# An Overview of Insurance Uses of Fuzzy Logic

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It has been twenty-five years since DeWit(1982) first applied fuzzy logic (FL) to insurance. That article sought to quantify the fuzziness in underwriting. Since then, the universe of discourse has expanded considerably and now also includes FL applications involving classification, projected liabilities, future and present values, pricing, asset allocations and cash flows, and investments. This article presents an overview of these studies. The two specific purposes of the article are to document the FL technologies have been employed in insurance-related areas and to review the FL applications so as to document the unique characteristics of insurance as an application area.

**Key words:** Actuarial, Fuzzy Logic, Fuzzy Sets, Fuzzy Arithmetic, Fuzzy Inference Systems, Fuzzy Clustering, Insurance

## 1 Introduction

The first article to use fuzzy logic (FL) in insurance was [29] [1], which sought to quantify the fuzziness in underwriting. Since then, the universe of discourse has expanded considerably and now includes FL applications involving classification, underwriting, projected liabilities, future and present values, pricing, asset allocations and cash flows, and investments.

This article presents an overview of these FL applications in insurance. The specific purposes of the article are twofold: first, to document the FL technologies have been employed in insurance-related areas; and, second, to review the FL applications so as to document the unique characteristics of insurance as an application area.

[1] While DeWit was the first to write an article that gave an explicit example of the use of FL in insurance, FL, as it related to insurance, was a topic of discussion at the time. Reference [43], for example, remarked that "... not all expert knowledge is a set of "black and white" logic facts - much expert knowledge is codifiable only as alternatives, possibles, guesses and opinions (i.e., as fuzzy heuristics)."

Before continuing, the term FL needs to be clarified. In this article, we generally follow the lead of Zadeh and use the term FL in its wide sense. According to [85],

> Fuzzy logic (FL), in its wide sense, has four principal facets. First, the logical facet, FL/L, [fuzzy logic in its narrow sense], is a logical system which underlies approximate reasoning and inference from imprecisely defined premises. Second, the set-theoretic facet, FL/S, is focused on the theory of sets which have unsharp boundaries, rather than on issues which relate to logical inference, [examples of which are fuzzy sets and fuzzy mathematics]. Third is the relational facet, FL/R, which is concerned in the main with representation and analysis of imprecise dependencies. Of central importance in FL/R are the concepts of a linguistic variable and the calculus of fuzzy if-then rules. Most of the applications of fuzzy logic in control and systems analysis relate to this facet of fuzzy logic. Fourth is the epistemic facet of fuzzy logic, FL/E, which is focused on knowledge, meaning and imprecise information. Possibility theory is a part of this facet.

The methodologies of the studies reviewed in this article cover all of these FL facets. The term "fuzzy systems" also is used to denote these concepts, as indicated by some of the titles in the reference section of this paper, and will be used interchangeably with the term FL.

The next section of this article contains a brief overview of insurance application areas. Thereafter, the article is subdivided by the fuzzy techniques[2] shown in Fig. 1.

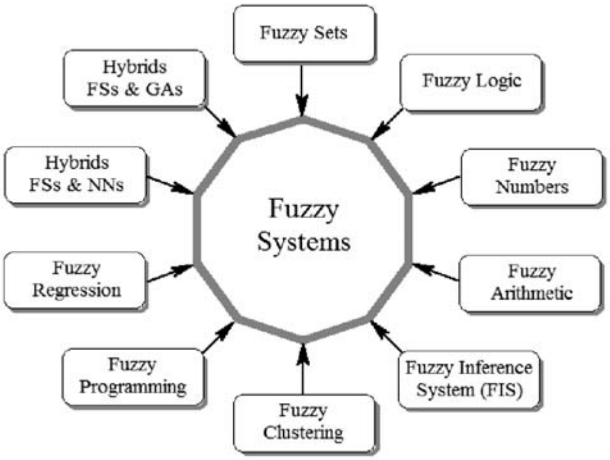


**Fig. 1.** Fuzzy Logic

[2] This article could have been structured by fuzzy technique, as was done by [75] or by insurance topic, as was done by [28] and [66]. Given the anticipated audience, the former structure was adopted.

As indicated, the topics covered include fuzzy set theory, fuzzy numbers, fuzzy arithmetic, fuzzy inference systems, fuzzy clustering, fuzzy programming, fuzzy regression, and soft computing. Each section begins with a brief description of the technique[3] and is followed by a chronological review of the insurance applications of that technique. When an application involves more than one technique, it is only discussed in one section. The article ends with a comment regarding future insurance applications of FL.

## 2 Insurance Application Areas

The major application areas of insurance include classification, underwriting, projected liabilities, ratemaking and pricing, and asset allocations and investments. In this section, we briefly describe each of these areas so that readers who are unfamiliar with the insurance field will have a context for the rest of the paper.

### 2.1 Classification

Classification is fundamental to insurance. On the one hand, classification is the prelude to the underwriting of potential coverage, while on the other hand, risks need to be properly classified and segregated for pricing purposes. Operationally, risk may be viewed from the perspective of the four classes of assets (physical, financial, human, intangible) and their size, type, and location.

### 2.2 Underwriting

Underwriting is the process of selection through which an insurer determines which of the risks offered to it should be accepted, and the conditions and amounts of the accepted risks. The goal of underwriting is to obtain a safe, yet profitable, distribution of risks. Operationally, underwriting determines the risk associated with an applicant and either assigns the appropriate rating class for an insurance policy or declines to offer a policy.

### 2.3 Projected Liabilities

In the context of this article, projected liabilities are future financial obligations that arise either because of a claim against and insurance company or a contractual benefit agreement between employers and their employees. The evaluation of projected liabilities is fundamental to the insurance and employee benefit industry, so it is not surprising that we are beginning to see SC technologies applied in this area.

[3] Only a cursory review of the FL methodologies is discussed in this paper. Readers who prefer a more extensive introduction to the topic, with an insurance perspective, are referred to [56]. Those who are interested in a comprehensive introduction to the topic are referred to [90] and [32]. Readers interested in a grand tour of the first 30 years of fuzzy logic are urged to read the collection of Zadeh's papers contained in [74] and [45].

### 2.4 Ratemaking and Pricing

Ratemaking and pricing refer to the process of establishing rates used in insurance or other risk transfer mechanisms. This process involves a number of considerations including marketing goals, competition and legal restrictions to the extent they affect the estimation of future costs associated with the transfer of risk. Such future costs include claims, claim settlement expenses, operational and administrative expenses, and the cost of capital.

### 2.5 Asset Allocation and Investments

The analysis of assets and investments is a major component in the management of an insurance enterprise. Of course, this is true of any financial intermediary, and many of the functions performed are uniform across financial companies. Thus, insurers are involved with market and individual stock price forecasting, the forecasting of currency futures, credit decision-making, forecasting direction and magnitude of changes in stock indexes, and so on.

## 3 Linguistic Variables and Fuzzy Set Theory

Linguistic variables are the building blocks of FL. They may be defined ([82], [83]) as variables whose values are expressed as words or sentences. Risk capacity, for example, a common concern in insurance, may be viewed both as a numerical value ranging over the interval [0,100%], and a linguistic variable that can take on values like high, not very high, and so on. Each of these linguistic values may be interpreted as a label of a fuzzy subset of the universe of discourse X = [0,100%], whose base variable, x, is the generic numerical value risk capacity. Such a set, an example of which is shown in Fig. 2, is characterized by a membership function (MF), $\mu_{high}(x)$ here, which assigns to each object a grade of membership ranging between zero and one.

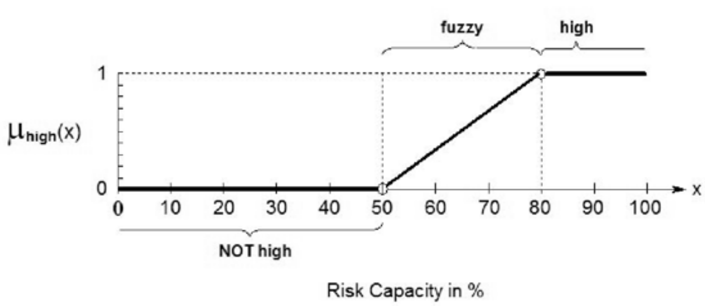


**Fig. 2.** (Fuzzy) Set of Clients with High Risk Capacity

In this case, which represents the set of clients with a high risk capacity, individuals with a risk capacity of 50 percent, or less, are assigned a membership grade of zero and those with a risk capacity of 80 percent, or more, are assigned a grade of one. Between those risk capacities, (50%, 80%), the grade of membership is fuzzy.

In addition to the S-shaped MF depicted in Fig. 2, insurance applications also employ the triangular, trapezoidal, Gaussian, and generalized bell classes of MFs. As with other areas of application, fuzzy sets are implemented by extending many of the basic identities that hold for ordinary sets.

### 3.1 Applications

This subsection presents an overview of some insurance applications of linguistic variables and fuzzy set theory. The topics addressed include: earthquake insurance, optimal excess of loss retention in a reinsurance program, the selection of a "good" forecast, where goodness is defined using multiple criteria that may be vague or fuzzy, resolve statistical problems involving sparse, high dimensional data with categorical responses, the definition and measurement of risk from the perspective of a risk manager, and deriving an overall disability Index.

An early study was by [7], who used pattern recognition and FL in the evaluation of seismic intensity and damage forecasting, and for the development of models to estimate earthquake insurance premium rates and insurance strategies. The influences on the performance of structures include quantifiable factors, which can be captured by probability models, and nonquantifiable factors, such as construction quality and architectural details, which are best formulated using fuzzy set models. For example, he defined the percentage of a building damaged by an earthquake by fuzzy terms such as medium, severe and total, and represented the membership functions of these terms as shown in Fig. 3.[4]

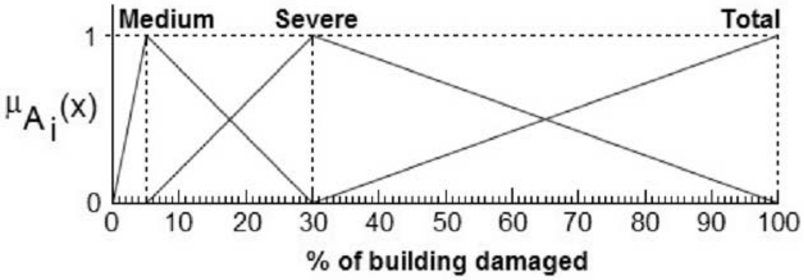


**Fig. 3.** MFs of Building Damage

Two methods of identifying earthquake intensity were presented and compared. The first method was based on the theory of pattern recognition where a discrimina-

[4] Adapted from [7, Figure 6.3].

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