EXHIBIT 1009:

WILLIAM LICHTEN, DATA AND ERROR ANALYSIS IN THE

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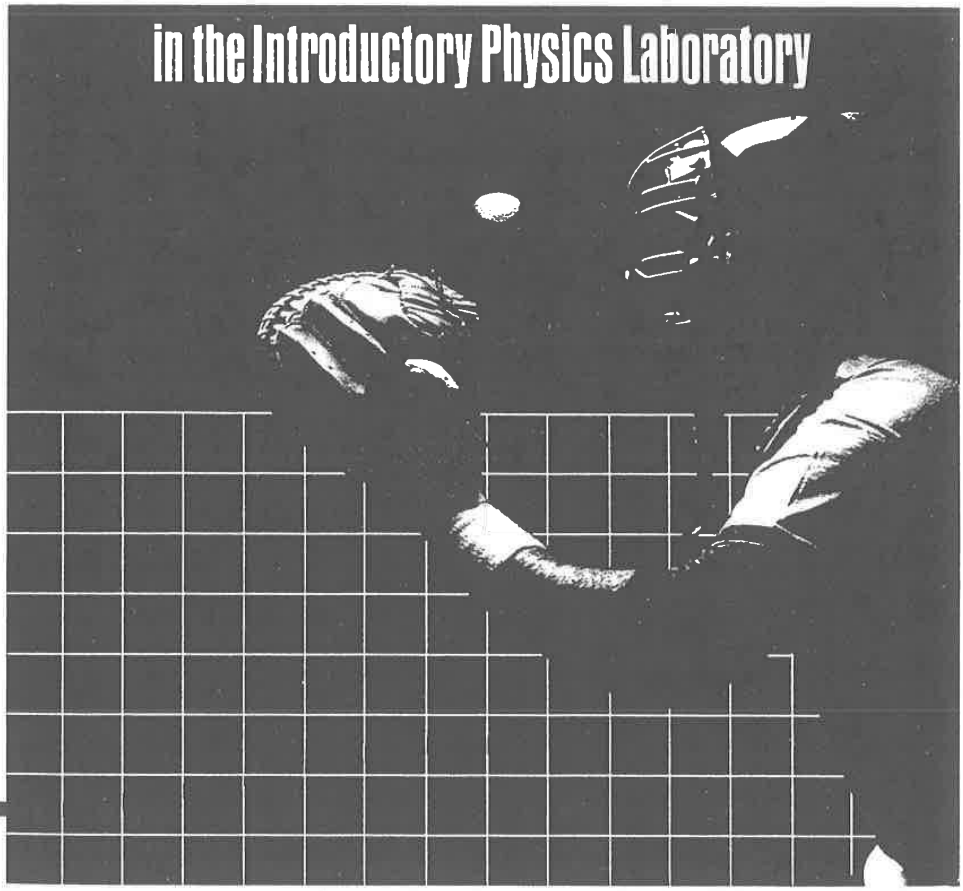
Data and Error Analysis

in the Introductory Physics Laboratory



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Data and Error Analysis in the Introductory Physics Laboratory

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Precision Versus Accuracy: Random and Systematic Errors

Let's go back to the example of the pencil. Suppose everyone in the class uses the same ruler, measures the pencil to the nearest millimeter, and all agree it is 192 mm long. All say that it couldn't be either 191 or 193 mm long. We say that the class has measured the length of the ruler to a *precision* of 1 mm. *Precision* is the reliability or repeatability of a measurement.

Suppose that the instructor now points out, "You all have made the same mistake. You lined up one end of the pencil and one end of the ruler together. The end of the ruler is worn badly; it doesn't begin at zero. Try to remeasure the pencil by putting it in the middle of the ruler. Then find the position of both ends." (see Table 1-1.) "Subtract one value from the other to find the length." Now the class finds that the pencil is 187 mm long! How can this be?

Both measurements are equally precise. The second one is more *accurate* than the first, because a *systematic* error (caused by the worn end of the ruler) is no longer there. A *systematic error* is an effect that changes all measurements by the same amount or by the same percentage.

The class's experience with the ruler is a mirror of the history of science. Systematic errors have often hidden unsuspected in measurements. The only way to eliminate systematic errors is to look carefully for them and to understand well the nature of the experiment or measurement.

Random Errors. Can We Avoid Them? Let's return to the example of the class measurement of the length of a pencil; when measuring to the nearest millimeter, everyone got the same value. Let's try to push the precision further and ask each person to measure to the nearest *tenth* of a millimeter. Now disagreements appear. We find different values: 186.7, 187.0, 187.3 mm, as shown in Table 1-1.

Is someone making a mistake? No, even the most careful and skillful person will come up with values that vary by one- or two-tenths of a millimeter. Now we are at the limit of measurement by use of the naked eye and rulers. The unavoidable change in successive measurements, due to small irregularities in the ruler, difficulty in estimating precisely, and the like, is called a *random error,* or *error* for short.

Your Best Estimate, The Arithmetic Mean. What Is Your Error? At this point, you've been careful not to make any mistakes, you've avoided all systematic errors, and you've narrowed your uncertainty to the random error of measurement. What's next?

Common sense tells you to take the average of several measurements, called the *arithmetic mean* or *mean.* The algebraic expression for the average $\overline{X}$ of N numbers is

$$\overline{X} = \frac{\text{sum}}{N} = \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{\sum x}{N} \tag{1-1}$$

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