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A geographical model for the daily and weekly seasonal volatility in the foreign exchange market

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The daily and weekly seasonality of foreign exchange volatility is modeled by introducing an activity variable. This activity is explained by a simple model of the changing and sometimes overlapping market presence of geographical components (East Asia, Europe, and America).

Integrating this activity over time results in the new ϑ time scale, characterized by non-seasonal volatility. This scale, applied to dense datastreams of absolute price changes, succeeds in removing most of the seasonal heteroscedasticity in an autocorrelation study. Unexpectedly, the positive autocorrelation is found to decline hyperbolically rather than exponentially as a function of the lag.

The foreign exchange (FX) market is the largest financial market, encompassing billions of dollars traded daily by thousands of actors, the microanalysis of which is complicated by the decentralizing (and destandardizing) effects of far-flung trading centers and disparate time zones. FX prices are characterized by daily and weekly seasonal heteroscedasticity, which we showed in a recent paper (Müller *et al.*, 1990). In this paper we follow with a geographical model that not only resolves seasonality through an activity-based time scale that we call ϑ-*scale*, but also relates activity to physical location. The seasonality is strikingly well explained by the changing presence and the partially overlapping business hours of the main markets in the world. Deliberately, we do not try to model the complex intra-day behavior with a *full* model of the generation process. Our ambition is to analyze, explain, and model the seasonal heteroscedasticity.

Seasonal heteroscedasticity affects the results of most statistical studies so it must be treated as a first priority. Many researchers who study daily time series implicitly use, as a solution, a business time scale that differs from the physical scale in its omission of Saturdays, Sundays, and holidays. With the ϑ-scale we

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extend this established concept to the intra-day domain, thereby allowing us to tackle a fundamental source of seasonality, the revolution of the earth.

There are, therefore, three main motivations for our model:

- to provide a tool for the analysis of market prices by extending the concept of business time scale to intra-day prices;
- to make a first step towards formulating a model for the generating process of market prices that also covers the intra-day movements;
- to gain an insight into the interactions of the main market centers around the world and their relevance to each particular FX rate.

The importance of intra-day prices lies both in the large number of independent observations, which enhances the significance of a statistical study, and in the increased ability to analyze finer details of the behavior of different market participants. The few authors who have reported intra-day analysis (Feinstone, 1987; Goodhart and Figliuoli, 1988, 1991; Ito and Roley, 1987; Wasserfallen and Zimmermann, 1985; Wasserfallen, 1989) limited themselves to certain periods of the day, generally the most active ones for a particular market center, so the problem of daily and weekly seasonality was avoided. The paper by Goodhart (1990) is an exception; its subject is the interaction of different market centers around the world with different opening hours. Moreover, the type of data he analyzes is very similar to the one used in this study but for a shorter period. Another paper by Baillie and Bollerslev (1989) treats the same data as Goodhart and reports a behavior of the intra-day market volatility similar to that in Müller *et al.* (1990).

The nature of the intra-day data analyzed in this paper is presented in Section I. High density data require automatic filtering because of the enormous number of prices to be treated. Our approach to it is described in the Appendix at the end of the paper.

In Section II, the seasonal heteroscedasticity is attributed to the active presence of traders on the FX markets, followed by the introduction of a new time scale, the ϑ-scale. In this time scale, price changes have a non-seasonal volatility. We call its derivative against physical time the *activity*. This new variable measures, for each time t, the active presence of traders on the FX market through the price changes they induce. The activity of the worldwide FX market is then decomposed into three submarkets geographically centered in East Asia, Europe, and North America. We show how the model parameters can be computed from the data and discuss some interesting results for particular FX rates. The definition of the market activity is discussed and compared with alternative definitions.[1]

The activity model is used in Section III to construct the ϑ-scale as the time integral of worldwide activity. This scale, when used for the analysis of time series with high-frequency data, yields results that are no longer overshadowed by seasonality. Adopting the base requirement of zero seasonality, a statistical measure of the ϑ-scale quality is also proposed. Some inevitable complications of the activity model and the ϑ-scale construction, such as business holidays and daylight saving time, are discussed.

To illustrate the usefulness of this approach, a comparative study of auto-correlation for absolute price changes in twenty minute intervals is presented in Section IV. We show that the autocorrelation peaks indicating seasonality vanish

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the success of this new time scale. The remaining autocorrelation reflects the conditional heteroscedasticity which can be studied with enhanced accuracy thanks to the high data density. We show that the behavior of the autocorrelation function on the ϑ-scale is more accurately described by a hyperbolic rather than a conventionally expected exponential decline.

Our conclusions are given in Section V.

I. Data

I.A. The FX market and the data sources

The bid and ask offers of major financial institutions are conveyed to customers' screens by large data suppliers such as Reuters, Telerate, or Knight Ridder and the deals are negotiated over the telephone. The FX market has no business hours limitations. Any market maker can submit new bid/ask prices; many larger institutions have branches worldwide so that trading is continuous. Nevertheless, the bid/ask prices do emanate from particular banks in particular locations and the deals are entered into dealers' books in particular institutions.

In this study, mainly Reuters data is analyzed. It is the same as that described in Müller *et al.* (1990) so our description here is limited to certain aspects that were not mentioned in the earlier paper. Data collected from Knight Ridder is also used to study the dependence of the results on the data supplier. The interbank spot exchange rates are published by Reuters in multiple contributor pages (FXFX, FXFY, and ASAQ). These three Reuters pages contain spot prices of 26 time series, including 24 FX rates against the US Dollar (USD) and two commodities, gold (XAU) and silver (XAG).[2]

In Table 1, the list of the 12 most frequently updated time series collected on the FXFX, FXFY, and ASAQ pages of Reuters are shown together with their tick frequency statistics. Besides the normal business day tick frequencies those for Saturday are also included, because they are so much lower than the average. Sunday frequencies are not so low, as late Sunday evening (GMT) is already Monday morning in East Asia. The number of ticks counted on a Reuters page is related to the market shares of the particular FX rates, but it depends on the actual market coverage of the Reuters information system. The four or five *major* FX rates are the most frequently updated.

Our database now contains more than *12 million ticks* for the 26 time series. It covers almost every day of the year except for rare failures of our system or of the data supplier which lead to *data holes*, over which we use linear interpolation, as in Müller *et al.* (1990). Data *filtering* is necessary; our approach is described in detail in the Appendix.

The results presented here are computed over a sample of four full years, starting March 3, 1986 and ending March 3, 1990. The main variable studied is the *logarithmic middle price* x_j, as in Müller *et al.* (1990),

$$\langle 1 \rangle \qquad x_j \equiv \frac{\log p_{\text{bid},j} + \log p_{\text{ask},j}}{2}$$

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TABLE 1. Characterization of the 12 most updated time series collected on the FXFX, FXFY, and ASAQ pages from Reuters.

Exchanged rate	Expressed in		Mean business day frequency	Mean Saturday frequency	First tick recorde
US Dollar	Deutsche Mark	USD-DEM	3100	49	01.02.8
US Dollar	Japanese Yen	USD-JPY	1900	42	31.01.8
US Dollar	Swiss Franc	USD-CHF	1550	45	01.02.8
British Pound	US Dollar	GBP-USD	1500	49	01.02.8
US Dollar	French Franc	USD-FRF	1000	38	31.08.8
Australian Dollar	US Dollar	AUD-USD	900	37	27.11.8
Gold	US Dollar	XAU-USD	800	70	01.02.8
US Dollar	Netherlands Guilder	USD-NLG	600	30	27.11.8
US Dollar	Italian Lira	USD-ITL	550	39	27.11.8
US Dollar	Canadian Dollar	USD-CAD	480	0	27.11.8
European Curr. Unit	US Dollar	ECU-USD	460	0	10.01.8
US Dollar	Spanish Peseta	USD-ESP	350	0	27.11.8

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I.B. Multiple contributor effect on very short-term price variations

The database prices used for our analysis are *quoted* prices and *not actual* trading prices.[3] FX traders we have interviewed find that the FXFX prices lag behind the real market prices and the quoted spreads are about twice as large as those actually used in the deals. According to their estimates, the lag of FXFX prices varies between a few seconds in periods of low volatility and one minute or more in highly volatile periods.

Goodhart and Figliuoli (1991) have reported a negative first order autocorrelation of price changes with a lag of one minute. We attribute these short-term oscillations (with amplitudes of about half the spread) to a *multiple contributor effect*: many market makers have current preferences for either selling or buying and publish 'new prices' attracting traders to make a deal in the desired direction. The sequence of quoted prices originates from market makers with *different* and *changing* preferences. A proper treatment of this problem would deserve a special study. We have not tried to make a model for the actual *traded* prices. In order to avoid the multiple contributor effect in our analysis, we do not use samples with resolutions finer than 10 minutes.

II. A model for the FX market activity

II.A. Seasonal patterns of the volatility and presence of markets

The behavior of a time series is called *seasonal* if it shows a periodic structure in addition to less regular movements. In Müller *et al.* (1990) we demonstrated daily and weekly *seasonal heteroscedasticity*, a seasonal behavior of FX price *volatility* rather than of FX prices themselves. The seasonality has been found in a study with intra-daily and intra-weekly sampling as well as in an autocorrelation analysis. Autocorrelation coefficients are significantly higher for time lags that are integer multiples of the seasonal period than for other lags. An extended autocorrelation study is included in this paper, in Section IV.

The intra-week analysis in Müller *et al.* (1990) shows that mean absolute price changes are much higher over working days than over Saturdays and Sundays, when the market actors are hardly present. The intra-day analysis in the same paper shows that the mean absolute hourly price changes have distinct seasonal patterns. These patterns are clearly correlated to the changing *presence* of main market places of the worldwide FX market. The lowest market presence outside the weekend happens during the lunch hour in Japan (noon break in Japan, night in America and Europe); it is at this time when the minimum of mean absolute hourly price changes is found.

Further evidence of a strong correlation between market presence and volatility is provided by another result of Müller *et al.* (1990): the intra-day behavior of the frequency of price quotes in the Reuters system. This variable obviously reflects the presence of markets and is positively correlated to volatility (in terms of mean absolute hourly price changes). Market presence is related to another variable which cannot be observed directly: the worldwide transaction volume. Many empirical studies give substantial evidence in favor of a positive correlation between price changes and volume in financial markets (see the survey of Karpoff, 1987).

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