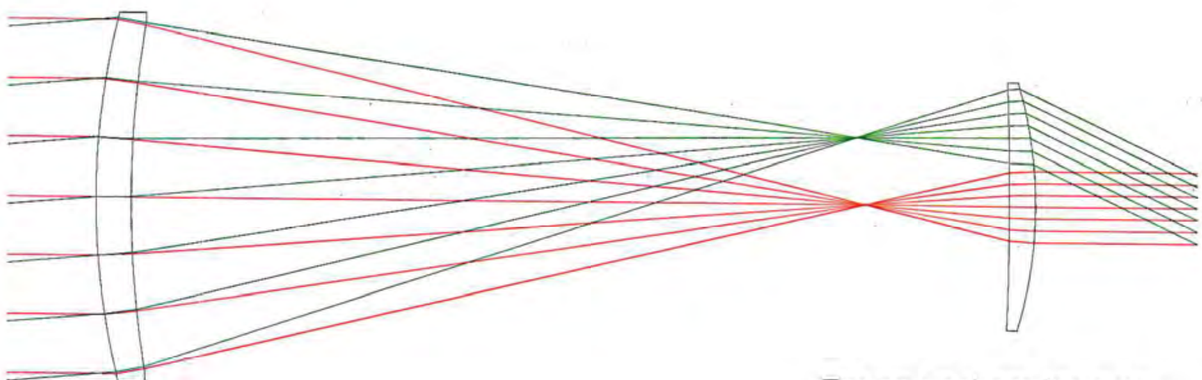
Aspheric Surfaces

- Aspheric surfaces technically are any surfaces which are not spherical, but usually refer to a polynomial deformation to a conic

$$z(r) = \frac{r^2/R}{1+\sqrt{1-(k+1)(r/R)^2}} + A\,r^4 + B\,r^6 + C\,r^8 + D\,r^{10} + \ldots$$

- The aspheric coefficients (A, B, C, D, …) can correct 3rd, 5th, 7th, 9th, … order spherical aberration
- When used near a pupil, aspherics are used primarily to correct spherical aberration
- When used far away (optically) from a pupil, they are primarily used to correct astigmatism by flattening the field
- Before using aspherics, be sure that they are necessary and the increased performance justifies the increased cost
  - Never use a higher-order asphere than justified by the ray aberration curves

Optimizing Aspherics

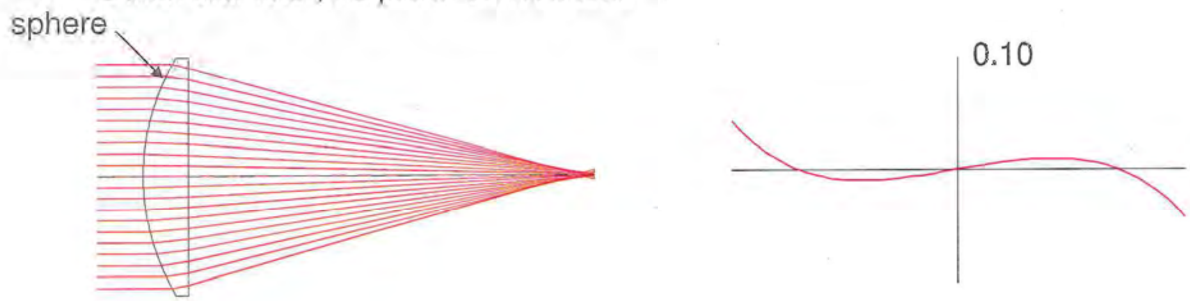


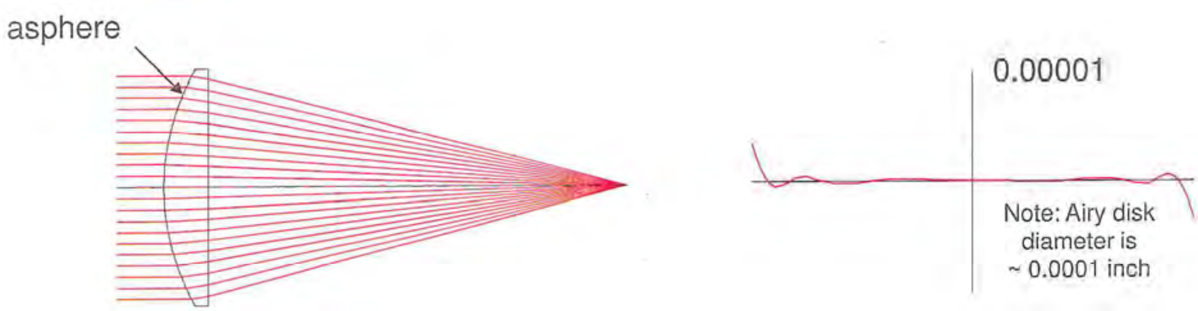
For an asphere at (or near) a pupil, there need to be enough rays to sample the pupil sufficiently.
This asphere primarily corrects spherical aberration.

For an asphere far away (optically) from a pupil, the ray density need not be high, but there must be a sufficient number of overlapping fields to sample the surface accurately.
This asphere primarily corrects field aberrations (e.g., astigmatism).

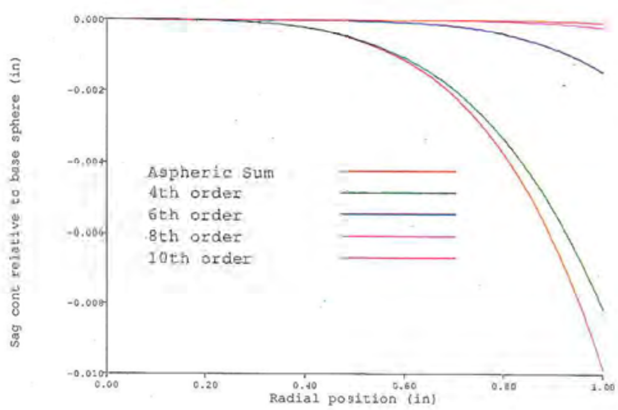
Asphere Example

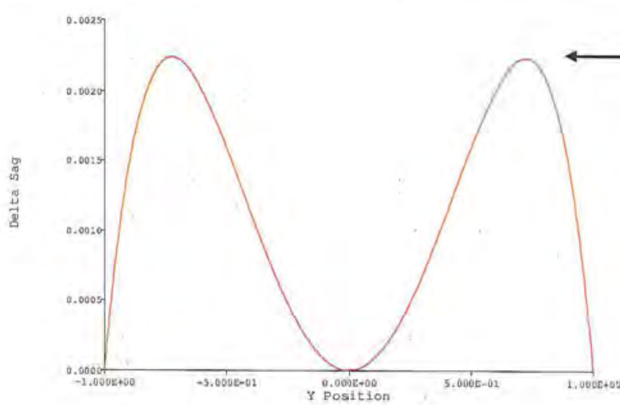
- 2 inch diameter, f/2 plano-convex lens





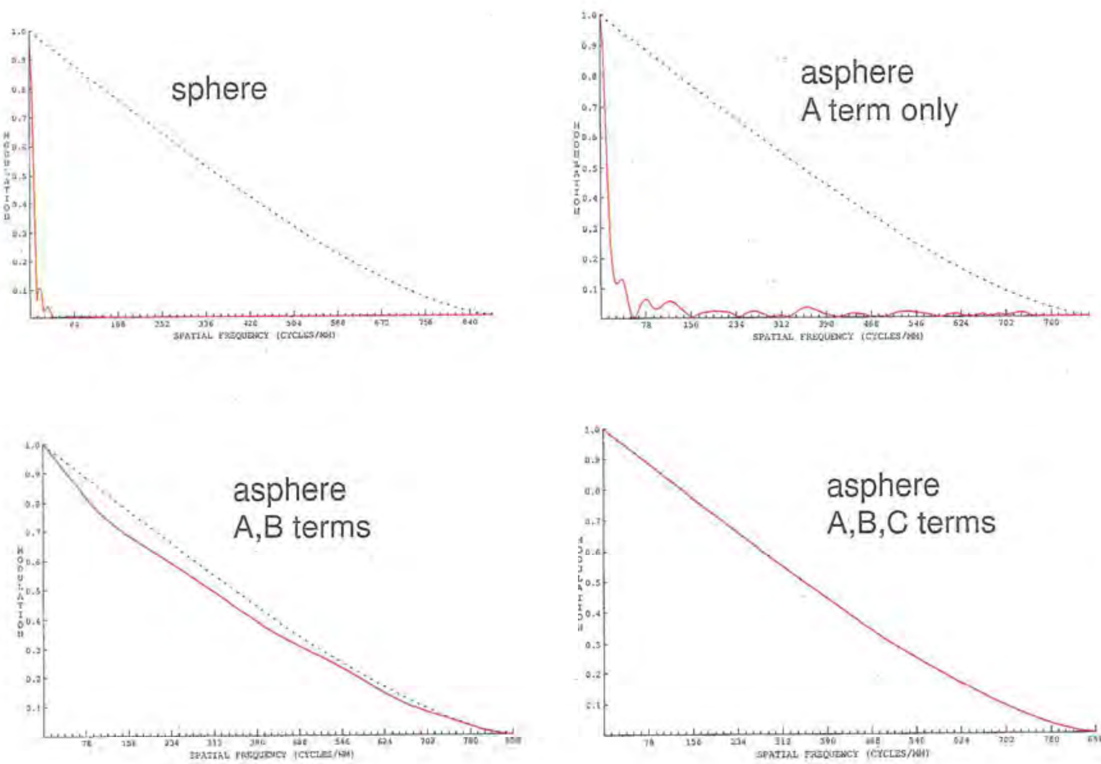
Aspheric Orders





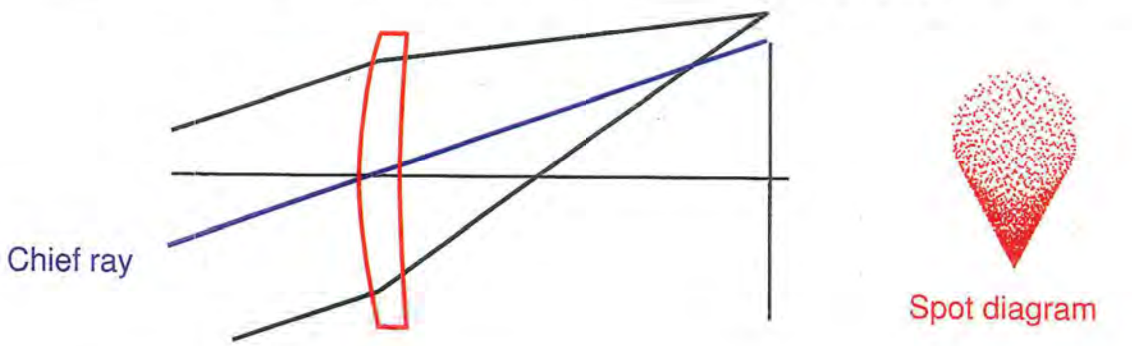
Corresponds to ~114 waves of asphericity

MTF vs. Aspheric Order



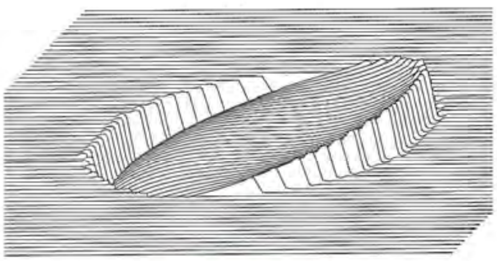
Coma

- Coma is an off-axis aberration
- It gets its name from the spot diagram which looks like a comet (coma is Latin for comet)
- A comatic image results when the periphery of the lens has a higher or lower magnification than the portion of the lens containing the chief ray



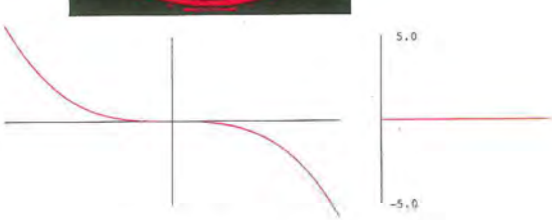
- The magnitude of the (third-order) coma is proportional to the square of the aperture and the first power of the field angle

Transverse vs. Wavefront 3rd-order Coma

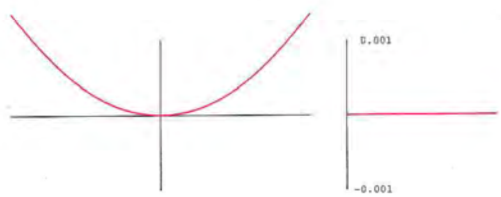


Wavefront map

Spot diagram



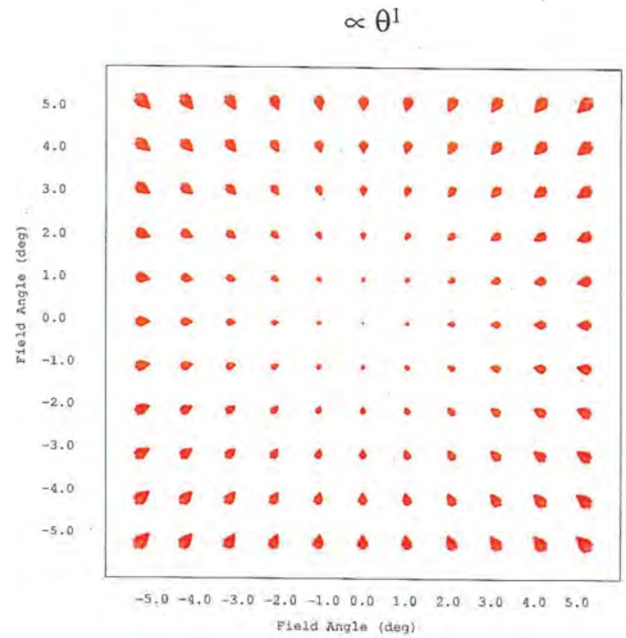
Wavefront error



Transverse ray aberration

Scaling Laws for Coma

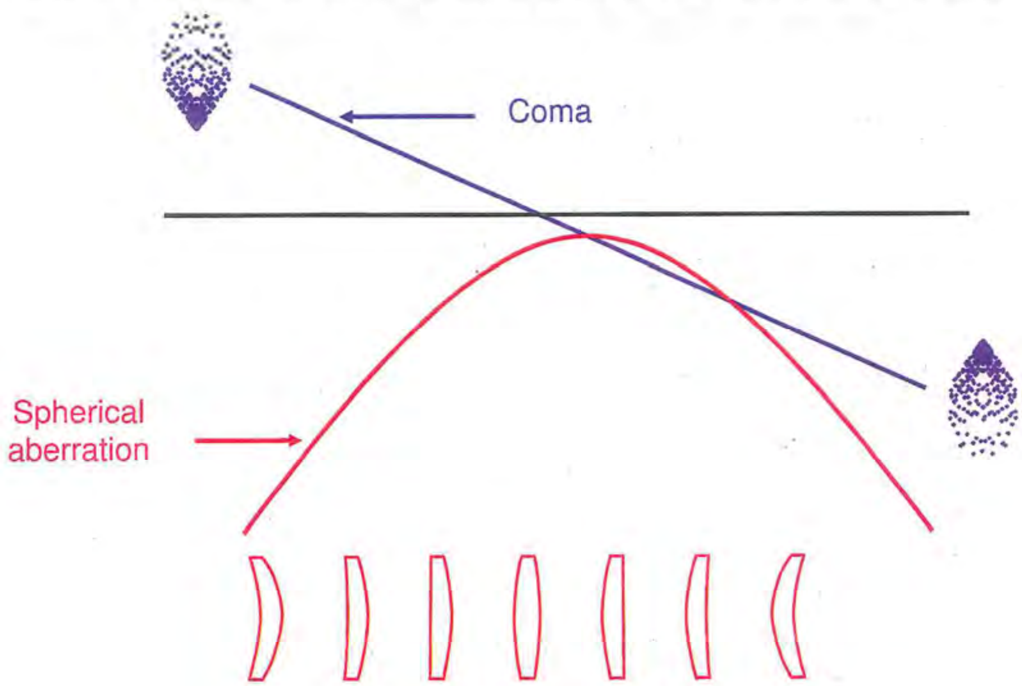
Spot size goes as the square of the EPD (or inverse square of the f/#)



Spot size is linearly dependent on field height

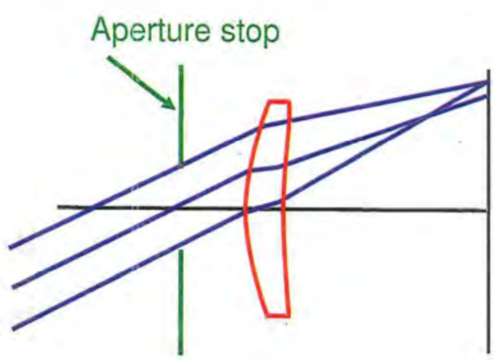
Coma vs. Lens Bending

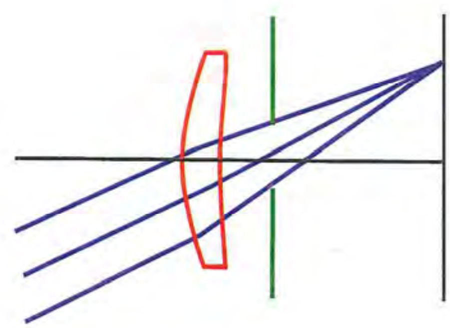
- Both spherical aberration and coma are a function of the lens bending



Coma vs. Stop Position

- The size of the coma is also a function of the stop location relative to the lens

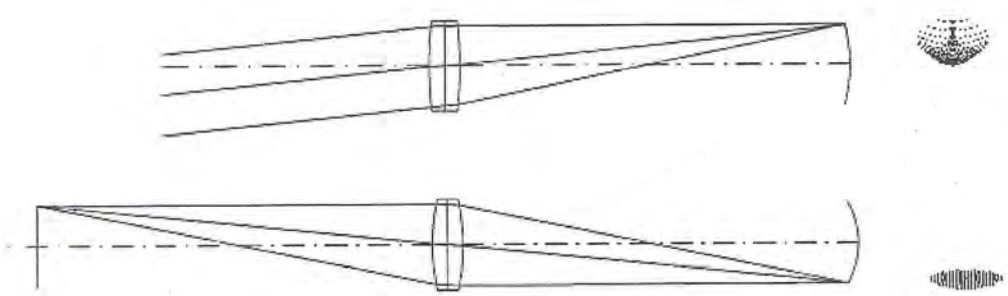




Coma is reduced due to increased lens symmetry around the stop

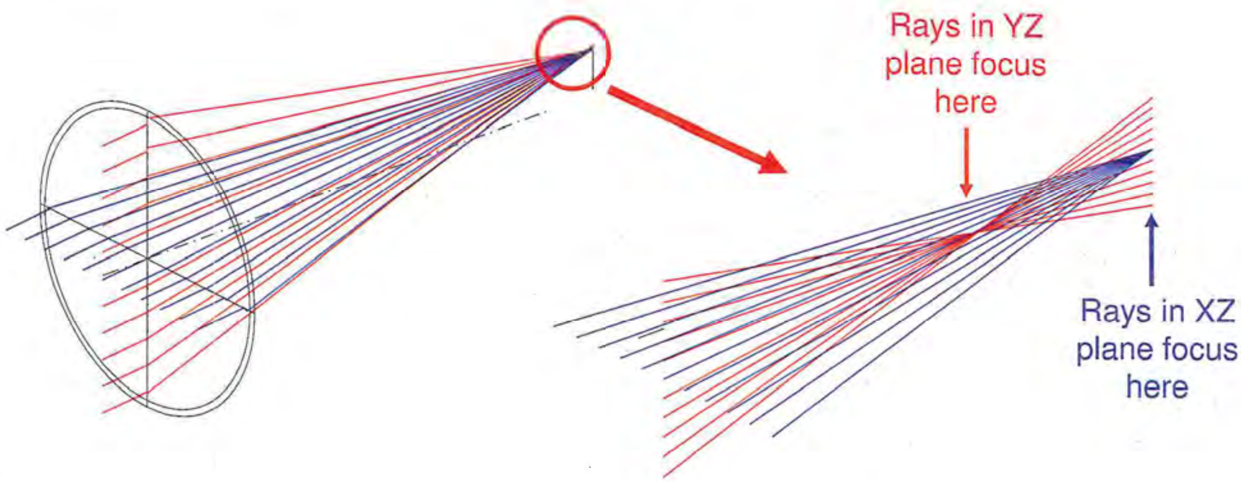
Coma is an Odd Aberration

- Any completely symmetric optical system (including the stop location) is free of all orders of odd field symmetry aberrations (coma and distortion)



Astigmatism

- Astigmatism is caused when the wavefront has a cylindrical component
  - The wavefront has different spherical power in one plane (e.g., tangential) vs. the other plane (e.g., sagittal)
- The result is different focal positions for tangential and sagittal rays



- The magnitude of the (third-order) astigmatism goes as the first power of the aperture and the square of the field angle

Cause of Astigmatism

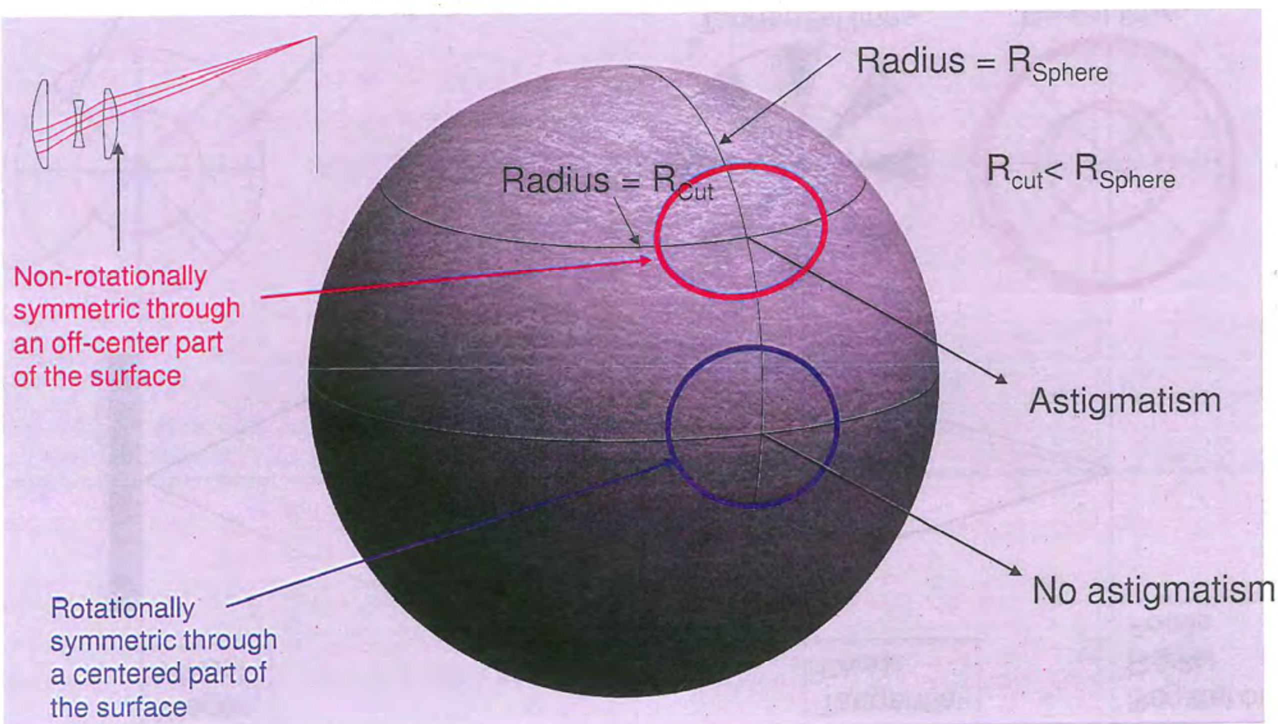
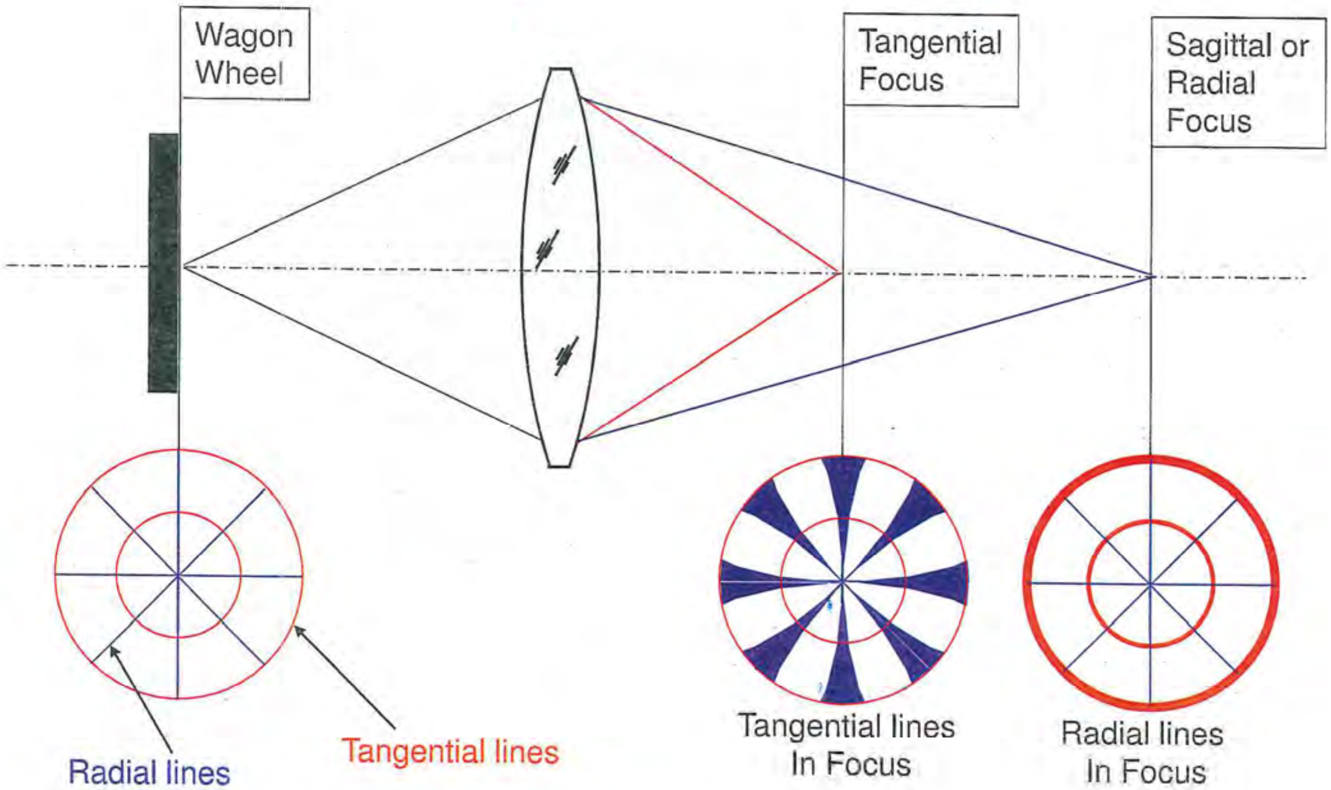
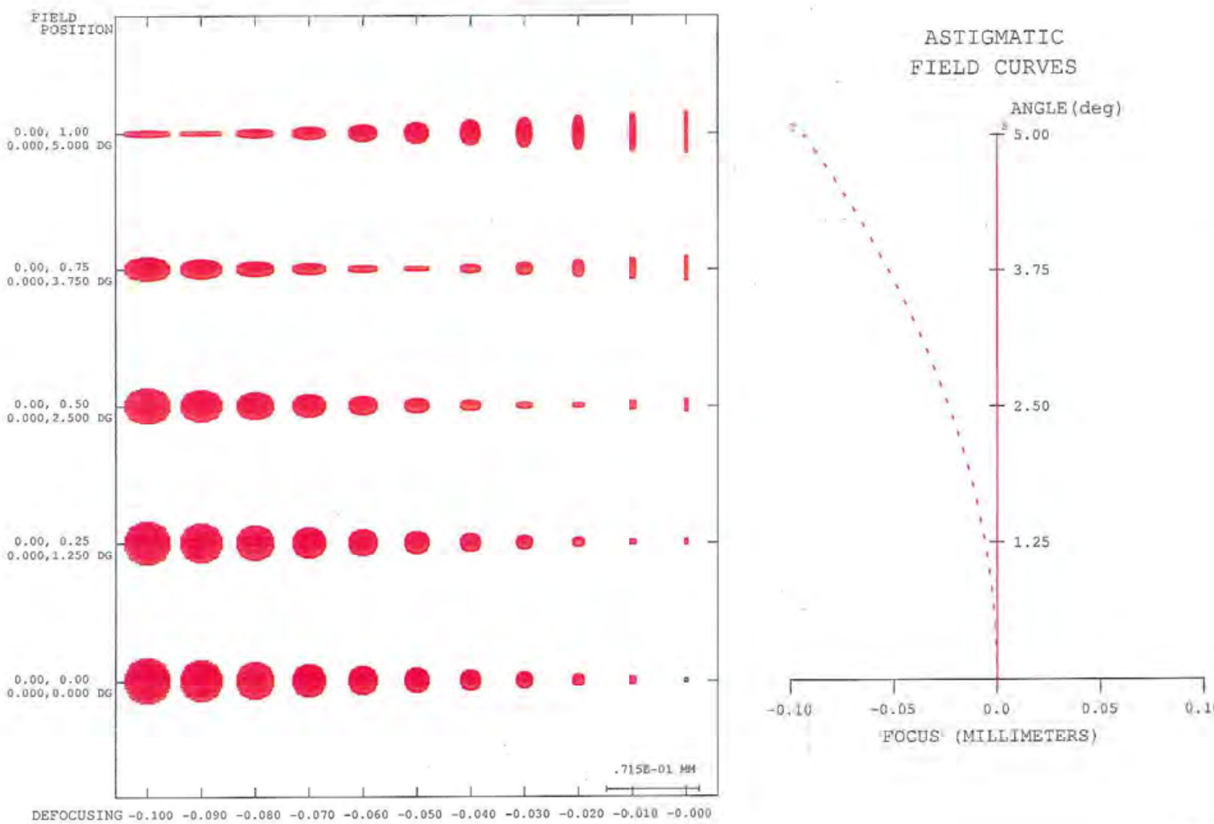


Image of a Wagon Wheel With Astigmatism



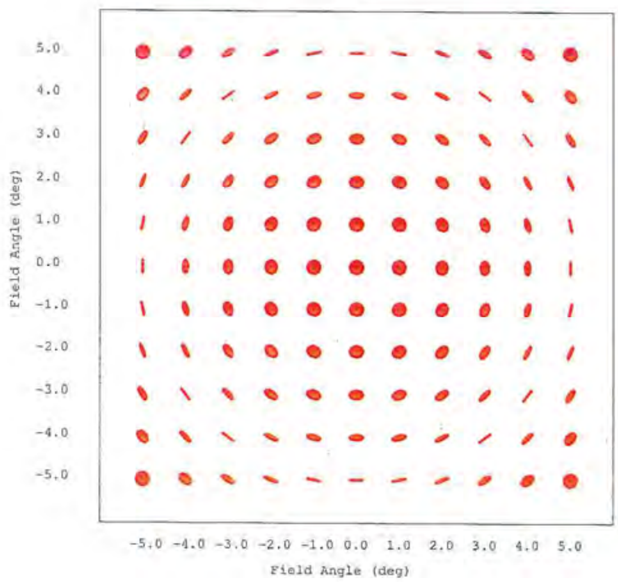
Astigmatism vs. Field



OPTI 517 91

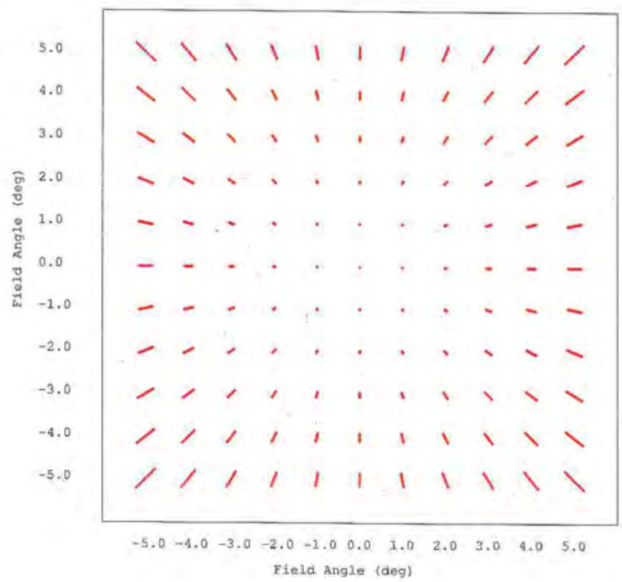
Scaling Laws for Astigmatism

$\propto (f/\#)^{-1}\,\theta^2$



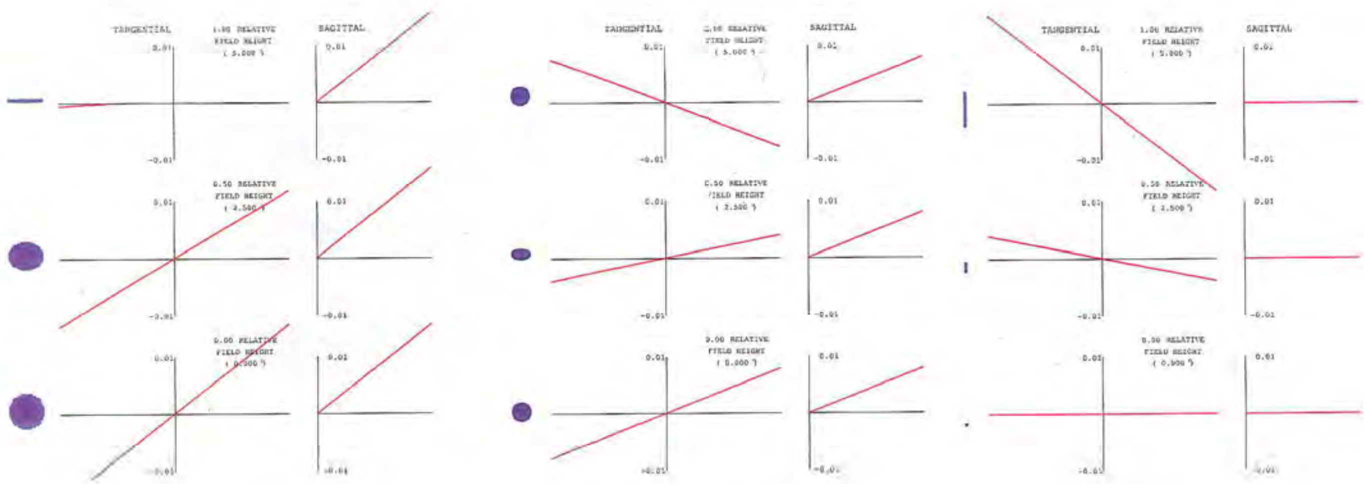
Tangential focus

$\propto (f/\#)^{-1}\,\theta^2$



Sagittal focus

Astigmatism Ray Aberration Plots



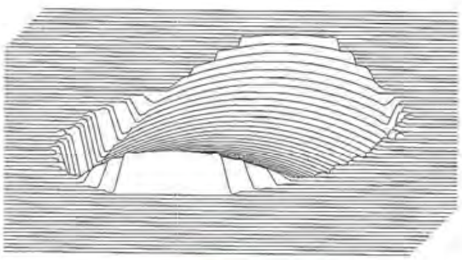
Tangential focus

Medial focus
(best diffraction focus)
Occurs halfway between sagittal and tangential foci

Sagittal focus

Note: the sagittal focus does not always occur at the paraxial focus

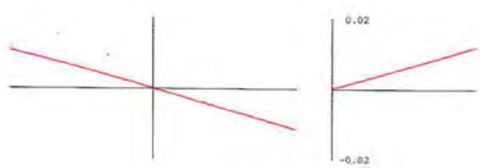
Transverse vs. Wavefront Astigmatism



At medial focus

Spot diagram

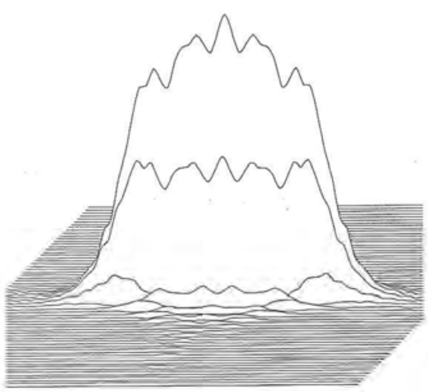
Wavefront map

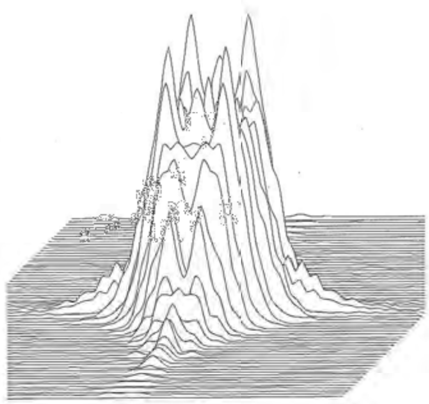


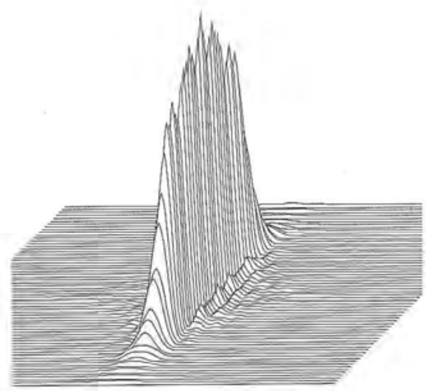
Wavefront error

Transverse ray aberration

PSF of Astigmatism vs. Focus Position







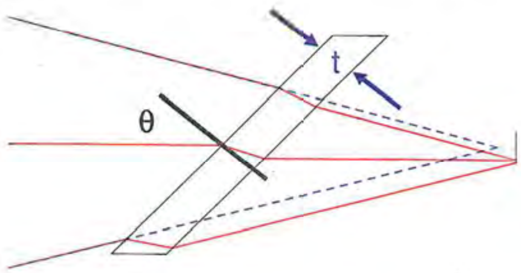
Tangential focus

Medial focus
(best diffraction focus)

Sagittal focus

Astigmatism of a Tilted Flat Plate

- Placing a tilted plane parallel plate into a diverging or converging beam will introduce astigmatism



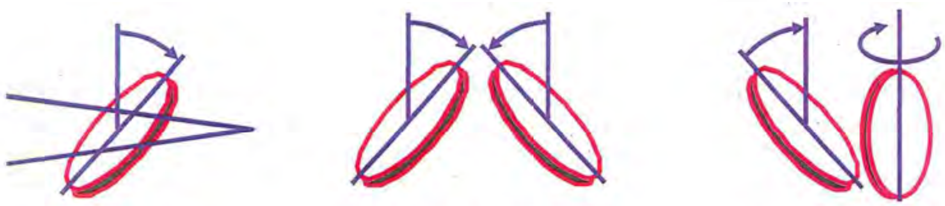
- The amount of the longitudinal astigmatism (focus shift between the tangential and sagittal foci) is given by

$$\text{Ast} = \frac{t}{\sqrt{n^2 - \sin^2\theta}}\left[\frac{n^2 \cos^2\theta}{n^2 - \sin^2\theta} - 1\right] \quad \text{Exact}$$

$$\text{Ast} = \frac{-t\,\theta^2\left(n^2 - 1\right)}{n^3} \quad \text{Third-order}$$

Correcting the Astigmatism of a Tilted Flat Plate

- The astigmatism introduced by a tilted flat plate can be corrected by
  - Adding cylindrical lenses
  - Adding tilted spherical lenses
  - Adding another plate tilted in the orthogonal plane



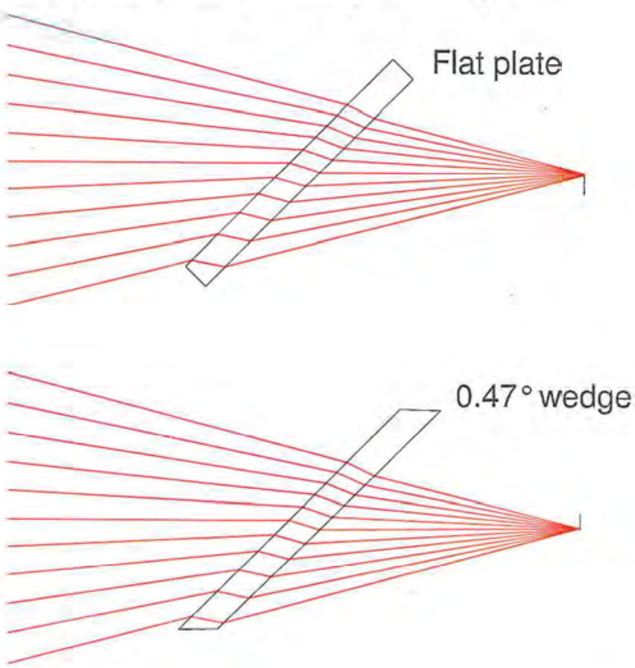
To correct for this

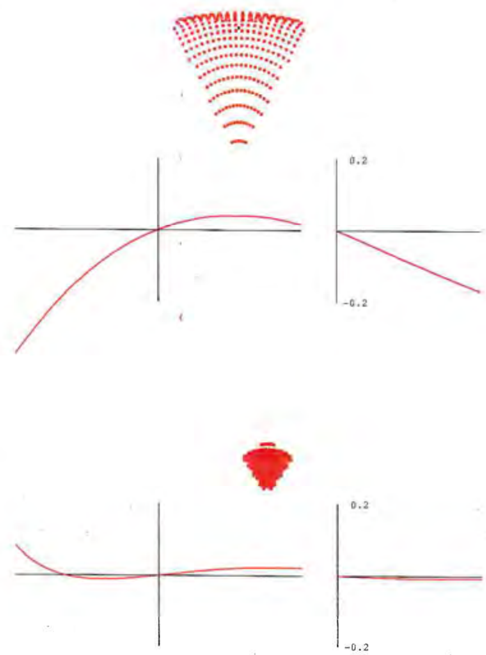
Do not do this (it will double the astigmatism)

Do this

Reducing the Astigmatism of a Tilted Flat Plate

- Astigmatism of a flat plate can be reduced by adding a slight wedge to the plate

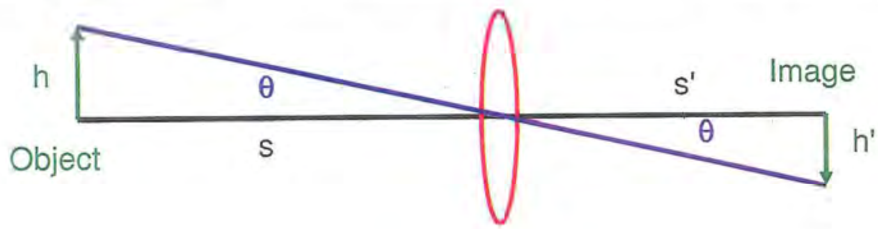




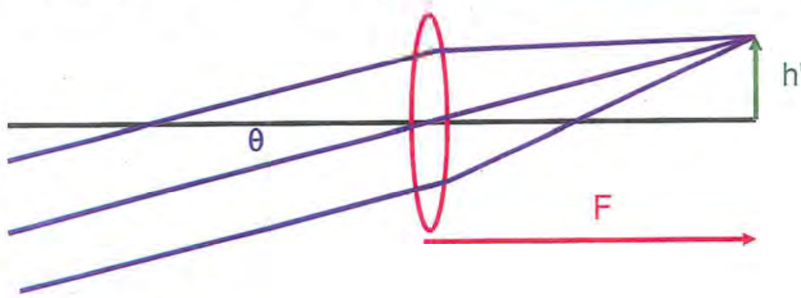
Transverse ray aberration

Rectilinear Imaging

- Most optical systems want to image rectilinear objects into rectilinear images

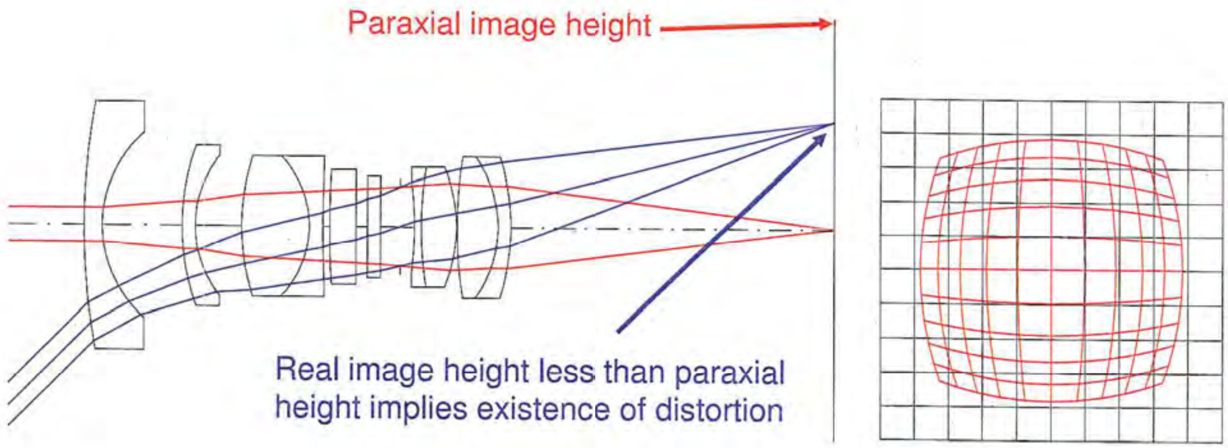


- This requires that $m = -s'/s = -h'/h = \text{constant}$ for the entire FOV
- For infinite conjugate lenses, this requires that $h' = F \tan\theta$ for all field angles



Distortion

- If rectilinear imaging is not met, then there is distortion in the lens
- Effectively, distortion is a change in magnification or focal length over the field of view

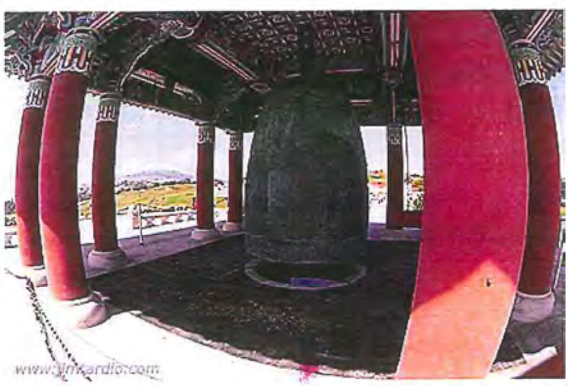


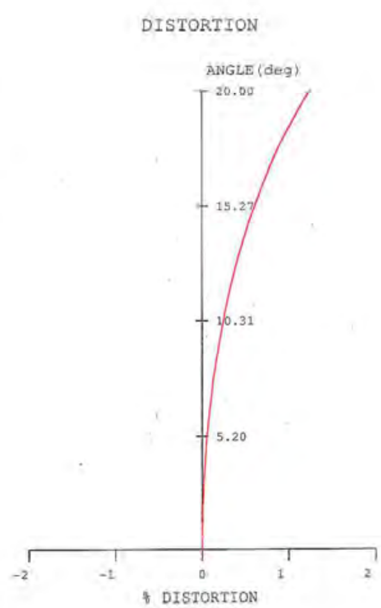
Plot of distorted FOV

- Negative distortion (shown) is often called barrel distortion
- Positive distortion (not shown) is often called pincushion distortion

More on Distortion

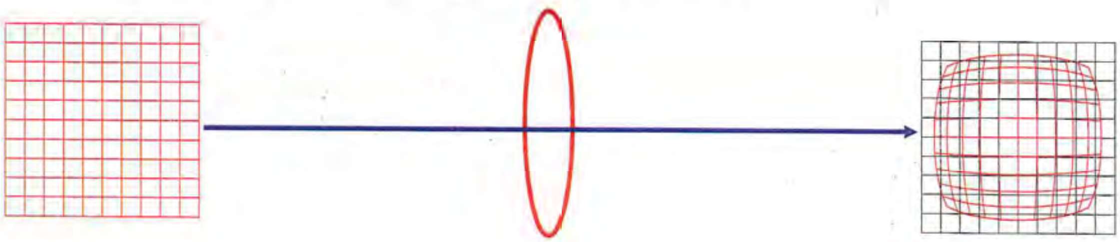
- Distortion does not result in a blurred image and does not cause a reduction in any measure of image quality such as MTF
- Distortion is a measure of the displacement of the image from its corresponding paraxial reference point
- Distortion is independent of f/number
- Linear distortion is proportional to the cube of the field angle
- Percent distortion is proportional to the square of the field angle



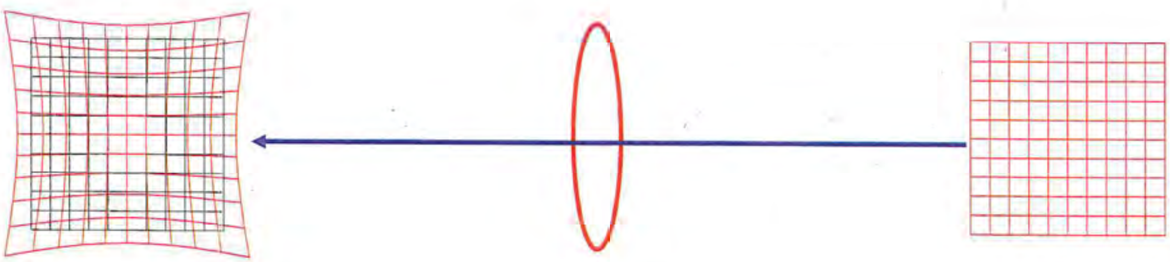


Implications of Distortion

- Consider negative distortion
  - A rectilinear object is imaged inside the detector

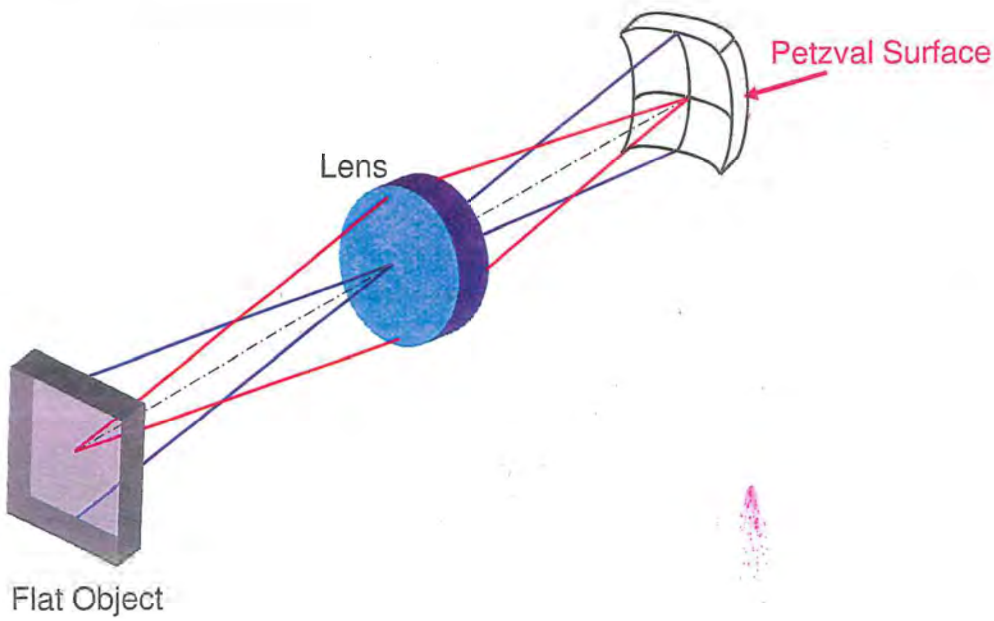


- This means a rectilinear detector sees a larger-than-rectilinear area in object space



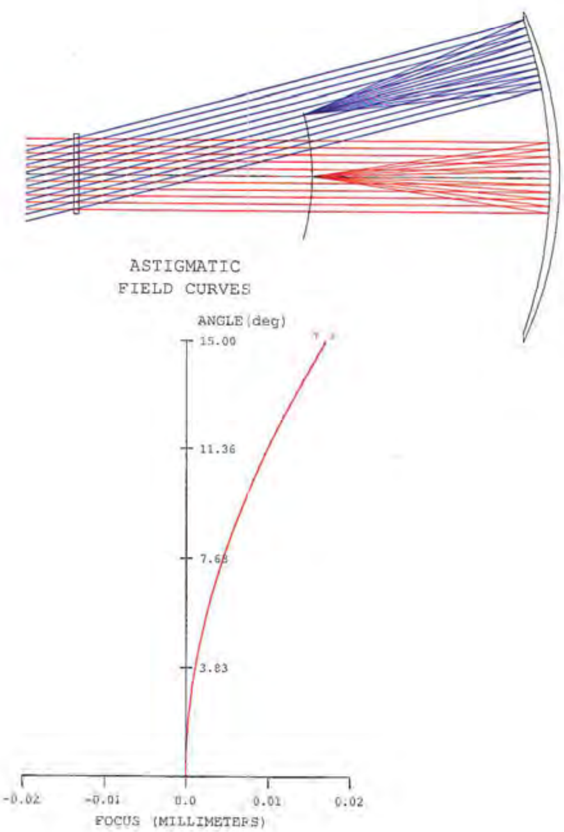
Curvature of Field

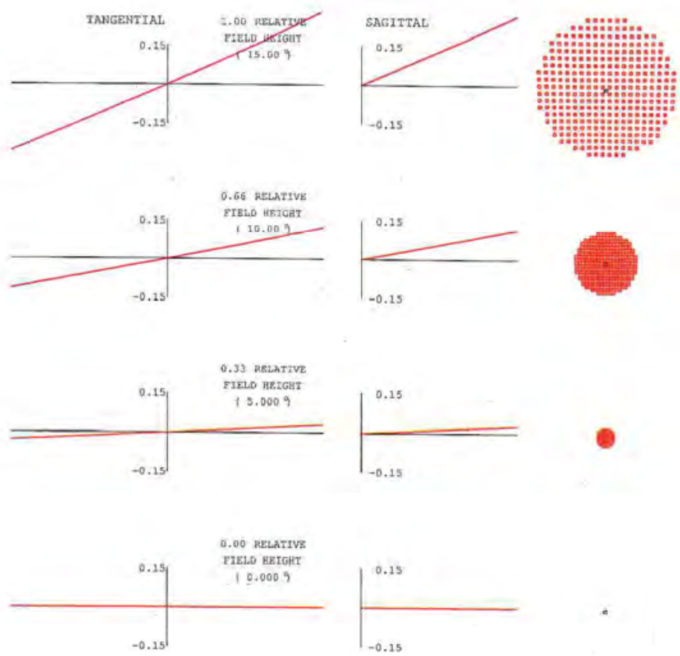
- In the absence of astigmatism, the focal surface is a curved surface called the Petzval surface



Third-order Field Curvature

$\propto (f/\#)^{-1}\,\theta^2$





Aberrations relative to a flat image surface

OPT 517 104

The Petzval Surface

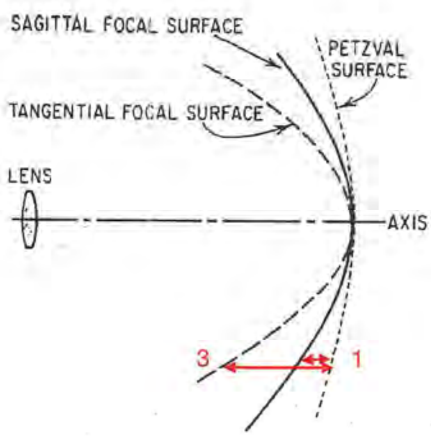
- The radius of the Petzval surface is given by

$$\frac{1}{R_{Petzval}} = \sum_i \left(\frac{1}{n_i F_i} \right)$$

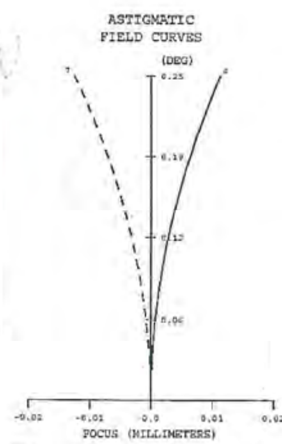
  - For a singlet lens, the Petzval radius = n F
- Obviously, if we have only positive lenses in an optical system, the Petzval radius will become very short
  - We need some negative lenses in the system to help make the Petzval radius longer (i.e., flatten the field)
- This, and chromatic aberration correction, is why optical systems need some negative lenses in addition to all the positive lenses

Field Curvature and Astigmatism

- As an aberration, field curvature is not very interesting
- As a design obstacle, it is the basic reason that optical design is still a challenge
- The astigmatic contribution starts from the Petzval surface
  - If the axial distance from the Petzval surface to the sagittal surface is 1 (arbitrary units), then the distance from the Petzval surface to the tangential surface is 3



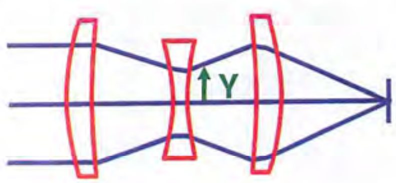
Field curvature and astigmatism can be used together to help flatten the image plane and improve the image quality



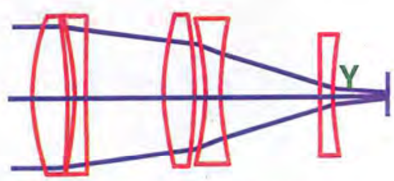
Flat Medial Field

Flattening the Field

- The contribution of a lens to the focal length is proportional to yF where F is lens power (1/F)
- The contribution of a lens to the Petzval sum is proportional to F/n
- Thus, if we include negative lenses in the system where y is small we can reduce the Petzval sum and flatten the field while holding the focal length

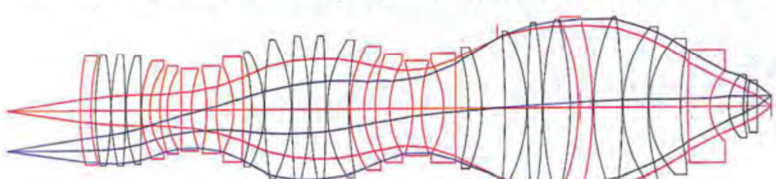


Cooke Triplet



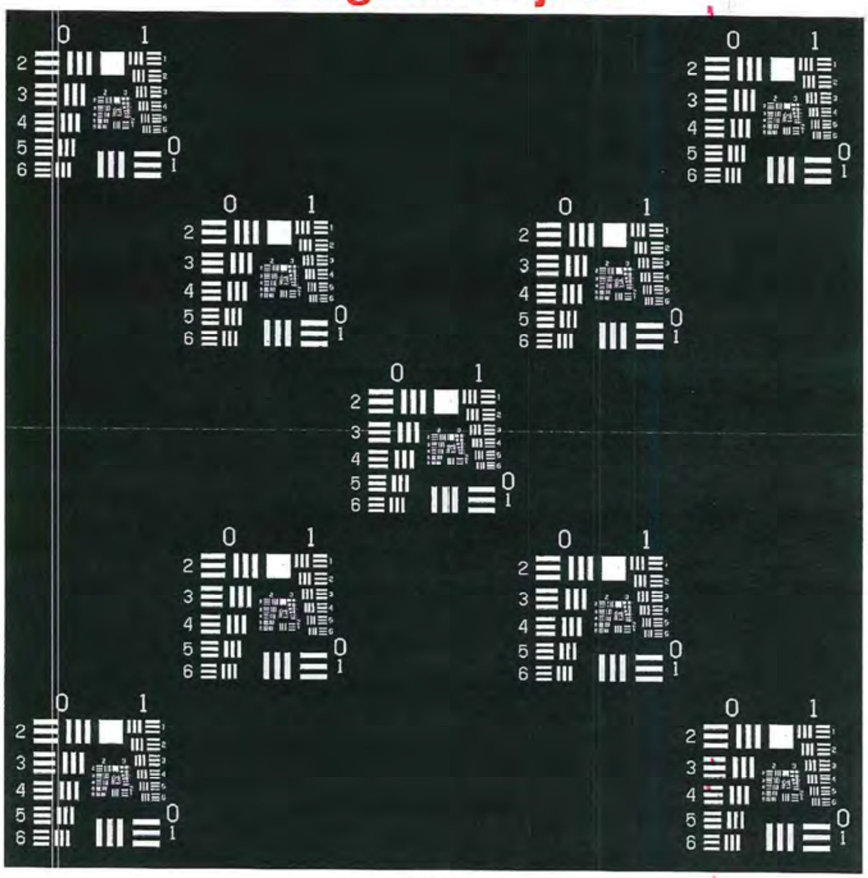
Lens With Field Flattener (Petzval Lens)

- Yet another reason why optical systems have so darn many lenses



Flat-field lithographic lens
Negative lenses in RED

Original Object



Spherical Aberration

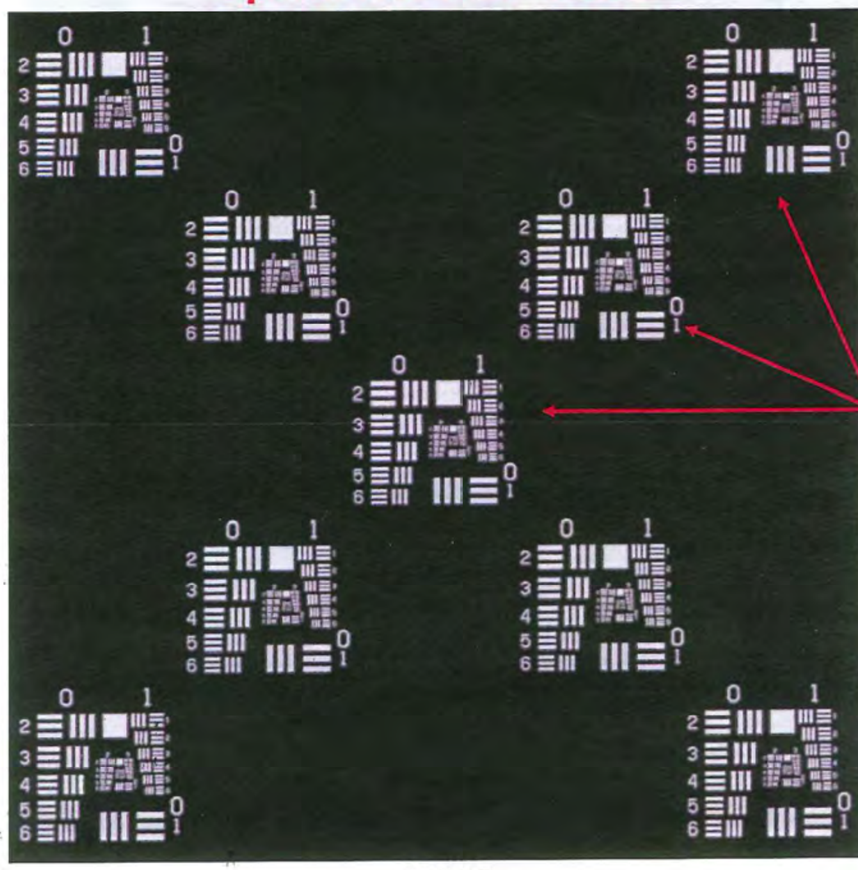
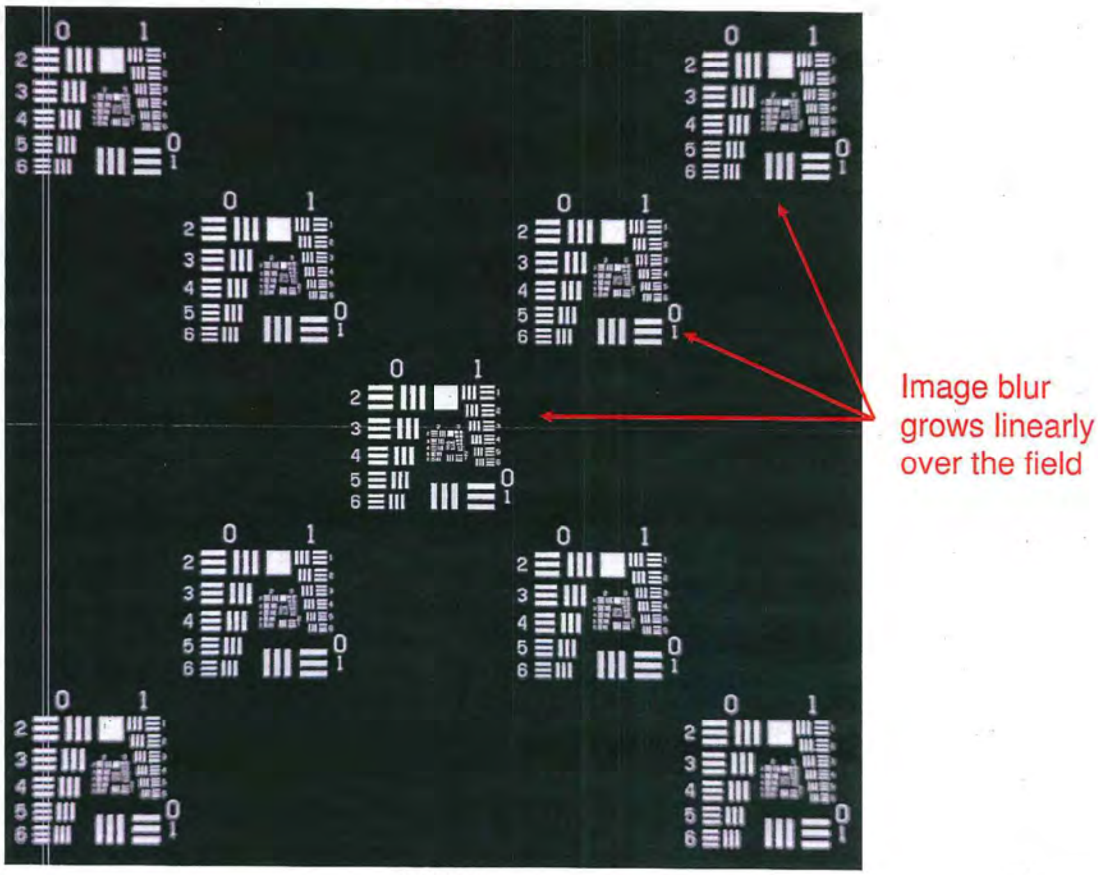


Image blur is constant over the field

Coma



Astigmatism

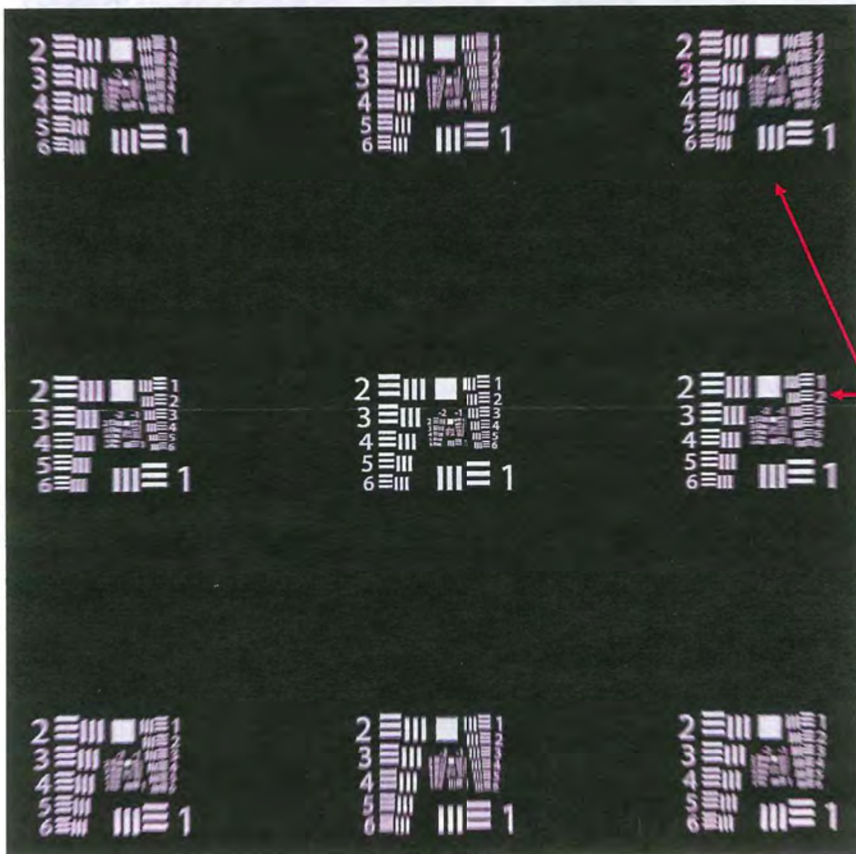
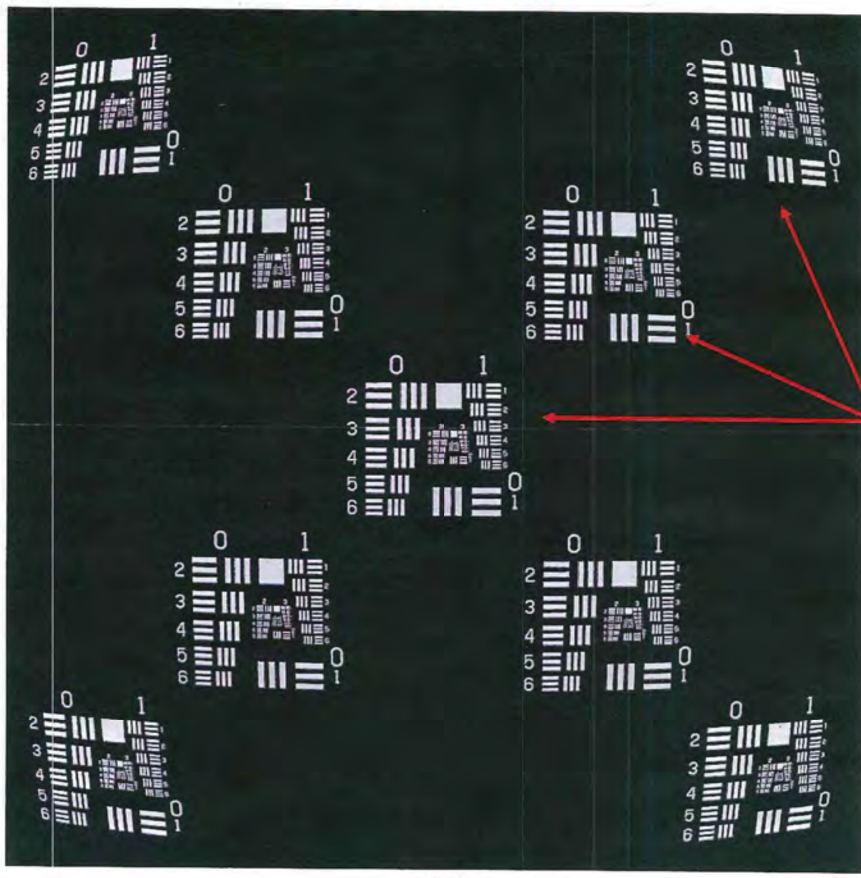


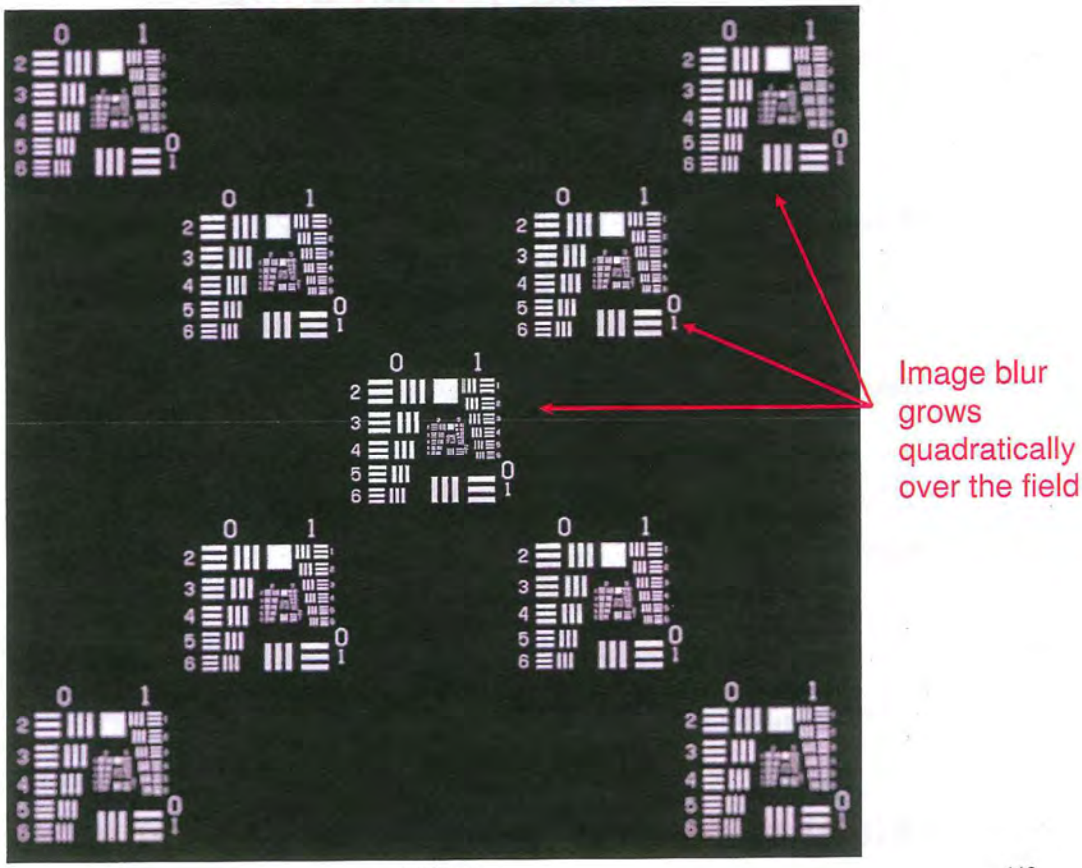
Image blurs more in one direction over the field

Distortion



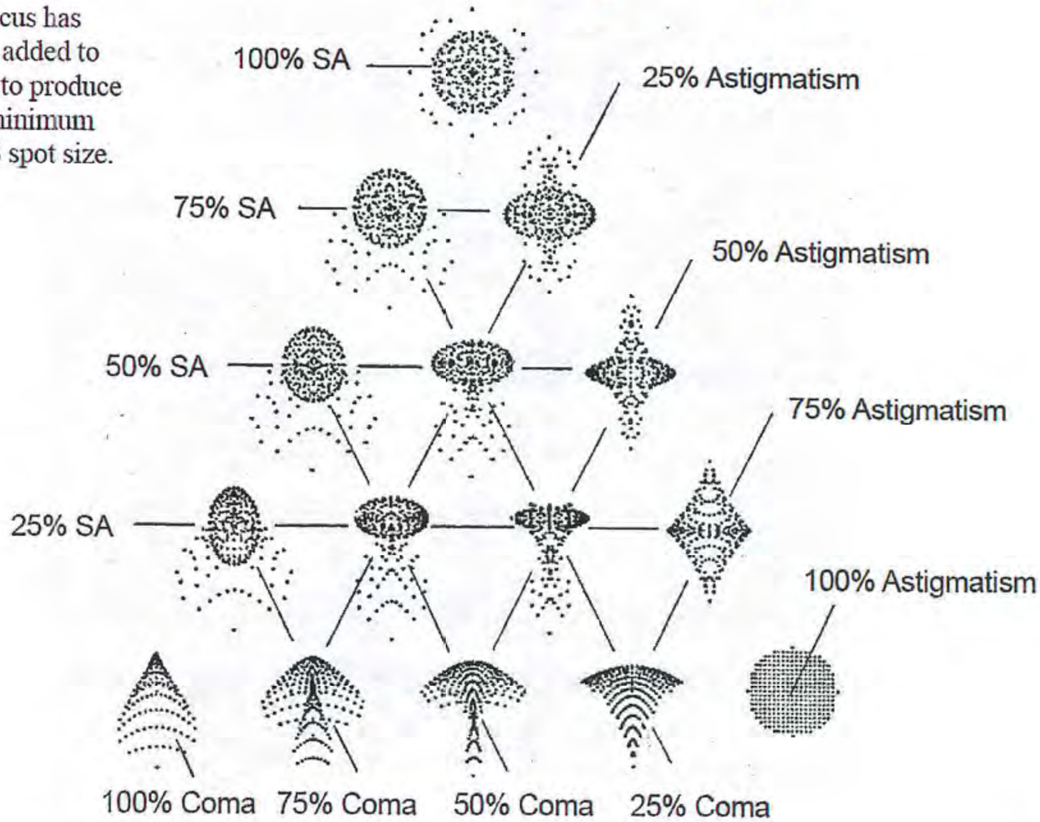
No image degradation but image locations are shifted

Curvature of Field



Combined Aberrations – Spot Diagrams

Defocus has been added to each to produce the minimum RMS spot size.

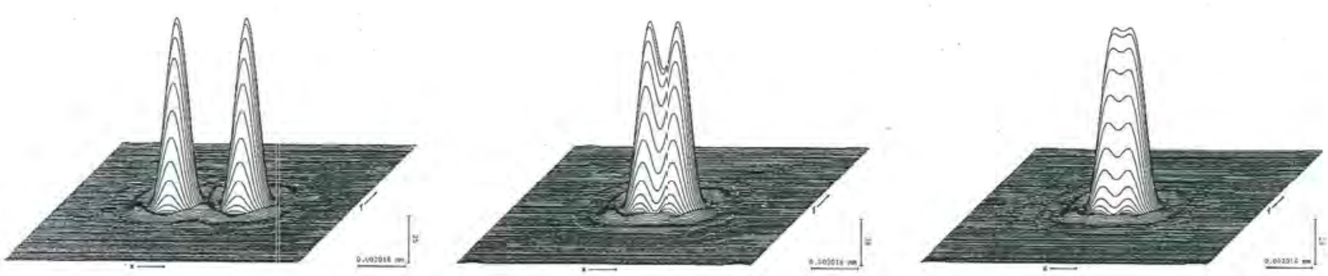


Balancing of Aberrations

- Different aberrations can be combined to improve the overall image quality
  - Spherical aberration and defocus
  - Astigmatism and field curvature
  - Third-order and fifth-order spherical aberration
  - Longitudinal color and spherochromatism
  - Etc.
- Lens design is the art (or science) of putting together a system so that the resulting image quality is acceptable over the field of view and range of wavelengths

Resolution

- Resolution is an important aspect of image quality
- Every image has some resolution associated with it, even if it is the Airy disk
  - In this case, the resolution is dependent on the aberrations of the system
- Resolution is the smallest detail you can resolve in the image
  - It determines whether you can resolve two closely spaced objects



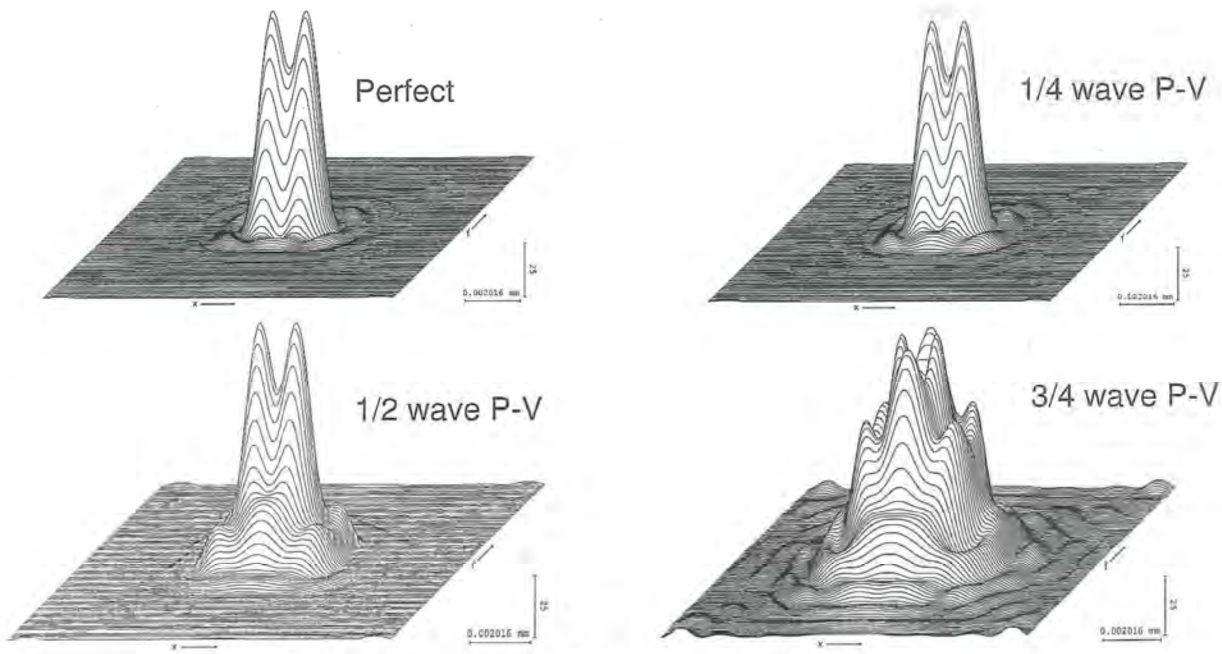
Well resolved

Rayleigh criterion peak of 2nd at 1st zero of first

Sparrow criterion overlap at FWHM

Resolution vs. P-V Wavefront Error

- The 1/4 wave rule was empirically developed by astronomers as the greatest amount of P-V wavefront error that a telescope could have and still resolve two stars separated by the Rayleigh spacing (peak of one at 1st zero of the other)



Resolution Examples

- Angular resolution is given by $\beta \approx 2.44\ \lambda/D$
  - Limited only by the diameter, not by the focal length or f/number

- U of A is building 8.4 meter diameter primary mirrors for astronomical telescopes
- For visible light (~0.5 μm), the Rayleigh spacing corresponds to an angular separation of $(2.44 * 0.5\times10^{-6} / 8.4)/2 = 0.073\times10^{-6}$ radian (~0.015 arc second)
- Assume a binary star at a distance of 200 light years (~1.2×10^{15} miles)
  - This would have a resolution of 90 million miles
  - Perhaps enough resolution to "split the binary"

- A typical cell phone camera has an aperture of about 0.070 inch
  - This gives a Rayleigh spacing of about 0.34 mrad (for reference, the human eye has an angular resolution of about 0.3 mrad)
  - For an object 10 feet away, this is an object resolution of about 1 mm

Film Resolution

- Due to the grain size of film, there is an MTF associated with films

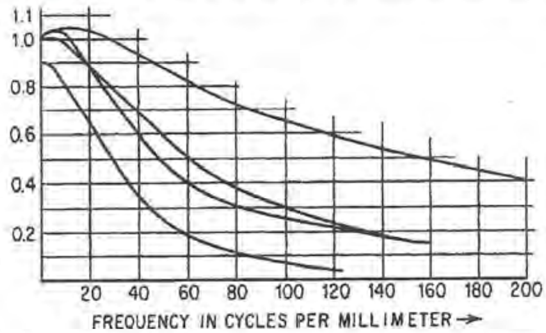
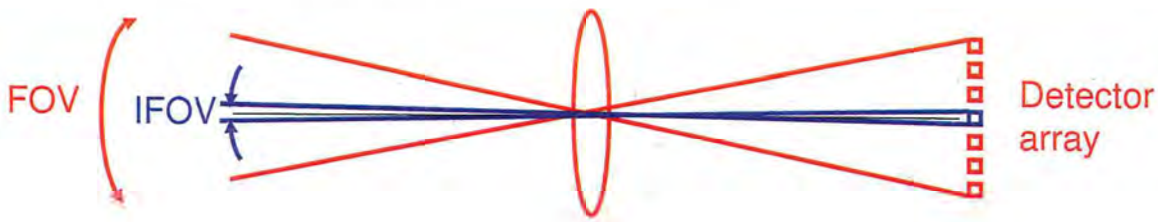


Figure 11.22 Modulation transfer functions of several photographic emulsions.

- A reasonable guide for MTF of a camera lens is the 30-50 rule: 50% at 30 lp/mm and 30% at 50 lp/mm
- For excellent performance of a camera lens, use 50% at 50 lp/mm
- Another criterion for 35 mm camera lenses is 20% at 30 lp/mm over 90% of the field (at full aperture)
- As a rough guide for the resolution required in a negative, use 200 lines divided by the square root of the long dimension in mm

Detectors

- All optical systems have some sort of detector
  - The most common is the human eye
  - Many optical systems use a 2D detector array (e.g., CCD)
- No matter what the detector is, there is always some small element of the detector which defines the detector resolution
  - This is referred to as a picture element (pixel)
- The size of the pixel divided by the focal length is called the Instantaneous FOV (IFOV)
  - The IFOV defines the angular limit of resolution in object space
  - IFOV is <u>always</u> expressed as a full angle



Implications of IFOV

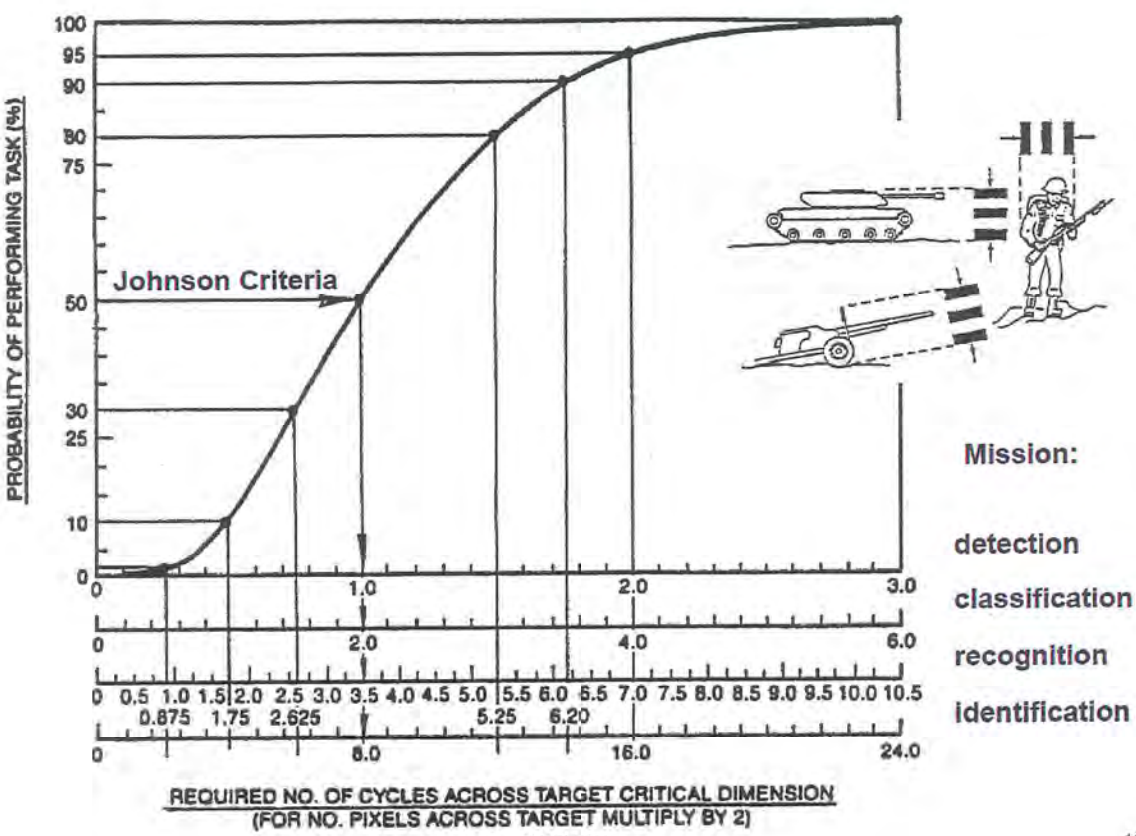
- If the target angular size is smaller than an IFOV, it is not resolved
  - It is essentially a point target
  - Example is a star
- If the target annular size is larger than an IFOV it may be resolved
  - This does not mean that you can always tell what the object is

Practical Resolution Considerations

- Resolution required to photograph written or printed copy:
  - Excellent reproduction (serifs, etc.) requires 8 line pairs per lower case e
  - Legible reproduction requires 5 line pairs per letter height
  - Decipherable (e, c, o partially closed) requires 3 line pairs per height
- The correlation between resolution in cycles/minimum dimension and certain functions (often referred to as the Johnson Criteria) is

– Detect	1.0 line pairs per dimension
– Orient	1.4 line pairs per dimension
– Aim	2.5 line pairs per dimension
– Recognize	4.0 line pairs per dimension
– Identify	6-8 line pairs per dimension
– Recognize with 50% accuracy	7.5 line pairs per height
– Recognize with 90% accuracy	12 line pairs per height

Johnson Resolution Criteria



OPTI 517

Examples of the Johnson Criteria

Detect
1 bar pair

Maybe something of military interest

Recognize
4 bar pairs

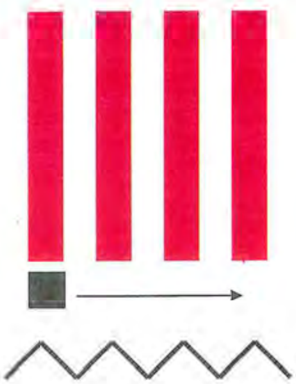
Tank

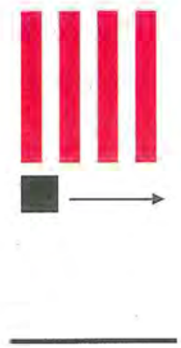
Identify
7 bar pairs

Abrams Tank

MTF of a Pixel

- Consider a fixed size pixel scanning across different sized bar targets





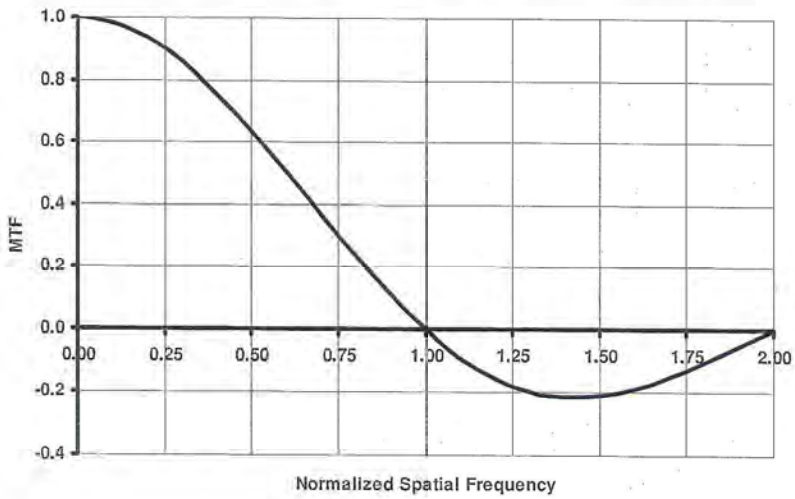
When the pixel size equals the width of a bar pair (light and dark) there is no more modulation

MTF of a Pixel

- If the pixel is of linear width Δ, the MTF of the pixel is given by

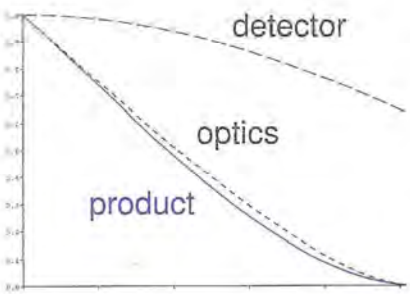
$$MTF(f) = \frac{\sin(\pi f \Delta)}{\pi f \Delta}$$

- The cutoff frequency (where the MTF goes to zero) is at a spatial frequency $1/\Delta$



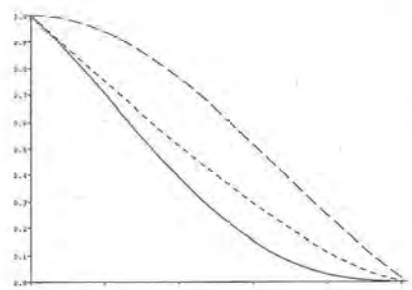
Optical MTF and Pixel MTF

- The total MTF is the product of the optical MTF and the pixel MTF



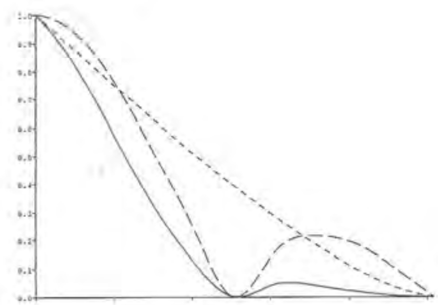
Case 1 - Optics limited

Best for high resolution over-sampling



Case 2 - Optics and detector are matched

Best for most FLIR-like mapping systems

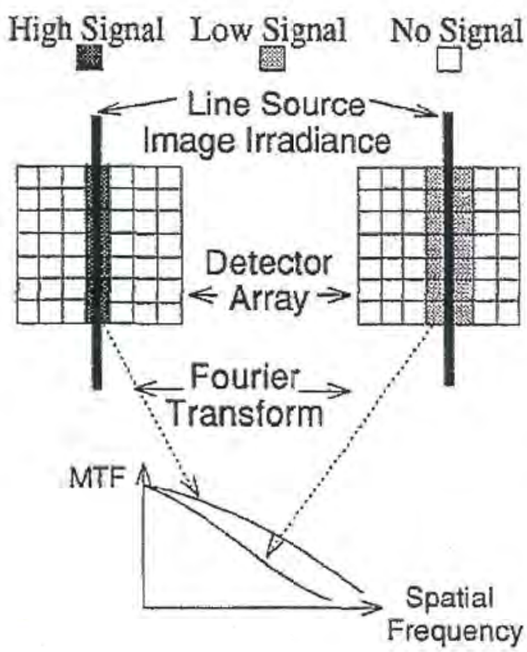


Case 3 - Detector limited

Best for detecting dim point targets

- Of course, there are other MTF contributors to total system MTF
  - Electronics, display, jitter, smear, eye, turbulence, etc.

Effects of Signal/CCD Alignment on MTF



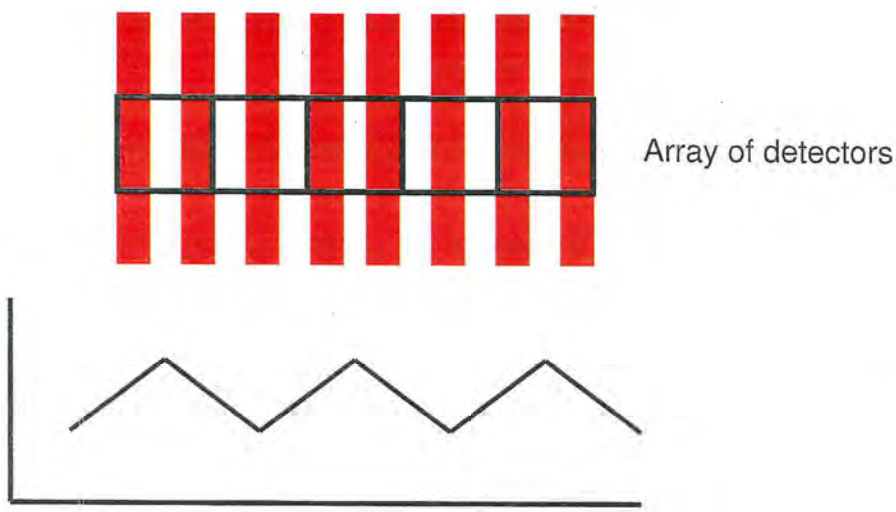
A sampled imaging system is not shift-invariant

MTF of Alignment

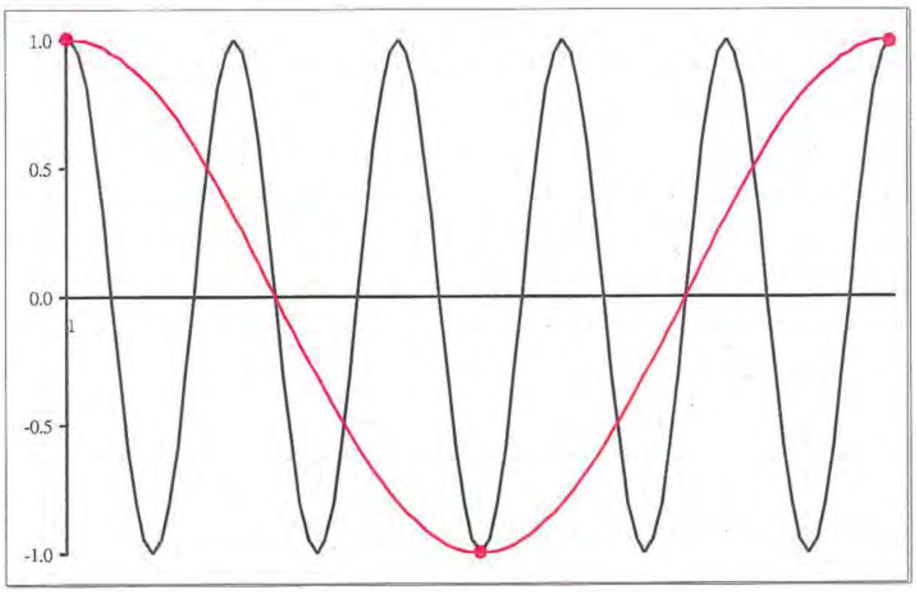
- When performing MTF testing, the user can align the image with respect to the imager to produce the best image
  - In this case, a sampling MTF might not apply
- A natural scene, however, has no net alignment with respect to the sampling sites
- To account for the average alignment of unaligned objects a sampling MTF must be added
  - $MTF_{sampling} = \sin(\pi f \Delta x)/(\pi f \Delta x)$ where Δx is the sampling interval
  - This MTF an ensemble average of individual alignments and hence is statistical in nature

Aliasing

- Aliasing is a very common effect but is not well understood by most people
- Aliasing is an image artifact that occurs when we insufficiently sample a waveform
  - It is evidenced as the imaging of high frequency objects as low frequency objects



Sampling of a Sine Wave

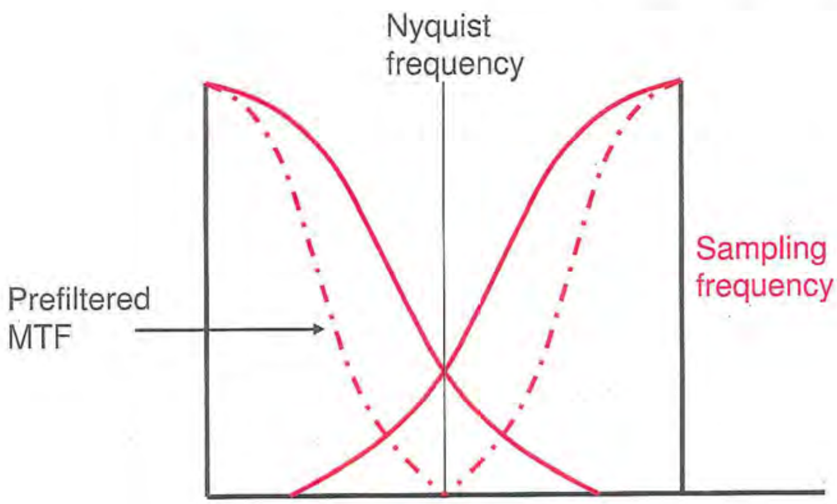


Nyquist Condition

- If we choose a sampling interval sufficiently fine to locate the peaks and valleys of a sine wave, then we can reconstruct that frequency from its sample values
- The Nyquist condition says we need at least two samples per cycle to reproduce a sine wave
  - For a sine wave period ξ, we need a sampling interval $\Delta x < \xi/2$

MTF Fold Over

- The effect of sampling is to replicate the MTF back from the sampling frequency
  - This will cause higher frequencies to appear as lower frequencies



- The solution to this is to prefilter the MTF so it goes to zero at the Nyquist frequency
  - This is often done by blurring

Conclusions

- Image quality is essentially a measure of how good an optical system is
- Different image quality metrics are needed for different systems
- The better the needed image quality, the more complex the optical system will be (and the harder it will be to design)
- The measures of image quality used by the optical designer during the design process are not necessarily the same as the final performance metrics
  - It's up to the optical designer to convert the needed system performance into image quality metrics as needed